This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Stanford University Libraries
3 6105 000 557 137

S STANFORD
ORD UNIVERSITY
SITY LIBRARIES
UNIVERSITY LIBRARIES STANFORD
LIBRARIES STANFORD UNIVERSITY
STANFORD UNIVERSITY LIBRARIES
ES STANFORD STANFORD
ORD UNIVERSITY LIBRARIES STANFORD UNIVERSITY
SITY LIBRARIES STANFORD UNIVERSITY LIBRARIES
UNIVERSITY LIBRARIES STANFORD UNIVERSITY LIBRARIES STANFORD
LIBRARIES STANFORD UNIVERSITY LIBRARIES STANFORD UNIVERSITY
STANFORD UNIVERSITY LIBRARIES STANFORD UNIVERSITY LIBRARIES
ES STANFORD UNIVERSITY LIBRARIES STANFORD UNIVERSITY LIBRARIES
ORD UNIVERSITY LIBRARIES STANFORD UNIVERSITY LIBRARIES
SITY LIBRARIES STANFORD UNIVERSITY LIBRARIES STANFORD
UNIVERSITY LIBRARIES STANFORD UNIVERSITY LIBRARIES STAN

PHILOSOPHICAL
TRANSACTIONS
OF THE
ROYAL SOCIETY
OF
LONDON.

FOR THE YEAR MDCCCLXXX.

VOL. 171.—PART I.

LONDON:
PRINTED BY HARRISON AND SONS, ST. MARTIN'S LANE, W.C.,
Printers in Ordinary to Her Majesty.
MDCCCLXXX.

ADVERTISEMENT.

The Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions* take this opportunity to acquaint the public that it fully appears, as well from the Council-books and Journals of the Society as from repeated declarations which have been made in several former *Transactions,* that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the public that their usual meetings were then continued, for the improvement of knowledge and benefit of mankind: the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions;* which was accordingly done upon the 26th of March, 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body,

64426

upon a subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

f Public Institutions and Individuals entitled to receive a Copy of the *Philosophical Transactions* ˚ each year, on making application for the same directly or through their respective agents, within ve years of the date of publication.

Observatories.

Kew.
od Hope. Liverpool.
Madras.
. Oxford (Radcliffe).
.

Institutions.

........Library and Museum.
m ......Free Central Library.
.........Asiatic Society.
Geological Museum.
........Philosophical Society.
ı........South African Library.
:ill ......Engineering College.
.........Royal Dublin Society.
Royal Irish Academy.
........Royal Society.
........Free Public Library.
.........Admiralty Library.
Chemical Society.
Entomological Society.
Geological Society.
Geological Survey of Great Britain.
Guildhall Library.
Institution of Civil Engineers.
Institution of Naval Architects.
Linnean Society.
London Institution.
Royal Asiatic Society.
Royal Astronomical Society.
Royal College of Physicians.
Royal College of Surgeons.
Royal Geographical Society.
Royal Engineers. Head Quarters Library.
Royal Horticultural Society.
Royal Institute of British Architects.
Royal Institution of Great Britain.
Royal Medical and Chirurgical Society.
Royal Society of Literature.
Society of Antiquaries.
Society of Arts.
The Queen's Library.
The Treasury Library.
The War Office.
United Service Museum.
Zoological Society.
.........Public Library.
r ....Literary and Philosophical Society.
........University Library.
.........McGill College.
.........Ashmolean Society.
Radcliffe Library.

Swansea ....Royal Institution.
Sydney ....Royal Society of New South Wales.
University Library.
Wellington ....New Zealand Institute.
Woolwich ....Royal Artillery Library.

America (South).

Buenos Ayres......Museo Publico.
Carácas ....Sociedad di Ciencias Fisicas y Naturales.

Austria.

Brünn............Naturforschender Verein.
Gratz ....Naturwissenschaftlicher Verein für Steierm
Hermannstadt ....Siebenbürgischer Verein für die Naturwis schaften.
Innsbruck ....Das Ferdinandeum.
Klausenburg ......Az Erdélyi Muzeum—Das siebenbürgisc Museum.
Prague ....Königliche böhmische Gesellschaft der Wis schaften.
Vienna............Kaiserliche Akademie der Wissenschaften.
Geographische Gesellschaft.
Geologische Reichsanstalt.

Belgium.

Brussels ....Académie Royale de Médecine.
Académie Royale des Sciences.
Musée Royal d'Histoire Naturelle de Belgi
Liége ....Société des Sciences.
Louvain ....L'Université.

Denmark.

Copenhagen ......Kongelige Danske Videnskabernes Selskab

Finland.

Helsingfors........Société des Sciences.

France.

Lyons..............Académie des Sciences, Belles-Lettres et A
Montpellier........Académie des Sciences et Lettres.
Faculté de Médecine.
Paris ....Académie des Sciences de l'Institut.
Dépôt de la Marine.
École des Mines.
École Normale Supérieure.
École Polytechnique.
Faculté des Sciences de la Sorbonne.
Jardin des Plantes.
L'Observatoire.
Société Entomologique.
Société de Géographie.
Société Géologique.
Société d'Encouragement pour l'Indus Nationale.
Toulouse ....Académie des Sciences.

Germany.

Berlin	Die Sternwarte.
	Königliche Akademie der Wissenschaften.
	Physikalische Gesellschaft.
Danzig	Naturforschende Gesellschaft.
Emden	Naturforschende Gesellschaft.
Frankfurt-a-M.	Senckenbergische naturforschende Gesellschaft.
Giessen	Grossherzogliche Universität.
Göttingen	Königliche Gesellschaft der Wissenschaften.
Halle	Kaiserliche Leopoldino-Carolinische deutsche Akademie der Naturforscher.
Hamburg	Naturwissenschaftlicher Verein.
Heidelberg	Universität.
Jena	Medicinisch - Naturwissenschaftliche Gesellschaft.
Karlsruhe	Grossherzogliche Sternwarte.
Kiel	Sternwarte.
	Universität.
Königsberg	Königliche - physikalisch - ökonomische Gesellschaft.
Leipzig	Astronomische Gesellschaft.
	Königliche Sächsische Gesellschaft der Wissenschaften.
Munich	Königliche-Bayerische Akademie der Wissenschaften.
Münster	Königliche theologische und philosophische Akademie.
Rostock	Universität.
Würzburg	Physikalisch-medicinische Gesellschaft.

Hungary.

Pesth	Á Magyar Tudós Társaság—Die Ungarische Akademie der Wissenschaften.

Italy.

Bologna	Accademia delle Scienze dell' Istituto.
Catanea	Accademia Gioenia di Scienze Naturali.
Florence	Reale Museo di Fisica.
Milan	Reale Istituto Lombardo di Scienze, Lettere ed Arti.
	Società Italiana di Scienze Naturali.
Naples	Società Reale, Accademia delle Scienze.
	Zoological Station (Dr. Dohrn).
Palermo	Consiglio di Perfezionamento (Accademia di Scienze Naturali ed Economiche).
Rome	Reale Accademia dei Lincei.
	Osservatorio del Collegio Romano.
	Società Italiana delle Scienze.
Turin	Reale Accademia delle Scienze.
Venice	Reale Istituto Veneto di Scienze, Lettere ed Arti.

Java.

Batavia	Bataviaasch Genootschap van Kunsten en Wetenschappen.

Netherlands.

Amsterdam	Koninklijke Akademie van Wetensch
Haarlem	Hollandsche Maatschappij der Wetens
Rotterdam	Bataafsch Genootschap der Proefond lijke Wijsbegeerte.

Portugal.

Coïmbra	Universidade.
Lisbon	Academia Real das Sciencias.

Russia.

Kazan	Imperatorsky Kazansky Universitet.
Moscow	Société Impériale des Naturalistes.
	Le Musée Publique.
Pulkowa	Nikolai-Hauptsternwarte.
St. Petersburg	Académie Impériale des Sciences.
	L'Observatoire Physique Central.

Spain.

Cadiz	Observatorio de S. Fernando.
Madrid	Real Academia de Ciencias.

Sweden and Norway.

Christiania	Kongelige Norske Frederiks Universit
Gottenburg	Kongl. Vetenskaps och Vitterhets Sam
Lund	Universitet.
Stockholm	Kongliga Svenska Vetenskaps-Akade
	Bureau de la Recherche Géologiqu Suède.
Trondhjem	Kongelige Norske Videnskabers Selsk
Upsala	Universitet.

Switzerland.

Bern	Allg. Schweizerische Gesellschaft.
Geneva	Société de Physique et d'Histoire Natu
	Institut National Genevois.
Zürich	Das Schweizerische Polytechnikum.

United States.

Albany	New York State Library.
Baltimore	Johns Hopkins University.
Boston	American Academy of Sciences.
	Boston Society of Natural History.
Cambridge	Harvard University.
Chicago	Academy of Sciences.
Newhaven (Conn.)	The Editors of the American Journal.
	Connecticut Academy of Arts and Scie
Philadelphia	Academy of Natural Sciences.
	American Philosophical Society.
Salem (Mass.)	Peabody Academy of Science.
Washington	Smithsonian Institution.
	U. S. Coast Survey.
	U. S. Geological Survey of the Territo
	U. S. Naval Observatory.

The *fifty* Foreign Members of the Royal Society.

ist of Public Institutions and Individuals entitled to receive a Copy of the Proceedings of the Royal Society.

.... South Australian Institute.
.... Queen's College.
.... Geographical Society.
.... Great Trigonometrical Survey of India.
Meteorological Committee's Office.
ge .... Union Society.
.... Philosophical Society.
.... Dudley and Midland Geological and Scientific Society.
h .... Geological Society.
Royal Medical Society.
Royal Physical Society.
Royal Scottish Society of Arts.
.... Queen's College.
.... Philosophical Society.
, Canada West.. Scientific Association.
own .... Royal Society of Tasmania.
.... Philosophical Society.
.... Historic Society of Lancashire and Cheshire.
Literary and Philosophical Society.
.... Anthropological Institute.
Institution of Mechanical Engineers.
Iron and Steel Institute.
London Library.
Mathematical Society.
Meteorological Office.
Meteorological Society.
National Association for the Promotion of Social Science.
Odontological Society.
Pharmaceutical Society.
Popular Science Review.
Quarterly Journal of Science.
Quekett Microscopical Club.
Royal Engineers (for Libraries abroad six copies).
Royal Microscopical Society.
Russell Institution.
Society of Biblical Archæology.
Standard Weights and Measures Department.
Statistical Society.
Victoria Institute.
r............. Geological Society.
.... Royal Society of Arts and Sciences.
.... Observatory.
Royal Society of Victoria.
.... Natural-History Society.
.... Royal Victoria Hospital.
-upon-Tyne .. Chemical Society.
North of England Institute of Mining Engineers.
.... Geological Society of Cornwall.
.... Plymouth Institution.

Poona .... United Service Institution.
Salford................ Royal Museum and Library.
Shanghai............... North China Branch of the Royal Asiat Society.
Stonyhurst .... The College.
Sydney (N.S.W.) ...... The Linnean Society of New South Wale
Windsor, Nova Scotia .. King's College Library.

Austria.

Innsbruck .... Naturwissenschaftlich - Medicinischer Verein.
Prague................ Spolek Chemikův Českých (Bohemi Chemical Society).
Schemnitz .... K. Ungarische Berg. und Först Akadem
Trieste................ Società Adriatica di Scienze Naturali.
Vienna................ Anthropologische Gesellschaft.
Œsterreichische Gesellschaft für Meteor logie.
Zoologisch-Botanische Gesellschaft.

Belgium.

Brussels .... Observatoire Royal.
Société Malacologique de Belgique.
Liége .... Société Géologique de Belgique.
Luxembourg .... Société des Sciences Naturelles.

Finland.

Helsingfors........... Societas pro Fauna et Flora Fennica.

France.

Apt (Vaucluse) .... Société Littéraire, Scientifique et Arti tique.
Bordeaux............. Académie des Sciences.
Faculté des Sciences.
Société de Médecine et de Chirurgie.
Société des Sciences Physiques et Naturelles.
Cherbourg .... Société des Sciences Naturelles.
Dijon .... Académie des Sciences.
Paris .... Association Française pour l'Avancemen des Sciences.
Société de Biologie.
Société de Physique.
Société Météorologique de France.
Les Mondes (Mons. l'Abbé Moigno).
Revue Scientifique (Mons. Alglave).

Germany.

Bremen .... Naturwissenschaftlicher Verein.
Breslau .... Schlesische Gessellschaft für Vaterlä dische Kultur.
Dresden .... Verein für Erdkunde.
Erlangen.............. Physikalisch-Medicinische Societät.
Frankfurt-a-M. .... Zoologische Gesellschaft.

reiburg im Breisgau	Naturforschende Gesellschaft.
örlitz	Naturforschende Gesellschaft.
alle	Naturwissenschaftlicher Verein für Sachsen und Thüringen.
eidelberg	Naturhistorisch-Medizinische Gesellschaft.
unich	Zeitschrift für Biologie.

Hungary.

esth	Kön. Ungarische Geologische Anstalt.

Italy.

lorence	R. Comitato Geologico d'Italia.
isa	Società Toscana di Scienze Naturali.
iena	R. Accademia de' Fisiocritici.
enice	Ateneo Veneto.

Japan.

okohama	Asiatic Society of Japan.

Netherlands.

msterdam	K. Zoologisch Genootschap Natura Artis Magistra.
trecht	Provinciaal Genootschap van Kunsten en Wetenschappen.

Russia.

t. Petersburg	Compass Observatory.

Spain.

Madrid	Comision del Mapa Geologico de España.

Switzerland.

Basel	Naturforschende Gesellschaft.
Bern	Naturforschende Gesellschaft.
Lausanne	Société Vaudoise des Sciences Naturelles.
Neuchâtel	Société des Sciences Naturelles.
Zürich	Naturforschende Gesellschaft.

United States.

harleston	Elliott Society of Natural History of South Carolina.
Davenport (Iowa)	Academy of Natural Sciences.
Madison	Wisconsin Academy of Sciences.
New York	American Geographical and Statistical Society. New York Medical Journal. New York Academy of Science. School of Mines, Columbia College.
Ohio	Kenyon College.
Philadelphia	Franklin Institute.
St. Louis	Academy of Science.
Salem (Mass.)	Essex Institute.
Vermont	Orleans County Society of History.
Virginia	Medical Society.
Washington	Department of Agriculture.

Individuals.

Brioschi, Il Signor Francesco	Milan.
Costa, Cavalier Achille	Naples.
Wartmann, Professor Élie	Geneva.
Wiedemann, Professor G.	Leipzig.
Wolf, Professor R.	Zürich.

CONTENTS.

PART I.

I. *On the Determination of the Rate of Vibration of Tuning-Forks. By* HERBERT M'LEOD, *F.C.S., and* GEORGE SYDENHAM CLARKE, *Lieut. R.E., Royal Indian Engineering College, Cooper's Hill. Communicated by* LORD RAYLEIGH, *M.A., F.R.S.* . . . . page 1

II. *The Contact Theory of Voltaic Action.*—Paper No. III. *By Professors* W. E. AYRTON *and* JOHN PERRY. *Communicated by Dr.* C. W. SIEMENS, *F.R.S.* 15

III. *Researches on the Comparative Structure of the Cortex Cerebri. By* W. BEVAN LEWIS, *L.R.C.P. (Lond.), Senior Assistant Medical Officer, West Riding Lunatic Asylum, Wakefield. Communicated by Dr.* FERRIER, *F.R.S., Professor of Forensic Medicine, King's College, London* 35

IV. *Experimental Researches on the Electric Discharge with the Chloride of Silver Battery. By* WARREN DE LA RUE, *M.A., D.C.L., F.R.S., and* HUGO W. MÜLLER, *Ph.D., F.R.S.*—Part III. *Tube-Potential; Potential at a Constant Distance and Various Pressures; Nature and Phenomena of the Electric Arc* 65

V. *On the Conduction of Heat in Ellipsoids of Revolution. By* C. NIVEN, *M.A., Professor of Mathematics in Queen's College, Cork. Communicated by* J. W. L. GLAISHER, *F.R.S.* 117

VI. *On some recent Improvements made in the Mountings of the Telescopes at Birr Castle. By the* EARL OF ROSSE, *F.R.S.* 153

VII. *Concluding Observations on the Locomotor System of Medusæ. By* GEORGE J. ROMANES, *M.A., F.L.S. Communicated by Professor* T. H. HUXLEY, *LL.D., Sec. R.S.* 161

VIII. *Researches on Explosives.*—No. II. *Fired Gunpowder. By Captain* NOBLE *(late R.A.), F.R.S., F.R.A.S., F.C.S., &c., and* F. A. ABEL, *C.B., F.R.S., V.P.C.S., &c., &c.* 203

IX. *English Reproduction Table. By Dr.* W. FARR, *F.R.S.* . . . . . page 281

X. *Agricultural, Botanical, and Chemical Results of Experiments on the Mixed Herbage of Permanent Meadow, conducted for more than Twenty Years in succession on the Same Land.*—Part I. *By* J. B. LAWES, *LL.D., F.R.S., F.C.S., and* J. H. GILBERT, *Ph.D., F.R.S., F.C.S., F.L.S.* 289

Index to Part I. 417

LIST OF ILLUSTRATIONS.

Plates 1 to 3.—Mr. H. M‘Leod and Lieutenant G. S. Clarke on the Determination of the Rate of Vibration of Tuning-Forks.

Plates 4 and 5.—Professors W. E. Ayrton and J. Perry on the Contact Theory of Voltaic Action.

Plates 6 and 7.—Mr. W. Bevan Lewis on the Comparative Structure of the Cortex Cerebri.

Plates 8 to 10.—Messrs. W. De La Rue and H. W. Müller on the Electric Discharge with the Chloride of Silver Battery.

Plates 11 to 13.—The Earl of Rosse on some recent Improvements made in the Mountings of the Telescopes at Birr Castle.

PHILOSOPHICAL TRANSACTIONS.

I. *On the Determination of the Rate of Vibration of Tuning-Forks.*

By HERBERT M'LEOD, *F.C.S., and* GEORGE SYDENHAM CLARKE, *Lieut. R.E., Royal Indian Engineering College, Cooper's Hill.*

Communicated by LORD RAYLEIGH, *M.A., F.R.S.*

Received January 16,—Read January 30, 1879.

[PLATES 1–3.]

IN a paper in the Proceedings of the Royal Society for 1877, XXVI., 162, when describing a new method for determining the speed of machines, we pointed out that by the employment of a graduated drum rotating with a known velocity the true vibration number of a tuning-fork could be accurately determined. The following is an account of the apparatus employed, and subjoined are records of some experiments which have been made on tuning-forks with it, which, in view of the attention now being given to the subject, may prove of interest.

One essential instrument proved to be a good time-measurer. It will be seen in the sequel that an electric current is required once a minute, and in the earlier experiments a two-day marine chronometer was employed, a key being depressed by hand at the end of each minute. So many difficulties, however, arose from its use that it was soon rejected and a clock substituted. Near the pendulum two springs were placed which, being pressed together at the end of each vibration, made a contact at every alternate second. About the 59th second of each minute, a key was depressed by hand and held down until the clock completed the circuit. This, though a great improvement, was not entirely satisfactory, and endeavours had to be made to eliminate entirely all contacts made by hand. This was done by means of a compensated pendulum worked by electricity and connected with an electric clock. The method of compensation was copied from that of a pendulum exhibited in the Loan Collection of Scientific Apparatus, as a model of the one belonging to the standard clock of the Royal Observatory at Greenwich. Originally, the impulse was given by an electro-magnet placed under the bob, the contact being made by a small trailing piece of steel, which, catching in a notch in a piece of steel below, depressed a spring at every alternate second. This method, which is frequently employed in small clocks, is generally arranged to give a powerful impulse to the pendulum, and this so much increases the arc of vibration that the trailing piece passes over the notch without again engaging in it until the arc has sufficiently diminished. Although such clocks are accurate enough for ordinary purposes, it is obvious that the method would be inadmissible in cases where the period of all vibrations must be the same. To ensure uniformity in this respect the battery power was diminished until the contact was made at each vibration, but after many trials the method was abandoned, for it was found that the friction between the steel trailing piece and the notched piece below was not constant, and, besides, the pendulum was often found to be swinging in an elliptic arc.

An endeavour was next made to work the pendulum by a driver depending on the action of gravity alone, the electric current being employed merely to lift the driving lever, so that the impulse given to the pendulum was entirely independent of the power of the battery. It will be unnecessary to detail the various experiments that were made with more or less success. The following is a description of the form of driving arrangement ultimately adopted.

On each side of the supporting spring of the pendulum is pivoted a branched lever *a*, *b*; Plate 1, fig. 1. At the lower end of each lever is a platinum pin which comes in contact with the rod of the pendulum as in many forms of gravity escapement. The distance of fall of each lever is regulated by means of screws, *c c*. The lever arm, *a*, is so adjusted that the pendulum rod touches the pin at the end of its swing, thus making an electric contact with a platinum collar placed on the pendulum rod, and causing the current to flow from the battery through the electro-magnet *d*, attracting the armature and raising the lever *e*. [This arrangement is not shown in the figure, which represents a slight modification to be described presently.] A pin, *p*, projecting from the lever *e*, lifts the driving lever *b*. On the return swing of the pendulum to the right the contact with *a* is broken and the lever *e* falls, leaving *b* suspended by a small steel catch, *f*. When the pendulum rod touches the platinum pin on the end of *b*, a current traverses the releasing magnet *g*, withdrawing the catch, and allowing the driving lever *b* to fall with the pendulum and thus to give the requisite impulse. Many accurate determinations were made with the contacts as above described, but it was found that the roughening of the pin and platinum collar by the spark produced variable friction; this led to the adoption of the arrangement shown in the figure, which consists in attaching (by an ivory pin, *h h*) a thin piece of watch spring to each lever *a* and *b*, so that the electric contacts are made between the platinum pin and a piece of gold soldered on to the spring. The pendulum rod thus presses against a piece of polished steel and all variable friction is prevented. The figure shows in the form of a diagram the arrangement of the connexions. The pendulum is driven by four cells of a gravity battery, but as the electro-magnet *d* has only a small resistance, the extra current at first occasionally fused together the contacts of lever *a*: this was prevented by connecting the wires on each side of the contact by a shunt consisting of a bobbin of fine wire of about 300 ohms resistance. This plan having, however, proved somewhat uncertain in action, the shunt has been replaced by a tube containing two platinum plates immersed in very dilute sulphuric acid.

At the back of the pendulum rod a short pin of chrysolite is fixed to collar *k*. This pin, passing under a small steel disk *a*, Plate 1, fig. 2, raises a thin brass spring fastened to the back of the pendulum case, and makes contact at each oscillation of the pendulum with a second spring carrying a platinum pin. The second spring is thereby raised sufficiently to make a contact with a third spring, and thus two different contacts are made at each second, one preceding the other by a small interval of time. The roughening and occasional sticking of the lower contact has been prevented by connecting a condenser, or two platinum plates in very dilute sulphuric acid, to the springs. The clock, placed at any distance from the pendulum, is actuated by the closing of the two lower springs, an electro-magnet raises a lever carrying a catch which turns a ratchet wheel with 60 teeth, the wheel being moved by the fall of the lever at the breaking of the pendulum contact. Plate 3, fig. 3, shows the arrangement of the minute wheel of the clock and the electric connexions. By an ingenious device of Mr. YEATES, who made the clock, the fall of the lever both turns and locks the wheel, the end of the catch falling on an inclined screw *s* which drives the point of the catch between the teeth of the wheel. On this minute wheel of the clock is a platinum pin *a*, which presses against a spring *f* once a minute. As this contact might possibly be irregular, since it is produced on the breaking of the pendulum contact and fall of the clock lever, whereby two elements of uncertainty are introduced, it is merely used as a switch, and the principal circuit is closed by the pendulum itself when the two upper springs are pressed together. The current then passes through the magnet of a relay and closes the circuit required in the tuning-fork apparatus. These minute contacts are made with great accuracy, for the upper contact is of very short duration, and it is made by the pendulum when moving with nearly its greatest velocity.

The rate of the clock was determined by observations with a small portable transit instrument. The arrangements were not as perfect as could be desired, but the time was determined from the mean obtained by the observations of at least four stars (generally more), and the rates of the clock and chronometer were thus both obtained; moreover the observations were made at such intervals that their inaccuracies did not much affect the determination of the rates. The clock was compared daily with the chronometer and the irregularities of the former corrected.

To prove that no great error can have crept into the calculations from any inaccuracy of the chronometer, the following table is given showing the dates of the observations and the chronometer rate in the interval.

	Gaining rate per day. seconds.
1878. Jan. 16	
	2·91
Jan. 19	
	2·527
Jan. 23	
	2·741
Feb. 25	
	2·657
Mar. 3	
	2·323
Mar. 10	
	2·270
Mar. 12	
	3·394
April 6	
	3·528
April 22	
	3·546
May 4	

	Gaining rate per day. seconds.
1878. May 4	
	2·463
May 14	
	2·232
June 3	
	2·282
June 11	
	2·297
June 19	
	2·673
June 22	
	2·476
June 27	
	3·098
July 15	
	2·504
Aug. 7	

The error of the clock rate when checked was not more, and probably less, than one second per day.

The apparatus used for the measurement of the tuning-forks was made from a portion of an astronomical chronograph constructed by Messrs. R. and J. Beck. The brass drum, which is two feet long and one foot in diameter, rests on friction wheels and is driven by a weight and train of wheels. The original regulator, consisting of a conical pendulum, was removed and its place supplied by a fan kindly designed for us by our friend and colleague, Professor W. C. Unwin, B.Sc. The fan and case are shown full size in Plate 2, fig. 4. The diameter of the fan is 3·8 inches, and it is attached to a vertical steel axis resting in an agate cup. The fan rotates in the lower part of a circular box 5 inches in diameter and 2 inches deep, the box being divided into two horizontal compartments by a fixed diaphragm *d d*, the upper compartment being ·75 inch and the lower 1·2 inch deep; the diaphragm is perforated by four segmental apertures *a a* which can be closed by a similarly perforated disk resting on the diaphragm and capable of being moved by an arm *r* on the top of the box. When the apertures in the diaphragm are opened, air is driven from the lower compartment into the upper, returning through the openings in the middle of the fan, and when the apertures are closed the air circulates with the fan in the lower half of the box. In the latter case the fan rotates about twice as fast as when the diaphragm is fully opened. The axis carries at the top a small pulley *p* by which it is connected to the drum of the chronograph by a piece of thread passing round the latter.

Round one end of the drum, strips of dark blue paper are wrapped, on which white equidistant lines parallel to the axis of the drum are ruled. The paper was graduated before being attached to the drum; as the division of a long strip of paper by means of compasses was found tedious and somewhat inaccurate, it was effected by rolling a toothed wheel along the paper, thus obtaining fine equidistant marks through which lines were ruled in Chinese white by means of a drawing pen. For convenience of counting, every fifth and tenth line was made longer than the others, and after attaching the strip to the drum, figures were placed opposite every tenth line. The two strips which are most generally used for the determinations have 486 and 985 lines respectively.

Opposite the graduated paper on the drum is fixed a microscope placed in a horizontal position; the instrument is suspended from a shelf above the table on which the chronograph rests, and the shelf also supports the clock. In the sub-stage a 2-inch objective is placed which produces an image of the graduations at the focus of the object glass of the microscope, the portion of the scale used being illuminated by a lamp and condensing lens. At the common focus of the lenses is placed the tuning-fork, the stem of which is surrounded by a piece of sheet lead and fixed in a vice attached to the table, the upper part of one of the prongs being in such a position that about one-third of the field of view is cut off by the prong, Plate 3, fig. 6. The lower part of the fork is surrounded by a glass case made of four strips of glass glued to a square piece of wood through which the stem of the fork passes. A hole is bored in one of the plates of glass and through this passes a thermometer graduated to ·2 C., the bulb being placed as close as possible to the fork without touching it. A plan of the arrangement is shown in Plate 3, fig. 8.

In the eyepiece of the microscope a horizontal hair and a vertical scale are fixed (Plate 3, fig. 6). A double-bass bow is suspended from the shelf by two parallel strings so that the operator may keep the fork in vibration while observing through the microscope. When fork and drum are at rest the white lines on the drum are seen across the field of the microscope but cut off on one side by the fork (Plate 3, fig. 6). When the drum rotates the lines can no longer be seen, but if at the same time the fork is in vibration wave-figures are produced extending from its edge. When the drum is rotating with such a velocity that a line passes over a distance equal to the interval between two adjacent lines during one vibration of the fork, a stationary wave (Plate 3, fig. 7) is seen of a length equal to the distance between two lines, and it only remains therefore to count the lines which pass in a known period of time to determine the number of vibrations of the fork. It was not found possible to obtain a perfectly uniform rotation of the drum by the regulator alone, as the slight irregularities of the clock-work and pulleys were sufficient to cause unequal rates of rotation. In order to obtain a final regulating power a piece of string connects the axis of the drum with a pulley, *g* (Plate 3, fig. 8), which can be turned by the operator's left hand. The motion of the wave upwards or downwards indicates that the drum is going too fast or too slow, and a gentle check, or assistance given by the hand to the axis of the pulley, is enough to keep the rotation uniform.

To count the number of lines passing across the horizontal hair of the eyepiece during an experiment it is necessary to know the number of complete revolutions, and also the additional fraction of a revolution which the drum has performed during the observation. The number of the revolutions is obtained by an electric counter attached to the axis of the drum, a pin on which, shown at *a* on the diagram, Plate 3, fig. 9, completes the circuit by pressing two springs together. To measure the fraction of the revolution, an electro-magnet, *b*, is made to produce a mark on the drum both at the commencement and conclusion of the experiment. Above the chronograph drum is a saddle supporting the electro-magnet; to the armature of the latter is attached a finely pointed tube, *t*, containing a solution of magenta mixed with an equal volume of glycerin to prevent it drying and so stopping up the jet. An electric current from the clock is made to pass through the electro-magnet, both at the beginning and end of the experiment, thus making two dots on the white paper covering the drum, the distance between the dots being the fraction of the revolution required. To find the number of lines in this fraction it is only necessary to place a straight-edge along the drum and to note the numbers of the lines corresponding to the two marks. The saddle carrying the electro-magnet is moved forwards or backwards parallel to the axis of the drum by a screw, which can be turned by means of a cord and pulley near the operator's hand, so that two consecutive dots may not be made on the same line on the drum, and the paper round the drum thus serves as a permanent record of the observations made. The electric current which makes the marks also starts and stops the counter. The circuit of the counter is broken at two mercury cups, *c*, which are connected by a copper wire attached to a small magnet resting between the poles of an electro-magnet, *d*. By means of the reversing switch, *e*, which is turned in one direction before the commencement of the experiment, the copper wire is dipped into the mercury cups, so that when the current is sent by the clock relay the counter is automatically started, and before the end of the experiment the reverser is turned over, so that the contact is broken and the counter stopped also automatically when the second mark is made on the drum.

In each determination of a fork, five experiments are usually made, lasting one, two, three, four, and five minutes respectively. The object of this is to diminish any error which may be due to variation in the duration of movement of the electro-magnets, and also to reduce to a minimum the errors of observation. The forks are placed in the vice many hours before the determinations are made so that they may attain a uniform temperature throughout, and that this temperature, as shown by the thermometer, may be as correct as possible. The glass case surrounding the fork prevents air currents from affecting the thermometer, and a large glass plate placed vertical on the table diminishes the rise of temperature due to radiation from the observer's body.

When a determination is to be made the counter is set at 0, and the velocity of rotation of the drum regulated by means of driving weights and by the fan until the wave figure is nearly steady: when this is accomplished the observer keeps one wave in the field of view by judicious manipulation of the regulating string. He then begins to count seconds by the clock and turns the reverser so that the current sent at the 60th second will start the electric revolution counter. When the sound of the pen striking the drum is heard, he observes what part of a wave is cut by the hair in the eyepiece, and continuing to count the seconds he maintains as far as possible this part of the wave near the centre of the field. It is difficult to keep the rotation absolutely uniform, but it suffices not to allow the wave to escape from the field of view, or if it does, to bring it back as soon as possible. If the observation is to last for one minute the reverser is turned in the opposite direction immediately after the first dot has been made; but if for a longer period, the contact remains broken by the reverser until after the end of the first, second, third, or fourth minute, when the connexions are so made that at the end of the next minute the counter is stopped. As soon as the first dot is made the screw carrying the electro-magnet is slightly turned so as to shift the position of the pen. If at the sound of the marking of the second dot on the drum the particular part of the wave which was on the hair at the beginning of the experiment is in another part of the field of view, the alteration of position is estimated as accurately as possible in wave-lengths or portions of a wave-length. A correction corresponding to this alteration of position is subsequently applied. The only object of counting the seconds throughout is to direct the observer's attention to the approach of the minute intervals. The five experiments are made consecutively and without stopping the drum. After each experiment the alteration of position of the wave (if any), the temperature indicated by the thermometer, and the number shown by the revolution counter are noted. When the experiments are finished the dots on the drum are marked by consecutive numbers, two belonging to each measurement. A long straight-edge is then placed horizontally against the drum and the latter turned until the centre of the dot is at the edge of the straight-edge, the number belonging to the line of the graduated slip of paper opposite the ruler is then read off.

A determination made on May 8, 1878, with a 256 fork of Kœnig's, will best illustrate the method of recording and calculating the results, and will also serve to show the accuracy of which the method is capable.

	Number of line on circle.
No. of 1st mark 163	301
" 2nd " 164	199·5

Duration of experiment, 5 minutes.
Temperature by thermometer, 16°·1 C.
Number shown by revolution counter, 158.
Number of lines on circle, 486.

Drum was going a little too fast, the part of the wave under observation had risen ·5 of a wave-length above the hair.

158 × 486=76,788 lines passed in 158 revolutions.
301−199·5=101·5 lines between the two dots.
101·5−·5=101 lines corrected for movement of the wave.
76,788+101=76,889 vibrations of fork in 5 minutes.

$$\frac{76889}{300}=256{\cdot}2966 \text{ vibrations per second.}$$

It frequently happens that the two numbers corresponding to the dots are on the opposite sides of the 0, in which case the counter indicates one more than the number of revolutions, and 486 must be added to the number corresponding to the first dot before subtracting the number corresponding to the second dot.

The remaining entries in the note-book serve to show the whole of these cases.

Mark	165	..	405·25	2 min.	
	166	..	268	16°·9	30618
			137·25	221 (counter)	137·75
			+ ·5	158 (at last experiment)	120)30755·75
			137·75	63 × 486=30618	256·298
Mark	167	407·25	407·25	3 min.	
	168	444·5	486	17°·0	45684
					448·75
			893·25	316	180)46132·75
			444·5	221	256·293
			448·75	95−1=94 × 486=45684	
Mark	169	84	84		
	170	258	486	1 min.	
			570	17°·0	15066
			258		311·5
			312	348	60)15377·5
			− ·5	316	256·292
			311·5	32−1=31 × 486=15066	
Mark	171	252	252	4 min.	
	172	464	486	17°·2	61236
					274
			738	475	240)61510
			464	348	256·2916
			274	127−1=126 × 486=61236	
			256·297	16·8	
			256·298	16·9	
			256·293	17·0	
			256·292	17·0	
			256·292	17·2	
			5) 22	5)34·9	
			Mean=256·294	16·98	
				− 1·4 (correction for error of thermometer.)	
				15·58	

When this determination was made the rate of the clock was taken as ·788 second per day, losing; correcting for this rate, we have $256{\cdot}294 \times \frac{86399{\cdot}212}{86400} = 256{\cdot}292$ vibrations per second at 15°·58 C.

The maximum difference between the above numbers is ·006 or ·00234 per cent.

An error of reading of one line, or mistaking one wave for the next, will produce, in a five-minute observation, a difference of ·0033 in the determination of a 256 fork, while the same error in an observation lasting one minute will produce a difference of ·0166; the value of an experiment lasting several minutes is therefore apparent. The method is of such a nature that no bias in the mind of the operator can have the slightest effect on the result, as the final numbers show no indication of their direction until the positions of the dots are read off on the drum; and, to prove that special training is not required for the observations, it may be mentioned that a sharp boy of 16 succeeded in keeping the figure sufficiently steady after two trials with the machine.

Three elements of possible error in the method above described suggested themselves —1st, the effect of the frequent bowing; 2nd, the variation of amplitude; and 3rd, the holding of the fork in a vice instead of fixing it, as usually employed, on a sounding box. These matters will now be dealt with *seriatim*.

1. *Effect of bowing.*

In order to dispense with the use of the bow, and to keep the amplitude constant, we had contemplated the employment of HELMHOLTZ's method of maintaining the forks in vibration by means of an electro-magnet excited by an intermittent current from a contact maker worked by a large tuning-fork, the vibration number of which would bear a simple relation to the vibration number of the fork to be measured. The experiment, however, was not tried, as we feared that the current might to some extent control the fork, which would, of course, be inadmissible; the bow was therefore retained.

If the bowing altered the phase of the vibration, which seemed at first not improbable, there would be a sudden start of the wave figure, in one direction or the other, at each stroke of the bow; this has never been observed. Again, if the bowing altered the phase, even to an extent inappreciable by the motion of the wave, its frequent occurrence at each application of the bow would necessarily alter the number obtained in the determination. In order to put this to the test six experiments were made, in three of which the fork was bowed every 20 seconds, two strokes of the bow being generally made on the prong, and in the three others the fork was bowed every five seconds. The determinations were made alternately, and the first four lasted five minutes, and the others three minutes. The following numbers were obtained:—

	Bowed every 20 seconds.	Temp.	Bowed every 5 seconds.	Temp.
	512·470	16·1	512·460	16·2
	512·460	16·25	512·452	16·25
	512·460	16·3	512·456	16·4
Mean	512·463	16·22	512·456	16·28

(These numbers are not corrected for the rate of the clock)

The effect of frequent bowing, therefore, appears to be to make the fork vibrate a little more slowly, the diminution of rate being ·00137 per cent. The fork was one of Kœnig's, and of stout metal; lighter made forks may be affected to a greater extent. Lissajous found that no change of form was exhibited by his figures when the forks were bowed ('Comptes Rendus,' 1855, xli., 816); although a different result was obtained by Poske (Pogg. Ann., 1874, clii., 449), who observed that the increase of amplitude produced by bowing seriously altered the rate of vibration. He, however, seems to have used a vibroscope, in which the ends of the prongs were weighted by a lens and counterpoise.* We have recently endeavoured to settle this question in another manner. The microscope being placed horizontally, a 256 fork was fixed vertically at the focus of the objective, and another fork was placed horizontally in such a position that an image of its edge was thrown by the lens in the substage on to the edge of the vertical fork. In this way the Lissajous' figure was seen in the angle made by the two images. No change of phase was perceptible on bowing either of the forks. Where two 60 forks were tested in a somewhat similar manner, a change of phase was seen; but one of these forks is thin in comparison with its length, and the prongs are weighted by a lens and counterpoise. It is evident, however, that the unskilful bowing of a light fork may alter the phase, by first damping and then re-starting the fork.

2. *Effect of the variation of amplitude.*

To determine the effect of different amplitudes, the scale in the eyepiece of the microscope was placed horizontally, and the amplitude kept as constant as possible by suitable bowing. Four experiments were made, each of five minutes' duration. In the first and third, the movement of the end of the prong was about ·37 millim.; in the second and fourth, about ·15 millim. The results are as follows:—

	Large amplitude.	Temp.	Small amplitude.	Temp.
	256·277	16·2	256·275	16·25
	256·273	16·4	256·274	16·45
Mean	256·275	16·3	256·2745	16·35

---

* Kayser, also, in his determinations of the effects of temperature, employed weighted forks (Ann. der Physik u. Chemie, 1879, N. F. viii. 444). As far as our experiments go, such forks seem to behave, in many respects, differently from those of the ordinary form.—(Note added December 31, 1879).

Consequently, this difference of amplitude does not appreciably affect the vibration number.

3. *Effect of the mode of fixing the fork.*

To determine whether any difference was perceptible when the fork was attached to the sounding box, the latter was supported on a piece of wood and a heavy leaden weight placed within it, the top being left without any pressure upon it. The experiment was not easy, as the pressure of the bow slightly moved the fork in the field of view, and the amplitude was small, which made the waves somewhat difficult to see. Three five-minute measurements were made, giving—

	512·489	at	16·2
	512·482		16·25
	512·477		16·35
Mean	512·483		16·26

The results previously given of the effect of bowing were made with the same fork at the same temperature; a comparison with these would seem to indicate that fixing in a vice slightly diminishes the vibration number, to an extent of about ·005 per cent. This might have been expected, as the fork communicates a slight motion to the body to which it is attached, and the heavier vice retards the fork.

Effect of temperature on the vibration number of forks.

In the previous paper some experiments were described by which the effect of a rise of temperature on a large fork, giving 60 vibrations per second, was found to be a diminution of the rate of vibration amounting to ·011 per cent. for each degree centigrade. It will be better to express this by the coefficient ·00011 for 1° C. This number has been confirmed by some determinations of a 256 fork at various temperatures. Since it is not easy to devise a thoroughly satisfactory means for altering the temperature of the fork whilst in the apparatus, the determinations were made when the temperatures of the surrounding air were very different. The following numbers were thus obtained:—

1.	256·310	at	15·01 C.
2.	256·273		16·31
3.	256·260		16·69
4.	256·175		19·75
5.	256·137		21·16
6.	256·052		24·29
7.	256·008		25·69

Calculating the coefficient for each degree centigrade from the determination made at the lowest temperature as the starting point, the following values were found:—

	For 1° C.	From determinations at
	·000116	15·01 and 16·31
	·000111	„ 16·69
	·000111	„ 19·75
	·000110	„ 21·16
	·000109	„ 24·29
	·000110	„ 25·69
Mean	·000111	

Starting from the determinations made at the highest temperature, we have :—

	For 1° C.	From determinations at
	·000110	25·69 and 15·01
	·000110	„ 16·31
	·000109	„ 16·69
	·000110	„ 19·75
	·000111	„ 21·16
	·000123	„ 24·29
Mean	·000112	

Rejecting the last number as being obviously too high, the mean of the other five is ·000110.

Determinations have been made with a new set of KŒNIG's forks, obtained for us by Mr. LADD last year, and stated to be correct at 15° C.* The following vibration numbers were found, and the corrections for temperature were made by the use of the coefficient ·00011 for each degree centigrade :—

---

* We afterwards heard that Mr. KŒNIG considered them to be correct between 19° and 20° C.

Date.	Mark on fork.	Number of measurements.	Percentage difference between highest and lowest numbers.	Temperature. ° C.	Vibration number.	Corrected to 15° C.	Mean.
1878. May 2	Sol$_3$ 758 V.S. RK. .	7	·00755	17·6	384·330	384·440	384·437
„ 3	„ „ . .	6	·00885	15·9	384·394	384·432	
„ 3	„ „ . .	6	·00338	16·9	384·349	384·429	
„ 4	„ „ . .	5	·00156	15·5	384·418	384·439	
May 4	Mi$_3$ 640 V.S. RK. .	4	·00125	16·27	320·321	320·366	320·372
„ 6	„ „ . .	5	·00219	15·12	320·365	320·369	
„ 6	„ „ . .	4	·00156	16·27	320·327	320·372	
„ 7	„ „ . .	5	00312	18·05	320·273	320·380	
May 8	Ut$_3$ 512 V.S. RK (I)	5	·00234	16·48	256·266	256·308	256·309
„ 8	„ „ .	5	·00234	15·58	256·292	256·308	
„ 9	„ „ .	5	·00195	17·39	256·217	[256·284]	
„ 10	„ „ .	2	·00156	16 30	256·273	256·310	
„ 10	„ „ .	2	·00039	16·35	256·272	256·310	
May 11	Ut$_3$ 512 V.S. RK (II)	4	·00195	17·99	256·223	256·307	256·306
„ 13	„ „	4	·00078	17·3	256·236	256·301	
„ 14	„ „	4	·00234	16·14	256·278	256·310	
May 14	Ut$_4$ 1024 V.S. RK .	4	·00078	16·82	512·446	512·548	512·551
June 6	„ „ . .	5	·00664	15·9	512·487	512·538	
„ 7	„ „ . .	4	·00137	18·07	512·386	512·559	
„ 8	„ „ . .	5	·00312	17·78	512·399	512·556	
„ 10	„ „ . .	6	·00615	17·05	512·451	512·567	
„ 11	„ „ . .	3	·00195	16·22	512·479	512·548	
„ 11	„ „ . .	3	·00156	16·28	512·472	512·544	
June 17	Ut$_3$ 512 V.S. RK (I)	4	·00117	15·01	256·310	256·310	256·310
„ 17	„ „ .	4	·00078	16·69	256·260	256·308	
„ 18	„ „ .	4	·00234	16·31	256·273	256·310	
„ 21	„ „ .	4	·00312	19·75	256·175	256·309	
„ 22	„ „ .	5	·00391	21·16	256·137	256·310	
„ 25	„ „ .	5	·00195	24·29	256·052	256·314	
„ 26	„ „ .	4	·00039	25·69	256·008	256·309	
Oct. 21	„ „ .	4	·00195	17·0	256·254	256·310	
„ 22	„ „ .	5	·00117	15·07	256·302	256·304	
„ 23	„ „ .	4	·00156	12·72	256·378	256·314	

A fork belonging to Sir WILLIAM THOMSON, and made by MARLOYE, gave the following numbers :—

Date.	Number of measurements.	Percentage difference between highest and lowest numbers.	Temperature ° C.	Vibration number.	Corrected to 15° C.	Mean.
1878. June 13 . . .	5	·00391	17·48	255·938	255·945	255·953
„ 15 . . .	5	·00430	16·12	255·962	255·962	

Method of comparing two forks.

Two forks, the vibration numbers of which differ very slightly from one another, may be conveniently compared by the method previously mentioned, namely, by placing them at right angles with one another, and in such positions that the images of their edges are seen to cross one another in the field of view of the microscope. When the forks are vibrating, a modification of LISSAJOUS' figure is seen in the angle made by the two images. If the forks are nearly of the same pitch, the corresponding LISSAJOUS' figure, obtained in the usual way, is, as is well known, a straight line changing to an ellipse which passes into a circle, then to another ellipse, and finally to another straight line inclined the opposite way to the first. When such a pair of forks, nearly in unison, is under observation, one of the lines is represented by a bright line bisecting the angle (due apparently to a diffraction phenomenon at the edge of each fork); this line then becomes double, and changes into an ellipse and finally to a circle. Directly the change from the line commences, the angle between the forks becomes filled by a dark shadow. The circle appears surrounded by a bright line, but the second ellipse is bounded on one side by a dark shadow, and on the other side by a bright line. At that part of the figure corresponding to the line lying across the angle, the angle itself appears to be filled up by a dark shadow with a well-defined edge. The figure then passes through the changes in the reverse order. The instant when the angle is free from shadow is very sharply defined, and, starting from this point, it is quite easy to count the number of optical beats which take place in a given time. The two forks may be kept in vibration by careful bowing, an assistant being required to bow the horizontal fork.

With two forks, one the octave of the other, the parabola seems to rest like a dark shadow alternately on the two forks. At these instants the angle is free from shadow; the figure of 8 is also perceptible surrounded by a bright line.

We found that the fifth could also be compared in a similar manner, but the observation was not quite so easy. In this case the sharpest indication was the filling up of the angle by shadow. With more complex relations between the two forks no sufficiently visible phenomena were perceptible.

Although, undoubtedly, great accuracy may be attained in counting beats by the ear, in some respects the optical method is more serviceable; it also permits of an extended observation, as the forks may be bowed without altering their phase.

As the question of absolute pitch has recently attracted much attention, and as tuning-forks are becoming more and more employed for measuring small intervals of time, we hope that the experiments above described may prove a useful contribution to the knowledge of the subject.

II. *The Contact Theory of Voltaic Action.*—Paper No. III.

By Professors W. E. AYRTON *and* JOHN PERRY.

Communicated by Dr. C. W. SIEMENS, *F.R.S.*

Received February 19,—Read March 13, 1879.

[PLATES 4–5.]

I.

UP to the commencement of 1876 direct experiments on the difference of potentials of solids in contact with liquids had been made by VOLTA,* Sir HUMPHRY DAVY,† MM. BUFF,‡ BECQUEREL,§ PÉCLET,|| PFAFF,¶ Sir WILLIAM THOMSON,** HANKEL,†† and GERLAND,‡‡ and of liquids in contact with liquids by KOHLRAUSCH;§§ but although extremely interesting they were all incomplete, for the following reasons:—

1. The earlier experiments were not carried out with apparatus susceptible of giving accurate results.

2. Owing to the incompleteness of the apparatus, assumptions had to be made not justified by the experiments.

3. *No direct* experiments had been performed to determine the difference of potential of two liquids in contact, with the exception of a few by KOHLRAUSCH, using a method which appeared to us quite inadmissible as regards accuracy of result.

With regard to the objection No. 1, it is needless to do more than to remark that some of the earlier experimenters thought it permissible to touch a liquid under test with the fingers, or assumed without experiment that a piece of blotting-paper, previously soaked in a liquid, acted inductively on a distant metallic surface as a layer of the liquid would.

---

* VOLTA, Gehler's Wörterbuch, Bd. iv., s. 616.

† DAVY, Ann. de Chim., t. lxiii., p. 230, 1807.

‡ BUFF, Ann. d. Chim. u. Pharm., Bd. xlii., s. 5, 1842; u. Bd. xlv., s. 137, 1844.

§ BECQUEREL, Ann. de Chim. et Phys., t. xxv., p. 405, 1824; Pogg. Ann., Bd. ii., s. 169.

|| PÉCLET, Ann. de Chim. et Phys. [3], t. ii., p. 233; Arch. de l'El., t. i., p. 621, 1841.

¶ PFAFF, Pogg. Ann., Bd. li., s. 110 und 197, 1840.

** THOMSON, Proc. Lit. and Phil. Soc. of Manchester, Jan. 21, 1862: Papers on Elec. and Mag., § 400, p. 317.

†† HANKEL, Elektr. Untersuchungen, vi.; Abhandl. der Math.-Phys., Classe der Königl. Sächs. Gesellschaft, Bd. vii., s. 585, 1865.

‡‡ GERLAND, Pogg. Ann., Bd. cxxxiii., s. 513, 1868.

§§ KOHLRAUSCH, Pogg. Ann., Bd. lxxix., s. 200, 1850.

Regarding objection No. 2, great vagueness existed as to whether the contact difference of potentials between two substances, A and B, which we shall call for brevity $\overline{AB}$, was a constant depending only on the substances A and B and the temperature, or whether, one or both being a liquid, it was a variable dependent upon what other substance was in contact with either. Some writers regarded it as a variable;[*] GERLAND[†] considered he had proved it constant when one or both of the substances was a solid. But, first, the agreement of the value of the electro-motive force of each of his cells with the algebraical sum of the separate differences of potential at the various surfaces of separation, which was the test of the accuracy of his theory, is so striking, and so much greater than polarisation, &c., usually allows one to obtain in experiments of such delicacy, that one cannot help feeling doubtful regarding his conclusions; secondly, his apparatus did not allow of his experimenting with two liquids in contact, consequently he could not legitimately draw any conclusion in this latter case. And although KOHLRAUSCH had made some few experiments on the difference of potentials of liquids in contact, still, since he employed moist blotting-paper surfaces instead of the surfaces of the liquids themselves, we felt, for this reason if for no other, that his results did not carry the conviction the distinguished position of the experimenter might have led us to anticipate.

In fact, the inability of all experimenters to *measure directly* and *accurately* with their apparatus the contact difference of potentials of a solid and a liquid, and especially of *two liquids*, caused the whole theory of voltaic action to remain in a vague and incomplete state. It therefore appeared to us, in 1875, desirable to design an apparatus and carry out a series of experiments on this subject; and the results obtained during the following year will be found in our papers Nos. I. and II. (Proc. Roy. Soc., No. 186, 1878). Our method consisted in measuring directly the difference of potentials in volts at each separate contact of dissimilar substances in the ordinary galvanic cells, and then ascertaining whether the algebraical sum of all the contact differences of potential was or was not equal to the electro-motive force of the particular cell in question. The result obtained was that, within the limits of experiment, if $\overline{AB}$,[‡] $\overline{BC}$, $\overline{CD}$, &c., were separately measured (any one or more of the substances being solid or *liquid*), then if any number A, B, C, D . . . . K were joined together, and the electromotive force of the combination $\overline{AK}$ measured, the following equation was found true:—

$$\overline{AK}=\overline{AB}+\overline{BC}+\overline{CD}+\ldots\ldots+\overline{JK}.$$

Which proved that each surface of separation produced its effect independently of any other.

---

[*] F. JENKIN, Elec. and Mag., p. 44, 1873

[†] GERLAND, Pogg. Ann., Bd. cxxxvii., s. 552, 1869.

[‡] $\overline{AB}$, as previously defined, being the difference of potentials between A and B in contact with one another, but neither in contact with any other conductor.

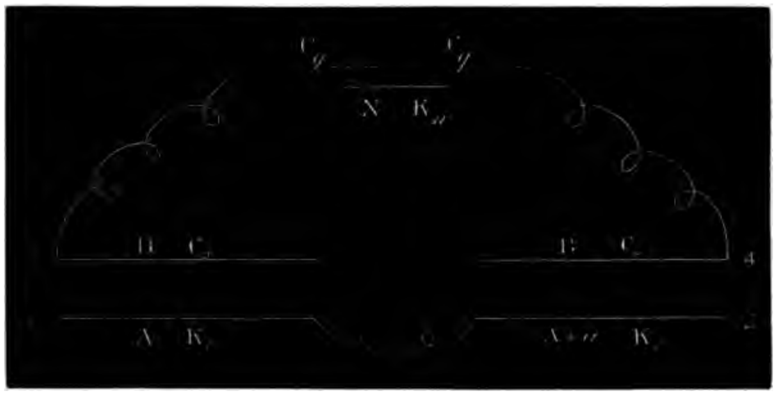
Since a preliminary notice of our first paper was communicated by Sir **William Thomson** to the British Association in 1876, Professor **Clifton** has published one "On the Difference of Potential produced by the Contact of Different Substances;"* but in spite of the exact workmanship of Professor **Clifton**'s apparatus and the probable accuracy of the results attainable with it, it nevertheless possesses the same objection as the instrument employed by **Gerland**, namely, that it was not possible to measure directly with it the difference of potential of two liquids in contact. The results given by Professor **Clifton** are, as far as they go, generally in accord with those contained in our paper.

II.

The experiments of which the results form the subject of the present communication were made during 1877 and 1878, and the new apparatus employed, although generally the same in principle as that previously used, possessed many important modifications suggested by experience.

The method of measurement was as follows† :—Let 3 and 4 be two insulated gilt-brass plates connected with the electrodes of a delicate quadrant electrometer. Let 1 under 3 and 2 under 4 be the surfaces whose contact difference of potential is to be measured. 3 and 4 are first connected together and then insulated, but remain connected with their respective electrometer quadrants. Now, 1 and 2 are made to change places with one another, 1 being now under 4 and 2 under 3, then the deflection of the electrometer needle will give a measure of the difference of potentials between 1 and 2.

Fig. 1.



For before reversal let (fig. 1) the potential of the electrometer needle be N, of the induction plates and of the quadrants be B, of the substance 1 be A, and of the substance 2 be $A+a$, so that a is what we desire to measure;

---

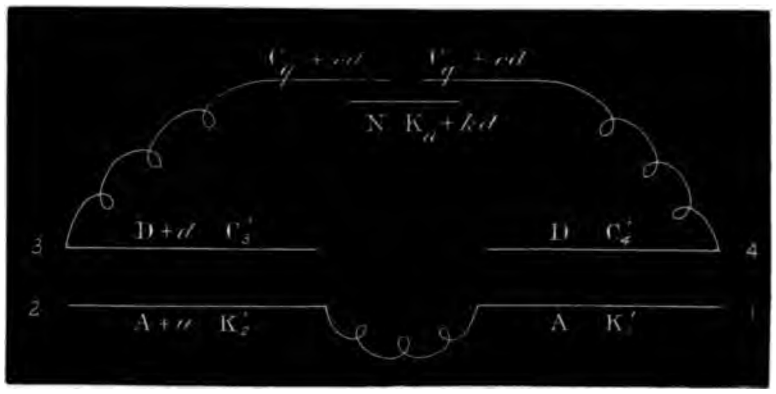
* Proc. Roy. Soc., 1877, vol. xxvi., p. 299.

† It need hardly be noticed that an *inductive* and not a *conductive* method of experimenting must necessarily be employed since the latter would introduce always unknown differences of potential.

Let the capacity of each pair of quadrants be C_q, of 3 be C_3, and of 4 be C_4;

Let the coefficient of induction of the electrometer needle on each pair of quadrants be K_n, of substance 1 on plate 3 be K_1, and of substance 2 on plate 4 be K_2.

Fig. 2.



Now on reversal (fig. 2) the needle will be deflected and the change of the capacity of each pair of electrometer quadrants, as well as the change of the coefficient of induction of the needle on each, will be proportional to the deflection, that is, proportional to d the difference of potentials now existing between the two sets of quadrants, therefore let the potential of 3 become $D+d$, and of 4 become D;

Let the capacity of each pair of quadrants become C_q+cd, and of 3 become C'_3, of 4 become C'_4;

Let the coefficient of induction of the electrometer needle on each pair of quadrants become K_n+kd, of substance 2 on plate 3 become K'_2, and of substance 1 on plate 4 become K'_1; then—

Before reversal, fig. 1,

$$\left.\begin{array}{l}\text{The charge in 3 and in}\\ \quad\text{the pair of attached}\\ \quad\text{quadrants}\end{array}\right\}=AK_1+BC_3+NK_n+BC_q$$

$$\left.\begin{array}{l}\text{The charge in 4 and in}\\ \quad\text{the pair of attached}\\ \quad\text{quadrants}\end{array}\right\}=(A+a)K_2+BC_4+NK_n+BC_q$$

After reversal, fig. 2,

$$\left.\begin{array}{l}\text{The charge in 3 and in}\\ \quad\text{the pair of attached}\\ \quad\text{quadrants}\end{array}\right\}=(A+a)K'_2+(D+d)C'_3+N(K_n+kd)+(D+d)(C_q+cd)$$

$$\left.\begin{array}{l}\text{The charge in 4 and in}\\ \quad\text{the pair of attached}\\ \quad\text{quadrants}\end{array}\right\}=AK'_1+DC'_4+N(K_n+kd)+D(C_q+cd)$$

This is the most general case, it only assumes the electrometer quadrants to be symmetrical with reference to the needle, but allows any amount of unsymmetry in our induction apparatus. In practice, the motion of the electrometer needle is usually in these experiments so small that we may neglect the change in the capacity of the quadrants, and in the coefficient of induction of the needle on them produced by this motion; consequently since the charge in the plate 3 and the attached pair of quadrants is not altered by the reversal

$$AK_1+BC_3+NK_n+BC_q=(A+a)K'_2+(D+d)C'_3+NK_n+(D+d)C_q,$$

similarly

$$AK'_1+DC'_4+NK_n+DC_q=(A+a)K_2+BC_4+NK_n+BC_q$$

$$\therefore aK_2=A(K'_1-K_2)+D(C'_4+C_q)-B(C_4+C_q)$$

$$d(C'_3-C_q)=A(K_1-K'_2)+B(C_3+C_q)-D(C'_3+C_q)$$

In order therefore that *a*, which is the difference of potentials we desire to measure, shall be proportional to *d*, the observed difference of potentials, it is necessary that

$$\text{either } C_3 = C_4 \text{ or } B=0$$

$$\text{and either } C'_3 = C'_4 \text{ or } D=0$$

$$\text{and either} \left\{ \begin{array}{l} K_1 = K'_2 \\ K'_1 = K_2 \end{array} \right. \text{ or } A=0$$

Now the conditions on the left hand side mean perfect symmetry of our induction apparatus before and after reversal. But as this, at any rate in Japan, is practically unattainable, it is desirable to make B, D, and A each nought. B can be made nought and also D very small by not merely connecting plates 3 and 4 together at the commencement of each experiment, but also with the outside of the electrometer or the earth, and we can also make A small by connecting either substance 1 or 2 also with the earth. Such connexions were always made in the following experiments.

In order to rotate the lower substances 1 and 2, it is necessary, if one or both of them be a liquid, to first increase the distance between 1 and 2 and between 3 and 4, in order that 3 and 4 may not strike against the sides of the vessels containing the liquids. In our former apparatus this was done by the table carrying 1 and 2 being supported and capable of revolution on a central leg, which itself was movable up and down (Proc. Roy. Soc., No. 186, vol. xxvii., pl. 7). But when the two substances of which it was desired to measure the difference of potentials were, say, a vessel of mercury and a comparatively thin plate of metal, the difference of weight on the two sides made it almost impossible with this method to obtain sufficient parallelism of the surfaces 1, 2, and 3, 4; so that our former experiments were necessarily confined to pairs of substances not having a great difference in weight. But, as it was obviously desirable that the present investigation should not be thus limited, it became necessary to construct a new apparatus, of which the end elevation is shown in Plate 5, and the side elevation in Plate 4.

Again, in the old apparatus it was necessary to open the metal cover enclosing the whole apparatus in order to lower, rotate, and raise plates 1 and 2. Now we consider that, although theoretically there was no objection to opening this case, provided it was shut again to screen the apparatus from exterior electrical induction before a reading was made, still that, as it introduced a possibility of the plates 3 and 4 being touched during an experiment, this opening was objectionable. In the present arrangement, the table A B, which carries the levelling screws *l l* for supporting the substances of which the contact differences of potential are to be measured (in our figure a liquid, L, and a solid plate, P, about 530 square centimetres in area), runs with three wheels, W, on a circular very rigid metallic horizontal railway, R; and to avoid lateral motion the table is kept centred by a stout iron pin, M, turning in a brass socket, S. The necessary vertical motion is produced in this apparatus by the plates 3 and 4, which are attached to the upper framework with an arrangement similar to that employed with a parallel ruler, being raised or lowered by a vertical brass rod, *r r*, capable of rotation, so as to allow of the pin, *p p*, passing through the slot, *s s* (shown in plan below), or being stopped by it when the rod is required to remain raised. The plates 3 and 4, which are each about 300 square centimetres in area, are supported by chemically clean glass rods, G, kept artificially dry by pumice-stone soaked in sulphuric acid, and resting in the leaden cups, U, sliding into the brass cylindrical tubes, B, with bayonet joints. These leaden cups may be lowered by handles, *h*, so as to rest on the brass plates 3 and 4 when the instrument is not in use, thus preventing access to B of the external air. The permanent adjustment for coincidence of the plane surfaces of the brass plates is effected by means of three screws, *w*, in each of the brass caps, C. Temporary adjustment of the horizontality of the brass plates is effected by means of a spirit level and the three large levelling screws, K, which support the whole apparatus, and the positions of which are fixed by hole, slot, and plane.

T T are strips of brass soldered to the induction plates 3 and 4, and connected by means of clips to fine platinum wires leading to the insulated electrodes of the quadrant electrometer, and to the insulated terminals of a highly insulated short circuit key, not shown in the drawing.

In addition to the whole framework being made very stout and of cherry-wood, one of the Japanese woods that warps least, it was further strengthened by various cross tie-pieces, as shown in the drawings.

The whole apparatus, including the short circuit key and electrometer, was, to avoid induction from outside, enclosed in a large zinc case connected with the earth, and was not opened at all during one complete experiment, consisting of some ten short circuitings of the upper plates, reversals of the table A B, and corresponding readings to the right and left of the electrometer needle.

We shall now describe a complete operation to obtain the contact difference of potentials, between a metal and liquid, for example. Suppose the permanent adjustments to have been made, and the gilt plates 3 and 4 to be quite bright. The plate P is cleaned with emery paper that has touched *no other metal*, and all traces of the emery removed by means of a clean cloth; it is then placed on the three levelling screws, *l*, and fixed in position by hole, slot, and plane. The porcelain dish containing the liquid is laid in a metal one just fitting it, and on the base of which is a hole, slot, and plane; this is now laid on the other levelling screws, *l*.

The rod, *r r*, is then lowered until the disk, *d d*, rests on a brass plate let into the top of the wooden framework at the top of the instrument—that is, until the induction plates 3 and 4 are in their lowest position. The levelling screws, *l l*, are now raised until a small metal ball, of a diameter of 8 millims., is in contact at three fixed points with the plate 4 and the plate P, or until when in contact with the plate 3 it and its reflection in the liquid L appear to meet. To avoid any harm arising from possible contact of the liquid with this gauge ball, it was made of a material not acted on by the particular liquid under experiment.

Before proceeding further, each pair of quadrants is in succession put to earth, the other pair remaining insulated in order to test for any possible leakage from the needles to the quadrants. Each pair of quadrants is now charged with a battery, the other pair being connected with the earth, in order to test for any leakage along the glass rods G, the small glass rods supporting the quadrants in the electrometer, or along the paraffined ebonite pillars of the short circuiting key. It having thus been ascertained that there is no leakage, the strip of metal which has been cut from the same sheet of metal as P itself, and temporarily attached to it by a binding screw soldered to P, is made quite bright with emery paper and a cloth, and its end dipped into the liquid L, as shown in Plate 4. The zinc case is then closed up, plates 3 and 4 connected together, and with the earth, by means of a key (the handle of which was a long thin ebonite rod projecting through the zinc case), and the electrometer reading taken. 3 and 4 are then insulated from one another, and from the earth, and raised by means of the rod, *r r*, projecting above through the zinc case; the table A B turned from below by means of a handle passing through the base of the instrument; 3 and 4 then lowered into exactly their former position, ensured by the parallel motion of the supporting beam and by the limiting stop, *d d*. The reading of the electrometer is now taken. Again short circuit, insulate, raise, reverse, and lower, and take a new electrometer reading, &c.

Some ten readings having been thus obtained, a fresh set of experiments is always made with the same two substances in the following way in order to compensate for the error introduced by defects in parallelism of the apparatus affecting the result obtained from two rigid surfaces (as that of copper and zinc), differently from the result found with one or with two liquid surfaces under test. Instead of commencing, as before, with the liquid L under 3 and the plate P under 4, we start with the plate under 3 and the liquid under 4, and readjust, by means of the levelling screws l, the heights of the surfaces, until their distance from the plates 3 and 4 is, as before, 8 millims. We then short circuit, insulate, raise, reverse, and lower, and take exactly as many readings as before; and the mean of the two sets of readings, obtained with the two modes of levelling, is regarded as the result of the particular experiment.

III.

The results we have already obtained in this present investigation group themselves under three heads:—

1st. The contact difference of potentials of metals and liquids at the same temperature.

2nd. The contact difference of potentials of metals and liquids when one of the substances is at a different temperature from the other in contact with it; for example, mercury at 20° C. in contact with mercury at 40° C.

3rd. The contact difference of potentials of carbon and of platinum with water, and with weak and strong sulphuric acid.

But those contained under head No. 1 are alone contained in the present investigation. The remainder of our completed experiments we desire to have the honour of submitting on a subsequent occasion to the Royal Society.

A fourth very important division of the subject, namely, the measurement of the contact difference of potential of substances in other gases than air, it has been our intention for some time to take in hand; but, although we commenced the working drawings of the apparatus necessary for that investigation at the beginning of 1877, we have not yet had an opportunity of constructing it.

The numbers which follow in the accompanying tables are those to which alone we attach importance, but it need hardly be said that, in such delicate experiments as the present, much time has to be spent in obtaining measurements which are afterwards found out to be wrong for a variety of reasons. All such we have not included in the following tables; and this therefore explains why, in some cases, one measurement only is apparently the result of a whole day's work.

In our previous papers Nos. I. and II. we used as our standard electro-motive force that of a LATIMER CLARK's standard mercurous-sulphate cell, since that was the most constant galvanic cell known to us; and we found experimentally, as Sir WILLIAM THOMSON previously ascertained, that zinc had to copper a contact difference of potentials 0·75 volt. Now, although Mr. LATIMER CLARK's cell is a very constant element, the difference of potentials of zinc and copper is even more constant; we have, therefore, in the present investigation compared all our measurements with this standard, which we have taken to be equal to 0·75 volt.

Date.	The following—	Has to the following—	At—	A contact difference of potentials, in volts.	Remarks.
			° C.		
30/1/78 .	Zinc sulphate, sp. gr. 1·125, at 16°·9 C.	Copper sulphate, sp. gr. 1·087, at 16°·6 C.	16·6	+0·090	Each of the numbers in the preceding column is the average of about ten observations.
	Copper.	„ „	17	+0·103	
	Zinc	Zinc sulphate, sp. gr. 1·125, at 16°·9 C.	17	−0·238	
8/2/78 .	Copper.	Distilled water	..	+0·269	
11/2/78 .	Zinc	„ „	13·5	+0·088	
	Zinc sulphate, saturated at 15°·3 C.	„ „	13·5	+0·120	
	Zinc	„ „	13·5	+0·156	
	Copper.	„ „	16	+0·100	
12/2/78 . 10 A.M.	Copper sulphate, saturated at 15° C.	„ „	13	+0·044	The contact difference of potentials of saturated copper sulphate with distilled water was here measured several times on different occasions, as a test of the efficiency of our method of measuring the difference of potentials of *two liquids in contact*. The comparative constancy of the numbers obtained on the different occasions, 0·044, 0·052, 0·040, 0·040, 0·04 volt (each being the mean of some ten observations), proves that our method is practically accurate.
	Zinc sulphate, saturated at 15°·3 C.	„ „	14·5	+0·190	
11 A.M. .	Copper sulphate, saturated at 15° C.	„ „	14·6	+0·052	
3.15 P.M.	„ „	„ „	16	+0·040	
7.35 P.M.	„ „	„ „	15	+0·040	
15/2/78 .	Copper sulphate, saturated at 15° C.	„ „	16	+0·041	
18/2/78 .	Zinc sulphate, saturated at 15°·3 C.	Copper sulphate, saturated at 15° C.	14	+0·090	
	„ „	Distilled water	15·5	+0·184	
19/2/78 .	Copper.	Copper sulphate, saturated at 5° C.	15·6	+0·070	
	„	Distilled water	16	+0·150	
	Zinc	„ „	17·4	−0·105	
	„	Zinc sulphate, saturated at 17° C.	17	−0·457	
	Copper.	Tin.	17	−0·456	
19/2/78 .	„	Lead	18	−0·542	
	Lead	Zinc	18·7	−0·210	
	Zinc	Tin.	19	+0·281	
	Copper	Iron	18	−0·146	
20/2/78 .	Lead	Tin.	17·8	+0·099	
21/2/78 .	Iron	Water	13	+0·148	
	Platinum	„	14	+0·310	
	„	Nitric acid	17	+0·672	Difficult to measure in consequence of the film which condensed on the gilt induction plates.
	Zinc	Distilled water with a slight trace of sulphuric acid.	17	−0·241	
	„	Distilled water with about ⅕ per cent. of sulphuric acid.	18	−0·289	
	„	Distilled water with about ⅓ per cent. of sulphuric acid.	19	−0·312	
	„	1 of sulphuric acid and 20 of distilled water by weight.	18·8	−0·344	The value slowly increased from the formation of a salt on the surface of the zinc.
	„	1 of sulphuric acid and 30 of distilled water by weight.	19	−0·344	The zinc slowly blackened, but not so quickly as before.
22/2/78 .	„	Solution sea-salt, sp. gr. 1·18, at 20°·5 C.	19	−0·506	
23/2/78 .	Copper	„ „	15	−0·475	
25/2/78 .	Lead	Distilled water	17	+0·184	
	„	Strong sulphuric acid.	17·8	+0·720	
26/2/78 .	Zinc	Sal-ammoniac, saturated at 15°·5 C.	14	−0·637	
	Copper	„ „	15	−0·396	
	„	Strong sulphuric acid.	17	+1·056	
	Lead	„ „	18	+0·720	
	Platinum	„ „	18	+1·424	
	„	Lead	18	−0·771	
27/2/78 .	Copper	Platinum	15	+0·238	
	Platinum	Zinc	16·4	−0·981	
	„	Iron	17	−0·369	
	„	Tin.	17	−0·690	

Date.	The following—	Has to the following—	At—	A contact difference of potentials, in volts.	Remarks.
			° C.		
28/2/78	Copper	Alum, saturated at 16°·5 C.	15	−0·243	Each of the numbers in the preceding column is the average of about ten observations.
	Zinc	" "	17·8	−0·536	
	Lead	Sal-ammoniac, saturated at 17° C.	18	−0·189	
	Iron	" "	18	−0·652	
	Lead	1 of sulphuric acid, 5 of distilled water.	17	−0·120	
1/3/78	"	Distilled water	17	+0·167	
	"	Alum, saturated at 16°·5 C.	16	−0·139	
	Iron	" "	16	−0·653	
4/3/78	Lead	Distilled water	15·5	+0·163	
	"	Solution sea-salt, saturated at 17° C.	17	−0·267	
	Iron	" "	18	−0·605	
	Zinc	" "	15·4	−0·624	
	Copper	" "	16	−0·524	
5/3/78	Lead	Alum, saturated at 16°·5 C.	16	−0·139	
	Iron	" "	16	−0·653	
6/3/78	"	Mercury	19·4	+0·501	
	Copper	"	19	+0·378	
7/3/78	"	Brass	19	−0·151	
	Zinc	"	18	+0·679	
	Iron	"	19	+0·064	
	Lead	"	18·5	+0·472	
	Tin	"	18·5	+0·372	
	Platinum	"	18·6	−0·287	
	"	Solution sea-salt, saturated at 17° C.	18	−0·856	
	Tin	" "	17·8	−0·304	
	"	Sal-ammoniac, saturated at 15°·5 C.	18	−0·364	
8/3/78	Platinum	" "	17	+0·057	
	"	Alum, saturated at 16°·5 C.	17	+0·246	
	Tin	" "	17	−0·225	
9/3/78	"	Distilled water	14·5	+0·177	
	Iron	Mercury	16	+0·503	
	Lead	"	15	+1·00 rapidly diminishing to 0	
11/3/78	Tin	1 of distilled water, 5 of sulphuric acid by weight.	16	−0·256	
	Brass	" "	15	+0·016	
	"	Distilled water	14	+0·207	
	"	Solution sea-salt, saturated	14·5	−0·435	
	"	Alum, saturated	14·5	−0·014	
	"	Sal-ammoniac, saturated	14	−0·335	
12/3/78	Tin	Distilled water	16	+0·177	
	Platinum	Mercury	17	+0·204	

Here a number of apparently very discordant results were obtained when experimenting with mercury. The explanation at length was found to be due to the apparent contact difference of potentials between a metal and mercury as measured inductively, varying much with small alterations of temperature. Now as the mercury was redistilled in the laboratory between every two experiments to remove all traces of impurities, probably in some cases it had not become perfectly cold before a new experiment was made, and hence the apparent irregularities in the results. As, however, this investigation forms the subject of a future paper, it is unnecessary to say more now than to remark that in all the results mentioned in this communication, the mercury was not used until its temperature had become approximately that of the air.

Date.	The following—	Has to the following—	At—	A contact difference of potentials, in volts.	Remarks.
13/3/78	Platinum	Mercury	16	+0·129	
	Copper.	"	16	+0·316	
14/3/78	"	"	15	+0·254	
	"	Very clean mercury	15	+0·312	
15/3/78	Zinc	Saturated zinc sulphate	16	−0·420	
	Saturated zinc sulphate	Saturated copper sulphate	16	+0·095	
	Zinc	1 of distilled water, 3 of saturated zinc sulphate.	16	−0·444	

Date.	The following—	Has to the following—	At—	A contact difference of potentials, in volts.	Remarks.
			° C.		
15/3/78 .	Saturated copper sulphate .	1 of distilled water with 3 of saturated solution of zinc sulphate.	16	−0·102	Each of the numbers in the preceding column is the average of about ten observations.
	„ „	1 of distilled water with 1 of saturated solution of zinc sulphate.	16	−0·093	
	Zinc	Saturated zinc sulphate . .	16	−0·140	
	Copper.	Alum, saturated	15	−0·109	

As a small leakage was now observed between the needle of the electrometer and the quadrants this instrument was taken to pieces and cleaned.

18/3/78 .	Copper.	Mercury	13·5	+0·351	
19/3/78 .	Iron	„	13·5	+0·473	
	Copper.	Alum, saturated	15	−0·145	
	Platinum.	Strong sulphuric acid . .	15	+1·587	
	Lead	„ „	14·5	+1·252	
	Copper.	„ „	15	+1·170	
20/3/78 .	Platinum.	Mercury	15·5	+0·198	
	Brass	Distilled water	15	+0·256	
26/3/78 .	„	Sal-ammoniac, saturated at 15°·5 C.	17	−0·361	From the 26th March to the 8th April occupied with experiments on the contact difference of potentials of hot and cold mercury.
8/4/78 .	„	Zinc	15·7	−0·697	

IV.

We now endeavoured to measure the contact differences of potential of each pair of constituents in a LATIMER CLARK'S mercurous sulphate cell. First we tried mercury and mercurous sulphate paste, the mercury being put into one of our porcelain dishes and the mercurous sulphate paste into the other, the connexion between the two being made by a small glass siphon filled with the mercurous sulphate paste and the earth connexion by a platinum wire dipping into the paste in the porcelain dish. Many days were spent unsuccessfully at this experiment, and at similar ones which follow, on account of the thin layer of water which floated on the top of the mercurous sulphate, even after great care had been taken to remove it with a pipette. Often, therefore, instead of measuring the contact difference of potentials between a substance A and the mercurous sulphate paste, we obtained that of A with the paste plus that of the paste with water. At length, however, we obtained the following, each number given being, as usual, the mean of many experiments. Very great care was taken to have the materials chemically pure, the mercury being frequently redistilled; also to prevent any discrepancies in these results being caused by any possible electrification of the porcelain vessels themselves, they were thoroughly de-electrified after cleaning and before using.

It may be noticed that this difficulty experienced by the thin layer of water which floated on the mercurous sulphate paste is a very good example of the inaccuracies that must have been introduced by former experimenters using a moist blotting paper surface instead of the surface of a liquid itself. (See page 15, "Reasons for Incompleteness of Early Experimenters.")

Date.	The following—	Has to the following—	At—	A contact difference of potentials, in volts.	Remarks.
			° C.		
12/4/78	Mercury	Mercurous sulphate paste	15	−0·092	Connexion between the mercury and the paste made by a small glass siphon tube filled with the paste. Connexion with the earth made by a platinum wire dipping in the paste in the last four, and with an iron wire dipping into the mercury in the first case.
23/4/78	"	" "	17	+0·405	
	"	" "	17	+0·420	
24/4/78	"	" "	19	+0·360	
25/4/78	"	" "	17	+0·470	
13/4/78	Saturated solution zinc sulphate	" "	16	+0·103	Connexion between the liquid and the paste formed by a tube filled with the paste, but neither were connected with the earth.
	" "	" "	17	+0·106	Zinc sulphate solution connected with the earth by a platinum wire, the connexion with the paste remaining as before.
23/4/78	" "	" "	18	+0·740	" " "
24/4/78	" "	" "	18	+0·900	" " "
	" "	" "	18	+0·720	" " "
27/4/78	" "	" "	19	+0·100	Connexion between the liquid and the paste formed by a very clean cotton wick previously soaked in the saturated solution of pure zinc sulphate. Connexion with the earth made by a platinum wire dipping into the porcelain vessels containing the zinc sulphate.
	" "	" "	19	+0·080	
	" "	" "	19	+0·120	
		" "	19	+0·090	

In order to ascertain whether the high or the low value for the constant difference of potentials of zinc sulphate with mercurous sulphate was correct, an *incomplete* cell of zinc sulphate solution, mercurous sulphate paste, and mercury was formed, and the electro-motive force measured, the arrangement employed consisting of the zinc sulphate solution being put in one of the porcelain dishes, and in the other the mercury, the two being connected with a small beaker containing mercurous sulphate paste, and which did not act inductively on the induction plates. The connexion between the zinc sulphate and the mercurous sulphate was made by a clean cotton wick previously soaked in a saturated solution of the former, and the connexion between the mercury and the paste in the beaker by a small glass siphon tube filled with the latter. Earth connexion in the first experiment recorded was made by a platinum wire dipping into the mercury, afterwards by a platinum wire dipping into the zinc sulphate instead of into the mercury.

SATURATED zinc sulphate and mercury connected by mercurous sulphate paste.

Date.	Temperature.	Difference of potentials in volts.
24/4/78	18°	+0·725
25/4/78	17°	−0·330
27/4/78	19°	+0·770

Now the mean of the low set of values obtained previously with zinc sulphate and mercurous sulphate is +0·100, and of the high values +0·787; the mean of the values previously obtained with mercury and mercurous sulphate is +0·414, and a single value −0·092. In order to obtain the electro-motive force of the incomplete cell we must subtract the two latter from the two former, giving us

−0·314 or +0·879,

which are practically equal to the extreme values obtained directly for the electro-motive force of the incomplete cell. It therefore seems to be tolerably certain that the variable results obtained when experimenting with mercurous sulphate paste are due to the uncertainty of the nature of the bounding surface.

The mercurous sulphate paste and the saturated zinc sulphate solution were now boiled together, as is done in preparing Mr. LATIMER CLARK's constant element. Using this mixture in one porcelain vessel and mercury in the other, and joining them with a small glass siphon filled with the mixture, we obtained

		Volts.	Mean of.
9/5/78	Mercury has to Mixture, 22° C.	−0·210	10
11/5/78	„ „ „	+0·072	10

Wetting the surface of the mixture with a little saturated zinc sulphate solution brings up the answer to +0·340, the latter being the mean of ten observations all giving nearly equal results. The following contact difference of potentials was now measured.

		Volts.
11/5/78	Amalgamated zinc has to Mixture	−0·840
	„ „ „	−0·930
	„ „ „	−0·740

A little saturated zinc sulphate solution being poured over the mixture and taken out again carefully with a pipette altered the result to +0·520.

The following incomplete cell consisting of mercury and amalgamated zinc joined by the mixture of saturated zinc sulphate and mercurous sulphate paste previously referred to was now made, and the mean electro-motive force was found to be about +0·200 volt, the mercury being positive to the amalgamated zinc; this is roughly equal to the first value obtained with mercury and the mixture added to the last value obtained with the mixture and amalgamated zinc.

It was found that

			Volts.
26/4/78	Amalgamated zinc had to Copper	18° C.	+0·894

From which it follows that

Amalgamated zinc had to Platinum „ =+1·125

Therefore the electro-motive force of a LATIMER CLARK'S cell should be

$$\begin{array}{lr} & \text{Volts.} \\ \overline{\text{(Platinum, Mercury)}} & =+0{\cdot}156 \\ \overline{\text{(Mercury, Mixture, Amalgamated zinc)}} & =+0{\cdot}200 \\ \overline{\text{(Amalgamated zinc, Platinum)}} & =+1{\cdot}125 \\ \hline & 1{\cdot}481 \end{array}$$

The real value of the electro-motive force of the cell being about 1·46 volts.

Date.	The following—	Has to the following—	At—	A contact difference of potentials, in volts.	Remarks.
			° C.		
26/4/78 .	Amalgamated zinc	Copper	18	+0·894	
	„ „	Tin	18	+0·463	
	„ „	1 of sulphuric acid with 5 of distilled water by weight.	18	−0·429	
	„ „	„ „		−0·431	
	„ „	1 of sulphuric acid with 10 of distilled water by volume.	18	−0·858	Wiping the zinc surface increases the reading, showing that the half-liquid surface of the amalgam introduces some indefiniteness.
	„ „	„ „	18	−0·424	
	„ „	Distilled water . . .	18	+0·090	Readings go down rapidly, but wiping the strip of amalgamated zinc dipping into the water brings them up again.
	„ „	Strong sulphuric acid .	18	+0·848	As the zinc was rapidly acted on slightly by the strong sulphuric acid, it required re-amalgamating after every few observations.
27/4/78 .	„ „	Saturated zinc sulphate	19	−0·284	No falling off in the readings.
The induction plates were now regilt.					
12/5/78 .	Carbon	Zinc	21·5	−0·045	
	„	Copper	21·5	−0·370	
	„	Lead	21·5	−0·858	
	„	Mercury	21·5	+0·092	
	Platinum . . .	„	21·5	−0·143	
15/5/78 .	Carbon	Zinc	22	−1·143	Carbon slightly warm, having been washed and dried by heating.
The experiments of carbon with liquids constitute a separate paper.					
21/5/78 .	Distilled water .	Sulphuric acid . . .		+1·319 rising to +1·600	

The connexion between the liquids was made by a glass rod dipped previously into distilled water, and the legs of which rested one in each of the porcelain dishes. During each of the experiments the readings gradually rose, but if the glass rods were taken out, redipped in distilled water, and replaced, they returned to their original value. In case this might be due to the drying up of the film of water on the surface of the connecting glass rod, the connexion between the liquids after the next experiment was made in the following way. In addition to the wetted glass rod, the legs of which dipped into the two porcelain dishes, there were twisted round it a few clean cotton threads, which had been previously soaked in distilled water or in the non-acid liquid employed, alum, copper sulphate, &c. The threads were of such a length that one of their ends dipped into the dish containing the non-acid liquid, while the other descended nearly, but not quite, to the acid.

Date.	The following—	Has to the following—	At— ° C.	A contact difference of potential, in volts.	Remarks.
21/5/78 .	Sulphuric acid . .	Saturated zinc sulphate		−1·699	After long delay goes up slightly.
22/5/78 .	Distilled water .	Sulphuric acid . . .	22	+1·278 rising slightly	
	Saturated solution alum.	„ „ . . .	22	+1·458	
	Saturated solution copper sulphate	„ „ . . .	22	+1·269	Increased a very little in two hours.

V.

As a test of the accuracy of the measurements we may see how nearly for any three metals A, B, C, the sum

$$\overline{AB}+\overline{BC}+\overline{CA}$$

equals nought, as it ought to, since it is well known that in any compound metallic circuit at uniform temperature there is no electro-motive force. In order that the test may be the more rigid we will employ only the measured contact differences of potential, and not the calculated ones or those marked with an asterisk in the table of contact differences of potential of solids with solids.

			Volts.	Sum.
1.	$\overline{(\text{Copper, Lead})}+\overline{(\text{Lead, Platinum})}$	=	+0·229	
	$\overline{(\text{Platinum, Copper})}$	=	−0·238	−0·009
2.	$\overline{(\text{Copper, Zinc})}+\overline{(\text{Zinc, Lead})}$	=	−0·540	
	$\overline{(\text{Lead, Copper})}$	=	+0·542	+0·002
3.	$\overline{(\text{Iron, Brass})}+\overline{(\text{Brass, Copper})}$	=	+0·151	
	$\overline{(\text{Copper, Iron})}$	=	−0·146	+0·005
4.	$\overline{(\text{Zinc, Platinum})}+\overline{(\text{Platinum, Copper})}$	=	+0·743	
	$\overline{(\text{Copper, Zinc})}$	=	−0·750	−0·007
5.	$\overline{(\text{Zinc, Platinum})}+\overline{(\text{Platinum, Lead})}$	=	+0·210	
	$\overline{(\text{Lead, Zinc})}$	=	−0·210	0·000
6.	$\overline{(\text{Iron, Copper})}+\overline{(\text{Copper, Brass})}$	=	+0·059	
	$\overline{(\text{Brass, Iron})}$		−0·064	−0·005

Now if it be remembered that the various experiments with the different metals, although made at an average temperature of 18° C., were, as mentioned in the various tables, made at somewhat different temperatures the one from the other, it will be seen that the previous six sums cannot be absolutely nought. We may therefore regard them as sufficiently small to prove a considerable amount of accuracy in the experiments.

The numbers given in the two large tables for the contact differences of potential of solids with solids, solids with liquids, and liquids with liquids might be employed to build up the electro-motive forces of well known one and two fluid galvanic cells. As however the summation law referred to at the commencement of this paper may be considered as having been established in our two previous papers on the contact theory of voltaic action, it is not necessary in the present instance to take further examples of it.

In all our experiments two air contacts enter into our measurements, and the summation law gives no indication of the value of the contact difference of potentials at these air contacts. For example, suppose it be desired to measure the difference of potentials between two substances A and B in contact, either or both of which may be solid or liquid, then in reality what we measure is

$$(\overline{\text{Air, A}})+\overline{\text{AB}}+(\overline{\text{B, Air}}).$$

If now we measure by the inductive method the difference of potentials between B and C in contact, then what we really obtain is

$$(\overline{\text{Air, B}})+\overline{\text{BC}}+(\overline{\text{C, Air}}),$$

and the sum of these two we have proved in all cases to be equal to what is obtained if A and C be joined by B and the difference of potentials between A and C measured. What really we have proved is therefore that

$$(\overline{\text{Air, A}})+\overline{\text{AB}}+(\overline{\text{B, Air}})+(\overline{\text{Air, B}})+\overline{\text{BC}}+(\overline{\text{C, Air}})=(\overline{\text{Air, A}})+\overline{\text{AB}}+\overline{\text{BC}}+(\overline{\text{C, Air}}).$$

And this would be the case whatever the values of the difference of potentials at the air contacts, provided they remained constant, so that

$$(\overline{\text{B, Air}})=-(\overline{\text{Air, B}}).$$

One way, and the way we hope shortly to employ, to enable us to determine the electro-motive force of contact, in volts, of a substance and a gas is by repeating all these exact contact experiments in different gases, for if we measure by our inductive method the contact difference of potentials of A and B in a gas G then we shall obtain

$$\overline{\text{GA}}+\overline{\text{AB}}+\overline{\text{BG}},$$

and the difference between this and the result of the first measurement will be

$$(\overline{\text{Air, A}})+(\overline{\text{B, Air}})-\overline{\text{GA}}-\overline{\text{BG}}.$$

Consequently, if such experiments be made and the exact electro-motive forces of contact measured in a number of gases, we shall ascertain whether the contact difference of potentials of a substance and a gas differs much with different gases. Qualitative experiments in this direction have already been made with very interesting results by Mr. Brown, of Belfast, but his experiments differ from ours in not being quantitative.

If the gas measurements such as we have indicated be extended to a good Crookes' vacuum we may then possibly approximate to the real value of $\overline{\text{AB}}$, a contact difference of potentials of A with B, the value that we should obtain by a measurement of the Peltier effect. It is therefore our purpose to extend these quantitative measurements not only to other gases than air but to a Crookes' vacuum, as well as to measure the Peltier effects for a number of contacts of dissimilar substances.

We have to thank two of the students of telegraphic engineering at the Imperial College of Engineering, Japan, Messrs. T. Iwata and N. Nakayama, for considerable assistance rendered us in the carrying out of this somewhat laborious investigation.

MEAN contact differences of potential in volts; solids with liquids, and liquids with liquids.

		Carbon.	Copper.	Iron.	Lead.	Platinum.	Tin.	Zinc.	Amalgamated zinc.	Brass.	Mercury.	Distilled water.	Alum solution, saturated at 15° C.	Copper sulphate solution, saturated at 15° C.	Zinc sulphate solution, saturated at 15°·3 C.	Zinc sulphate solution; specific gravity, 1·125 at 16°·9 C.	1 distilled water, 3 saturated solution zinc sulphate.	Strong nitric acid.
	Mercury	·092	·308	·502	..	·156	..	..	..	..	..	..	..	..	..	..	..	..
	Distilled water	·01 to ·17*	·269 to ·100	·148	·171	·285 to ·345	·177	−·105 to +·156	·100	·231	..	..	..	+·043	·164	..	..	..
Solutions.	Alum, saturated at 16°·5 C.	..	−·127	−·653	−·139	·246	−·225	−·536	..	−·014	..	..	..	..	..	..	..	..
	Copper sulphate, saturated at 15° C. . .	..	·070	..	..	..	..	..	..	..	..	−·043	..	..	·095	..	·102	..
	Copper sulphate solution; specific gravity, 1·087 at 16°·6 C. .	..	·103	..	..	..	..	..	..	..	..	..	..	..	..	·090	..	..
	Sea salt; specific gravity, 1·18 at 20°·5 C. . .	..	−·475	−·605	−·267	−·856	−·334	−·565	..	−·435	..	..	..	..	..	..	..	..
	Sal ammoniac, saturated at 15°·5 C. . . .	..	−·396	−·652	−·189	·057	−·364	−·637	..	−·348	..	..	..	..	..	..	..	..
	Zinc sulphate, saturated at 15°·3 C. . . .	..	..	..	..	..	..	−·430	−·284	..	..	−·164	..	−·095	..	..	..	..
	Zinc sulphate solution; specific gravity, 1·125 at 16°·9 C. . . .	..	..	..	..	..	..	−·238	..	..	..	..	..	..	..	..	..	..
	1 distilled water, mixed with saturated solution zinc sulphate .	..	..	..	..	..	..	..	..	..	..	..	..	−·098	..	..	..	..
	1 distilled water, mixed with 3 saturated solution zinc sulphate .	..	..	..	..	..	..	−·444	..	..	..	..	..	−·102	..	..	..	..

---

* Depending on the carbon.

Mean contact differences of potential in volts; solids with liquids, and liquids with liquids—*continued.*

		Carbon.	Copper.	Iron.	Lead.	Platinum.	Tin.	Zinc.	Amalgamated zinc.	Brass.	Mercury.	Distilled water.	Alum solution, saturated at 15° C.	Copper sulphate solution, saturated at 15° C.	Zinc sulphate solution, saturated at 15°·3 C.	Zinc sulphate solution; specific gravity, 1·125 at 16°·9 C.	1 distilled water, 3 saturated solution zinc sulphate.	Strong nitric acid.
Dilute sulphuric acid.	Distilled water, with a slight trace of sulphuric acid . . .	..	..	..	..	..	..	−·241	..	..	..	..	..	..	..	..	..	·078
	Distilled water, with about one-fifth per cent. of sulphuric acid	..	..	..	..	..	..	−·289	..	..	..	..	..	..	..	..	..	..
	Distilled water, with about one-third per cent. of sulphuric acid	..	..	..	..	..	..	−·312	..	..	..	..	..	..	..	..	..	..
	By weight— 20 distilled water, 1 strong sulphuric acid . . .	..	..	..	..	..	..	−·344	..	..	..	..	..	..	..	..	..	..
	By volume— 10 distilled water, 1 strong sulphuric acid . . .	About −·035	..	..	..	..	..	..	−·358	..	..	..	..	..	..	..	..	..
	By weight— 5 distilled water, 1 strong sulphuric acid . . .	..	..	..	..	..	..	..	−·429	..	..	..	..	..	..	..	..	..
	1 distilled water, 5 strong sulphuric acid . . .	·01 to ·3	..	..	−·120	..	−·256	..	..	·016	..	..	..	..	..	..	..	..
Concentrated.	Sulphuric acid . . .	·55 to ·85*	1·113	..	·720 to 1·252	1·600 to 1·300	..	..	·848	..	..	1·298	1·456	1·269	1·699	..	..	..
	Nitric acid	..	..	..	..	·672	..	..	..	..	..	..	..	..	..	..	..	..
	Mercurous sulphate paste	..	..	..	..	..	..	..	..	..	·475	..	..	..	..	..	..	..

* Depending on the carbon.

The average temperature at the time these experiments were made was about 16° C. All the liquids and salts employed were chemically pure; the solids, however, were only commercially pure. The numbers in a vertical column below the name of a substance are the differences of potential, in volts, between that substance and the substance in the same horizontal row as the number, the two substances being in contact. Thus lead is positive to distilled water, and the contact difference of potentials is 0·171 volt.

MEAN contact differences of potential in volts; solids with solids.

	Carbon.	Copper.	Iron.	Lead.	Platinum.	Tin.	Zinc.	Amalgamated zinc.	Brass.	
Carbon	0	·370	·485*	·858	·113	·795*	1·096	1·208*	·414*	The average temperature at the times these experiments were made was about 18° C.
Copper	— ·370	0	·146	·542	— ·238	·456	·750	·894	·087	
Iron	— ·485*	—·146	0	·401*	— ·369	·313*	·600*	·744*	—·064	
Lead	— ·858	—·542	—·401*	0	— ·771	—·099	·210	·357*	—·472	
Platinum . . .	— 113*	·238	·369	·771	0	·690	·981	1·125*	·287	
Tin	— ·795*	—·456	—·313*	·099	— ·690	0	·281	·463	—·372	
Zinc	—1·096	—·750	—·600*	—·210	— ·981	—·281	0	·144	—·679	
Amalgamated zinc	—1·208*	—·894	—·744	—·357*	—1·125*	—·463	—·144	0	—·822*	
Brass	— ·414	—·087	·064	·472	— ·287	·372	·679	·822*	0	

The numbers without an asterisk were obtained directly by experiment, those with an asterisk by calculation; using the well-known assumption that in a compound circuit of metals all at the same temperature there is no electromotive force.

The numbers in a vertical column below the name of a substance are the differences of potential, in volts, between that substance and the substance in the same horizontal row as the number, the two substances being in contact. Thus lead is positive to copper, the electro-motive force of contact being 0·542 volt.

The metals were those of commerce, and therefore not chemically pure.

III. *Researches on the Comparative Structure of the Cortex Cerebri.*

By W. BEVAN LEWIS, *L.R.C.P.* (*Lond.*), *Senior Assistant Medical Officer, West Riding Lunatic Asylum, Wakefield.*

Communicated by Dr. FERRIER, *F.R.S.*, *Professor of Forensic Medicine, King's College, London.*

Received June 18,—Read June 19, 1879.

[PLATES 6, 7.]

THE object and scope of this paper are to detail the results of a full investigation into the minute structure of the cerebral cortex in the Pig, and to add such notes upon the histology of the same structure in the Sheep and Cat as will suffice for a fair comparative view of those divergencies in fundamental structure which present themselves between the brain of these animals and that of the highest members of the Mammalian series. Attention has been especially directed to the greater mass formed by the parietal, frontal, and upper arc of the limbic lobe,* the inferior arc of the limbic lobe (gyrus hippocampi) and the olfactory lobe being left for subsequent examination. The method adopted has been that of slicing the hemispheres of fresh brain from end to end upon the freezing microtome, and examining each individual section, both in the fresh state and after preservation, by a method already described.† Tables containing details of the dimensions of cells and depth of layers accompany the paper, being collated at the end for convenience of reference.

THE CONVOLUTIONS OF THE BRAIN IN THE PIG.

The regional distribution of ganglionic cells in the cortex of this animal constitutes so important a portion of our inquiry that, in order to avoid any obscurity in the subsequent sketch, it will be advisable to review briefly the arrangement of the convolutions and sulci. In doing so I shall follow the terminology adopted by Professor BROCA, in his late important work on the comparative anatomy of the convolutions in Mammals.‡

---

* The limbic lobe consists of the convolution of the corpus callosum, together with the gyrus hippocampi (BROCA).

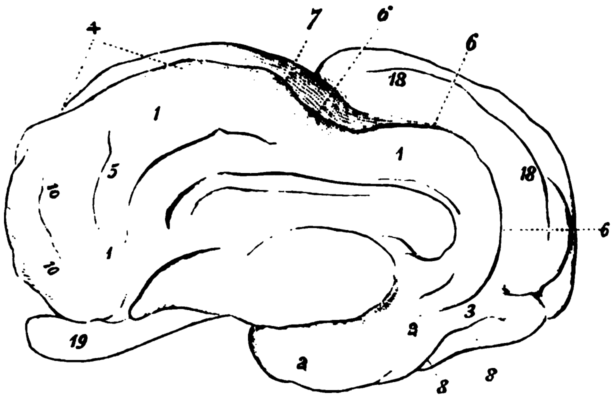
† "Application of Freezing Methods to the Microscopic Examination of the Brain." 'Brain,' part 3, October, 1878.

‡ "Anatomie Comparée des Circonvolutions," par M. PAUL BROCA. 'Revue d'Anthropologie.'

The brain of the Pig presents us with four primary divisions or lobes: 1, the great limbic lobe; 2, the parietal lobe; 3, the frontal lobe; 4, the olfactory lobe.

1. *The great limbic lobe.*—This lobe, seen chiefly on the internal aspect of the hemisphere, consists of a superior arc (convolution of the corpus callosum), fig. 1 (1, 1), commencing in front of the corpus callosum where the internal root of the olfactory unites with it, bends round the anterior extremity or genu of this great commissure, and borders its upper aspect as far as its posterior extremity, where it becomes continuous with the inferior arc of the same lobe, or the gyrus hippocampi, fig. 1 (2, 2), and from which it is marked off by a well-developed retro-limbic annectant (3). The anterior half of the upper arc becomes superficial and exposed upon the surface of the hemisphere, forming here the border of the great longitudinal fissure (4). The limbic fissure, separating this important lobe from the remainder of the hemisphere, may be traced in front as a somewhat obscure sub-frontal sulcus (5), and further back as a deeply-marked sub-parietal sulcus (6), interrupted posteriorly by the retro-limbic annectant (3). Beyond the latter annectant, it can be traced between the parietal lobe and the inferior arc of the limbic lobe, *i.e.*, the gyrus hippocampi (8). The anterior extremity of the sub-parietal sulcus turns upwards towards the surface of the hemisphere, and is continued obliquely forwards and outwards as a well-marked crucial sulcus, separating the superficial portion of the limbic lobe from the parietal lobe (7). The great comparative depth of the anterior portion of the superior arc, when contrasted with its posterior half, is well marked in this animal—a feature identified by **Broca** as of constant occurrence in "Osmatic" Mammals, or those in whom the olfactory apparatus is highly developed.

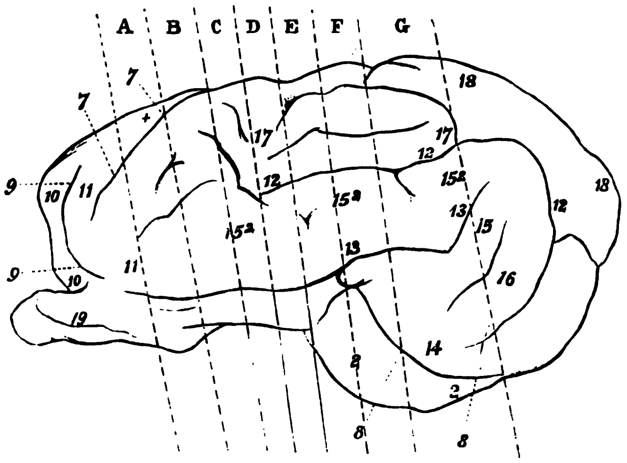
Fig. 1.



2. *The parietal lobe.*—Turning our attention to the external aspect of the hemispheres (extra-limbic mass of **Broca**), we find by far the greater portion enters into the constitution of the great parietal lobe. In this extra-limbic portion we recognise readily three primary fissures—the crucial, interparietal, and Sylvian. Still further forwards, at the extreme anterior pole of the hemisphere, is a shallow vertical fissure—

that of Rolando. The crucial sulcus, figs. 1 and 2 (7), separates the parietal from the anterior exposed part of the limbic lobe, figs. 1 and 2 (4); the fissure of Rolando, fig. 2 (9), separates the latter from the frontal lobe, fig. 2 (10); whilst the interparietal, fig. 2 (12), runs between the upper and lower series of parietal convolutions. The Sylvian fissure, fig. 2 (13), is directed obliquely upwards and backwards, the lower parietal gyri arching around the fissure. All the parietal convolutions take origin from a region termed by Broca the temporal lobule of the parietal lobe, and, after winding around the fissure of Sylvius, terminate near the fissure of Rolando, in a region corresponding to the ascending parietal convolution of higher Mammals, fig. 2 (11).

Fig. 2.



Above the interparietal fissure we find the third and fourth parietal convolutions, the latter being also called the sagittal, figs. 1 and 2 (18). The sagittal gyrus is almost invariably split up posteriorly by secondary sulci. The first parietal or Sylvian convolution, although separated from the second parietal behind the fissure of Sylvius, fig. 2 (15, 16), is in front of that fissure blended with the second parietal often as far as the gyrus post-Rolandique (11).

3. *The frontal lobe.*—This lobe is limited to the extreme anterior tip of the hemisphere and its inner aspect as far as the sub-frontal sulcus. The portion appearing upon the outer surface of the hemisphere anteriorly is the representative of the ascending frontal convolution (10), and is separated from the parietal lobe by the fissure of Rolando. Its inner surface is separated from the great limbic lobe by the sub-frontal sulcus, fig. 1 (5), or anterior extremity of the limbic fissure.

Lamination of the Cortex of the Brain in the Pig.

Number and arrangement of layers.—The general arrangement of the varied series of cells, constituting the greater portion of the cortex of the brain, is very similar to that which we find in animals which rank highest in the Mammalian series. The nerve-cells are distributed either in five or six layers, the five-laminated type being as characteristic of a certain constant area as the six-laminated arrangement is of another region of the brain. As in Primates and Carnivora, we find a limited portion of the cortex organically specialised by its disposition into five layers, and a more extensive realm distinguished by a six-fold lamination, so here in the Ungulata the same characters are impressed upon the cortex. A still more striking resemblance is presented by the character of the individual layers, which are not alone identical in number with those of the brain in higher groups of animals, but are likewise, to a great extent, identical in composition. Thus we find in the cortex of the Pig the first or pale external layer, the layer of small angular cells, the third or large pyramidal layer, the fourth layer of small angular and pyramidal cells (absent in five-laminated areas), the ganglionic series of cells, and, lastly, the sixth or spindle layer. Neither in the number of layers nor in the individual constitution of these layers must we expect to find any great distinctions between these distant groups of the Mammalian class. It is in the essential characters of the individual cells of these layers, in the relationship of these anatomical units the one to the other, and in their general distribution, that we detect divergence from the type normal to the higher Mammalia. These divergencies, we shall find, appear very strongly pronounced as further on we enter into a close examination of the constitution of each individual layer of the cortex. It is important that we bear in mind the homologous constitution of the cortical layers in distant groups of Mammals, and that we are alive to the fact that minute differences in elementary distribution may afford suggestive hints as to any existing analogy in functional endowments. It will be apparent, upon the most superficial examination of this subject, that it is of the highest importance that we define as accurately as possible the regions characterised by these diverse laminar arrangements; and the contrast established between the comparative area of these realms in different orders must be regarded as highly desirable. What is the relative area borne by the five- to the six-laminated region in individual groups of animals, and what is the relative extent of similarly laminated regions in different orders? These are questions which require careful and deliberate consideration, and towards the solution of which the following researches are devoted. As I have elsewhere indicated,* the great distinctions betwixt the two chief laminated types of the cerebral vault centre about the mid-regions of the cortex, and hence the ganglionic layer and the layer superimposed will invite in these investigations especial attention. Let it then be understood at the outset, that the ganglionic cells of the cortex are arranged upon two distinct and readily recognisable plans. These may be termed—1st, the grouped, nested, or clustered arrangement; 2nd, the laminar or solitary arrangement. The first is typical of a more limited area of the cortex than the latter, and is, as far as my investigations lead me, especially characteristic of the motor area of FERRIER—at least, as regards Man, the Monkey, the

* "On the Comparative Structure of the Cortex Cerebri." 'Brain,' part 1, April, 1878.

Cat, Dog, and allied animals. In these latter regions the ganglionic cells are clustered one above the other, occupying oval areas forming the well-known "nests" of **Betz**, and usually separated by short intervals from one another (Plate 7). The second arrangement, however, reminds one of the characters exhibited by this series of cells at the bottom of a sulcus. Here (in the sulcus) we find solitary cells at great comparative distances apart, arranged irregularly along the floor of the sulcus, forming in vertical sections a well-marked *linear* series (Plate 7). Not unfrequently two cells lie side by side, but no groups of such elements are discoverable in these localities; hence the depth of this layer is far less than the same layer at the summit or side of a convolution. Vertical sections will therefore show at the summit of the gyrus deep oval groups of cells, contrasted with a simple streak of solitary and distant cells at the bottom of the sulcus. In general terms, it may be said that the typical arrangement of the ganglionic series, at the base of a sulcus, is identical with the formation of the same layer over a very extensive area of the vault. When in examining vertical sections of the cortex we find the grouped cells give place at the *summit* and *sides* of a convolution to a shallow streak of distant cells, arranged as solitary cells or in twos or threes at most, we may assure ourselves we have passed from the region which **Ferrier** considers impressed with motor endowments, and have reached a territory possessing an equally characteristic formation. Further, the *transition* from the grouped arrangement into the solitary or laminar band here referred to resembles what is observed at the sides and base of a sulcus in the motor area. The groups, from being closely aggregated, become more distant and *discrete;* the groups themselves become shallower, include fewer cells, until one or two elements alone reappear at great distances apart, and characterise the region furnished with the solitary or laminar type. I do not mean to infer that the base of a sulcus in the motor area furnishes us with a lamination identical in its characters with that of non-motor realms. The distinctions are as follows: in the region just alluded to as possessing the laminar arrangement of ganglionic cells there is superimposed on it a distinct layer of small pyramidal and angular cells, whilst the horizontal distribution of spindle cells never occurs except at the base of a sulcus. To summarise, then, in a few words, we have two regions of the vault characterised by two very distinct types of lamination :—

A. The five-laminated type with its grouped ganglionic cells, exhibiting a laminar arrangement at the sulcus only.

B. The six-laminated type, with the solitary laminar arrangement of the sulcus *generally* distributed over the summit and sides of the gyrus ; and the introduction of a layer of small pyramidal elements immediately above the ganglionic group.

Transition realms.—Every intermediate state is clearly discernible betwixt these two types of lamination in neighbourhoods where they approach each other and blend. Thus, in studying the passage of a five- into a six-laminated region, we meet with territories where the five-laminated cortex presents us with very small ganglionic cells still arranged in nests, and forming but pigmy representatives of those larger groups believed to be characteristic of motor areas. Again, we come upon regions where a nested arrangement of small ganglionic cells is associated with a six-laminated cortex. In fact, the nested arrangement and size of the individual cells appear to be invariably in inverse ratio to the depth of the small angular formation superimposed, so that, as we approach motor realms, the latter thins out and gradually disappears, whilst the nests become more and more perfect and embrace larger and still larger cells. On the other hand, as we recede from motor regions the nests thin out, become less perfect as groups, include cells of small dimensions, become more widely separated from each other, whilst the superimposed stratum gains in depth and increases in density by the close aggregation of its angular and pyramidal elements. In studying the arrangement of the layers of the cortex the above points appear to me to demand careful attention, and the eye should become perfectly familiarised with the varied appearances presented by transition zones of the cerebrum.

First layer (Plate 6, A).—Over different regions of the cortex this stratum varies somewhat in depth: In the neighbourhood of the limbic lobe it averages ·619 mm., and nearer the Sylvian fissure it becomes ·418 mm. in depth. In all points except the relative proportion borne by the connective to the nervous elements, this layer closely resembles the corresponding layer in the higher Mammalia. The matrix is constituted of a fine areolation of connective fibrils, with which is associated an extremely delicate network of nerve fibrils. The latter are derived from the apex processes of nerve cells in subjacent layers, the long, delicate process of the large ganglionic cell being frequently traced up into this layer, where it is observed to divide into numerous subdivisions. The connective elements include cell-forms of two kinds—the perivascular, and the neuroglia corpuscles, both forms nucleated, and measuring respectively 5μ and 9μ in diameter. The larger elements are especially numerous just beneath the pia mater, and constitute a distinct belt of cells in many situations—irregular in form, and throwing off numerous delicate processes. These cells, known also as DEITER's corpuscles, are found in such vast numbers in the Sheep that attention will be directed to their mode of distribution and significance when describing the first cortical layer of that animal. In all fresh preparations this layer presents us with three distinct regions differing somewhat in appearance and corresponding very nearly to its upper, middle, and lower third. In the upper division we find a band of medullated nerve fibres following the course of the convoluted surface and surrounded on all sides by processes from the cells of DEITER. The lower division shows us a coarse network formed by the division and decussation of the apex processes of the second layer of cells, which are often thrown off at a very oblique angle from the cell.

Second layer.—A shallow belt of densely congregated cells forms the second layer of the cortex (Plate 6, B). Its superficial and deep limits terminate somewhat abruptly, so that a fair estimate may be taken of its depth, which is found to be decidedly less than the corresponding layer in the human brain. The average depth may be given at ·093 mm., increasing, however, in some regions to ·116 mm. or even ·138 mm.

In the human brain, on the other hand, the depth of this layer in the frontal lobes varies between ·139 mm. and ·186 mm., whilst posterior to the fissure of ROLANDO it may increase to ·279 mm. The nerve-cells vary greatly in form, appearing as angular, pyriform, pyramidal, oval, or globose elements, which throw off several delicate processes, one or more thrown off from the apex towards the surface of the cortex being a frequent but *not constant* arrangement. I have elsewhere alluded to a striking character as common to these elements in the human brain,* viz.: the relatively large size borne by the nucleus to the protoplasm of the cell as compared with larger nerve-cells of subjacent layers. We find the same character maintained by these cells in the cortex of the Pig. Another peculiarity is the frequent occurrence of the globose form of cell which throws off an apical process and several very delicate branches from its sides, into which branches, however, the cell protoplasm does not prolong itself as in other cells; hence its contour is wholly unaffected by the branching. This form of cell is peculiar, and I shall allude to its possible relationships further on. In this layer the angular form of cell is most frequent; its peripheral branches diverge obliquely to the radial fasciculi, cross each other in all directions, and by their repeated divisions and ramifications form a dense network of fine fibres quite characteristic of the deeper portions of the first layer. In delicate sections this network can be traced upwards for a distance of ·162 mm.—*i.e.*, one-fourth the whole depth of the superficial layer of the cortex. These angular cells are peculiar in form, and are met with frequently, as we shall see, at deeper levels. The gradual blending of the second and third layers of the cortex as seen in the human brain is far less marked in the Pig, as over most regions of the cortex one layer *abruptly* terminates in the other.

Third layer (Plate 6, C).—The depth of this layer, taken together with the second layer, averages ·913 mm. This average was struck from measurements at thirty-six different points of the cortex of the vault, the variations lying between ·744 mm. and 1·488 mm. The greatest depth is attained immediately below the summit of the convolution, whilst towards the sulci the layer gradually becomes shallower. The gradual increase in size of its constituent cells from above downwards, which is so prominent a feature in the human brain, is not observed here, the deeper cells of this layer being identical in point of size with the more superficial cells bordering upon the second layer. Not only are these cells very constant in size throughout the depth of this layer, but they vary very slightly in magnitude throughout the whole surface of the cortex. An approximate average from a very large number of measurements may be given as 23μ in length by 13μ in breadth, and the longest diameter of their nucleus as 9μ. On reference to the table of measurements we shall find that, whilst the upper series of cells in this layer in Man and the Carnivora do not reach this magnitude, the lower series in Man are far larger, whilst in the Cat the lower series corresponds exactly in size with those of the Pig. It is important to bear in mind the fact that the lowest stratum of the third layer in the Cat is formed of cells identical in size with those which constitute the whole thickness of this stratum in the Pig; whilst none of the smaller cells crowding the upper portion of this layer in Man, the Monkey, and the Cat are here present. The nerve-cells of this layer vary much in contour: we find them oval, oblong, pyriform, and pyramidal, and a few even elongate spindles. By far the more frequent form assumed is that of the pyramidal or a slight modification of the pyramid. The junction of every main branch with its cell is marked by the gradual increase in size of the branch where it blends with the cell, occasioned by the continuation of a funnel-shaped prolongation of protoplasm from the cell into its branch. This *gradual blending* of the cell with its branch is a safe criterion to follow if any doubt arise as to the actual junction betwixt the two. The average number of processes thrown off by these cells (inclusive of basal and apex process), as seen in vertical sections, is six. As many as eight or more are often seen; but, from reasons already adduced,* no approximate knowledge can be obtained of the absolute number of primary branches. The apex arising from the gradual attenuation of the cell lies more or less parallel with the radial fasciculi, and is therefore directed outwards or directly towards the superficial layer of the cortex. The basal process appears to conform in every respect to the corresponding process in the large ganglionic cell. The smaller branches arise from various points of the periphery of the cell, more especially near the base, and radiate outwards, dividing and subdividing into extremely delicate fibrils occupying an extensive zone around the cell. Each cell contains an oval nucleus (deeply stained by aniline and carmine), which is invariably centric in position. The cells of this layer frequently have a number of small nuclei arranged around or upon them: these elements pertain to the pericellular lymph spaces.† As to the general arrangement of the cells of the third layer, no regular grouping is apparent: they are scattered very irregularly through the greater depth of this layer except at its deeper portions, where they lie in dense clusters between the advancing sheaves of apical processes given off from the ganglionic cells of the subjacent stratum. These latter bundles of nerve-fibres consequently occupy somewhat pale spaces almost devoid of cells, forming well marked septa betwixt the cells of the deeper regions of this layer. The peculiar angular cell is found *sparsely* scattered throughout the whole depth of the third layer as in the cortex of higher animals.

---

* "The Cortical Lamination of the Motor Area of the Brain," Proc. Royal Soc., No. 185.

Ganglionic layer (Plate 6, D).—We now arrive at a region of the cortex where the most striking and characteristic features develop themselves, and where the distinctive characters impressed upon the architecture of the cortex in Man and the lower Mammalia come out most prominently. Any one accustomed to the microscopic appearance of the cortex cerebri in Man in or neighbouring upon the motor area cannot fail, on examining the cortex of the Pig for the first time, to be struck by one great differential character. In Man and the higher Mammalia the cells of the third layer increase in size with their depth, so that the lowest stratum consists of tolerably large cells, and the eye is thus, as it were, prepared for the enormous elements of the ganglionic layer below. In the Pig, however, the transition is most sudden from the small cells of the lowest stratum of the third layer to the dense nests of large ganglionic cells beneath. The transition is so abrupt as to form a characteristic feature in the brain of this group of animals. I will endeavour to give here a general idea of this formation by a description of the nerve-elements under the following heads:— 1. Depth of layer; 2. Form of cells; 3. Size of cells; 4. Processes.

---

* "Cortical Lamination of the Motor Area of the Brain," Proc. Royal Soc., No. 185.

† See on this subject OBERSTEINER'S views, "Ueber einige Lymphräume im Gehirne" (Sitzb. d. k. Akad. d. Wissensch., Abth. i., Jan. Heft, 1870). Also an article by the writer entitled, "The Relationships of the Nerve-Cells of the Cortex to the Lymphatic System of the Brain," Proc. Royal Soc., No. 182, 1877.

1. *Depth of ganglionic layer.*—The pale band corresponding to this layer in Man, and containing the large cells grouped in nests or clusters, is in the Pig crowded with very dense groupings of cells in certain regions, whilst at other sites, as above mentioned, the laminar or solitary arrangement prevails. The depth of this layer, therefore, varies with these two varieties of lamination. In the clustered arrangement the average depth is ·595 mm.; in the laminar arrangement it may be a simple streak of cells or attain a depth of from ·186 mm. to ·372 mm. The greatest depth in the clustered regions was ·697 mm. Further details respecting absolute depth of individual layers is given in a tabulated form for convenience of reference. (See p. 62.)

2. *Form of cells.*—It has been stated in a former memoir[*] that a great irregularity in marginal conformation is quite peculiar to these cells in Man. Now we can by no means state that such is the case with the corresponding elements in the Pig; in fact, a striking uniformity in the contour of these cells is observable, by far the greater proportion taking the form of an elongate pyramid, the few exceptions occurring being usually gigantic spindles. They resemble closely, both in size and form, the large pyramidal cells at the *deepest portion* of the third layer in Bimana, Quadrumana, and the large Carnivora, as also the *ganglionic cells* in the parietal and tempero-sphenoidal lobes of Man. Nowhere do we find the irregular, swollen, and at times almost globose cells so frequent in the motor area of the human brain. How is it possible, it may be asked, to discriminate between these cells and those of the third layer if they approach them so closely in size and contour? In the first place, their arrangement in nests is indicative of their true significance; in the next place, their *abrupt* commencement is indicative of a layer distinct from the third; and, lastly, of still greater import to my mind is the fact that in those regions marked by a six-laminated type the belt of angular cells becomes *interposed between this layer of cells and the third layer.* To summarise, therefore, we may state that the ganglionic layer in the Pig consists of cells which, whilst arranged in *clusters* corresponding to the nests of **Betz** in the human subject, conform in size and configuration to the lowest pyramidal cells of the third layer in Man (Plate 7).

---

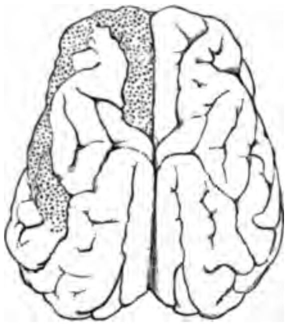
[*] *Loc. cit.*

3. *Dimensions of cells.*—The largest cells are found anterior to the crucial sulcus in the antero-superior portion of the great limbic lobe, where that lobe becomes superficial on the hemisphere, figs. 1 and 2 (4). Next to them in point of size are the ganglionic cells of the anterior portion of the parietal convolution immediately external to the crucial sulcus. In the exposed portion of the limbic lobe above referred to, the cells attain the dimensions of 48μ in length by 17μ in breadth, their nucleus measuring 15μ long. Such are the dimensions in the middle third of this region, fig. 2 (B C). Towards the frontal extremity of the hemisphere near the fissure of ROLANDO, fig. 2 (9) they decrease to 37μ × 16μ (nucleus 11μ), whilst posteriorly at the commencement of the crucial sulcus they average but 25μ × 19μ. The variations observed in the dimensions of these cells will be further dealt with in our remarks upon regional distribution, and will also be found in a tabulated form at the end of this article.

Regional Distribution.

That portion of the antero-superior arc of the great limbic lobe or convolution of the corpus callosum, which becomes in the Pig and other Mammals superficial on the upper aspect of the hemisphere, and which in this animal bounds the anterior half of the longitudinal fissure, is *par excellence* the region which exhibits the highest development attained by the ganglionic series of cells. From the commencement of the crucial sulcus where this lobe becomes superficial to its termination anteriorly, the five-laminated cortex and nested arrangement of the fourth layer of cells is maintained throughout. Local variations in the distribution of the ganglionic cells occur markedly in this lobe, and will require separate notice further on. For the present it is necessary that we trace the regions over which this characteristic formation is spread. The extreme anterior tip of the hemisphere participates in this formation, both convolutions bordering upon the fissure of ROLANDO exhibiting a five-laminated cortex and distinct clustered cells in the fourth layer. The same fundamental type of structure can be traced along the whole course of the lowest parietal convolutions (first and second), from their union with the ascending parietal (post-Rolandique, BROCA) towards and

Fig. 3.



around the Sylvian fissure. To speak generally, then, it may be stated that the arrangement of the elements of the fourth layer in clusters is more especially characteristic of the cortex in—

a. The great limbic lobe (anterior half of superior arc).

b. The frontal lobe.

c. The first and second parietal convolutions.

Embraced within this territory we have the third and fourth parietal convolutions lying above the primary parietal sulcus (inter-parietal) extending from the ascending parietal convolution to the opposite pole of the hemisphere. These convolutions, forming the greater mass of the upper and posterior aspects of the hemisphere, present us with an arrangement of the ganglionic series very distinct from that just dealt with: I allude to the solitary or laminar arrangement of cells. This mode of distribution is highly characteristic of this region. When referring to the transition realms of the cortex it was stated that the passage of one form into that of another is never *abrupt;* the change is a *gradual* one, and hence no exact boundary line can be drawn betwixt the clustered cells of the fourth layer in the *limbic* lobe and the solitary elements of the same layer in the adjacent parietal gyri. Vertical sections taken through these adjacent convolutions constantly exhibit the grouped arrangement overstepping the boundary of the crucial sulcus and spreading towards the summit of the uppermost parietal convolution, but thinning out into the stratiform or solitary arrangement beyond. On the other hand, numerous nests of these same elements occupy that aspect of the third parietal gyrus which dips into the inter-parietal sulcus. Bearing this fact in mind, we may state that the crucial and inter-parietal sulci roughly map out the boundary lines between two highly characteristic formations—the clustered and the solitary ganglionic series. We will now deal with the several regions alluded to seriatim, and describe the local variations in the conformation of the mid-regions of the cortex in—1, the limbic lobe; 2, frontal lobe; 3, parietal lobe.

1. *The great limbic lobe.*—We shall limit our examination to the antero-superior arc of this lobe, or the convolution of the corpus callosum, and first inquire into the special arrangement of the fourth layer of the cortex in that anterior portion which lies in front of the *sub-parietal* sulcus, and whose limits extend from the commencement of the crucial sulcus to the internal root of the olfactory nerve, fig. 1 (6–19). The superficial portion of this lobe exposed on the surface of the hemisphere is represented in the diagram as mapped out by several oblique lines into six divisions (fig. 2). The ganglion cells of the fourth layer are most densely grouped in the divisions B and C. At this site the nests are large, oval, and deep, and are rich in densely-packed cells; the groups themselves are so numerous and closely set as to be almost confluent. Reference to the table of measurements will show us that the cells of this immediate locality attain the greatest average dimensions—viz.: $48\mu \times 17\mu$, the largest cell observed being $69\mu \times 32\mu$. At D the dense confluent groups are again met with, but over a limited area, the outer aspect exhibiting chiefly distant or discrete clusters.

At E, and still more at F, the dense confluent groups re-appear, although the tendency to thin out into discrete clusters shows itself towards the crucial sulcus. A gradual reduction in the dimensions of these cells is noticed towards F, so that at the commencement of the crucial sulcus they average but $28\mu \times 17\mu$. Further forwards at A, dense groups appear on the internal aspect of the lobe just where it becomes superficial, but between this and the crucial sulcus the cells are scattered in distant groups, and average in size $37\mu \times 16\mu$. These densely-grouped confluent nests may be traced down along the whole internal aspect of the anterior arc of the limbic lobe as far forwards as the sub-frontal sulcus. In the divisions marked A C great fusiform cells are frequently met with measuring 55μ long $\times 13\mu$ in diameter, or $69\mu \times 11\mu$. In entering thus somewhat minutely into the details of examination of an important region of the cortex, I have been influenced by the desire of pointing out what may prove a significant fact: that these groups of ganglionic cells do not form an *equably* distributed layer, but are associated in dense masses towards various centres with intervening tracts formed by distant or discrete cell groupings. The remaining portion of the convolution of the corpus callosum, viz.: that part lying *beneath the sub-parietal sulcus* and limited posteriorly by the *retro-limbic annectant*, presents us with the most perfect example of the six-laminated cortex and solitary arrangement of ganglionic cells. The band of angular or pyramidal elements separating the third layer from the ganglionic is both deep and dense.

2. *Frontal lobe.*—The extreme tip of the hemisphere anteriorly constituting the frontal lobe presents us with the nested arrangement of cells, the nests being in the ascending frontal convolution thinly supplied with cells and distant in their grouping, whilst throughout the ascending parietal gyrus the nests are formed of densely-congregated cells. These rich nests are, however, invariably discrete and even distant. The inner aspect of the frontal lobe, limited behind by the sub-frontal sulcus, has numerous cell groups which form, however, but ill-defined nests. Throughout the whole extent of the frontal lobe no coalescence of groups or confluent nests were observed.

3. *Parietal lobe.*—The four convolutions constituting this lobe are by no means identical in structure throughout. The convolutions below the inter-parietal fissure will first engage our attention. The first or Sylvian convolution is usually blended with the second parietal in front of the fissure of SYLVIUS; a few faint markings indicating their line of union. Throughout these convolutions the general arrangement of ganglionic cells is in shallow, ill-formed nests, poor in cells, and the nests discrete and often distant. The richer aggregation of cells occurs along what corresponds to the second parietal gyrus bordering upon the inter-parietal fissure, and they thin out into distant nests towards the Sylvian fissure. The groups of cells are dense and almost confluent in the divisions A B C, fig. 2, of the second parietal convolution, as also they are throughout the whole extent of the first and second parietal convolutions at F. Still further back at G we find both gyri possess well-formed distinct nests of from

five to six cells, which groups are invariably distant from one another. Their maximum dimensions are obtained at B ($=40\mu \times 21\mu$) and at F ($=37\mu \times 27\mu$), the minimum at E ($= 28\mu \times 18\mu$).

Carrying our observations into the structure of the upper parietal region, immediately above the inter-parietal fissure, we find the solitary or laminar type of the ganglionic layer as most prevalent throughout. We have here, of course, two convolutions to deal with, viz.: the third parietal and the fourth or sagittal convolution. The anterior extremities of these gyri exhibit, as above stated, certain modifications in the immediate neighbourhood of the crucial and inter-parietal sulci; but with this exception the laminar arrangement is maintained. In the regions alluded to, the nested cells may be traced from the crucial and inter-parietal sulci some distance up the adjacent side of the upper parietal convolutions. They frequently reach the summit of these gyri as extremely rich nests, which are invariably discrete and often very distant. The laminar arrangement of the posterior parietal region is highly characteristic. It appears as a pale band, along the upper limit of which a line of solitary cells lie at distances of from ·139 mm. to ·379 mm. apart. Frequently, however, two or three of such cells appear side by side. In this region we recognise the tendency to the accumulation of angular and small pyramidal elements towards the lower limits of the third layer, which tendency becomes more marked the further back we carry our examination of the cortex. It must, however, be distinctly understood that we have here no distinct belt of small angular elements; no accumulation of cells so differentiated from the third and subjacent layers as to merit the name of an additional layer of the cortex. We find over the greater portion of the parietal region but the faint shadowing forth of the six-laminated type—an arrangement met with over a very limited region of the cortex in this animal.

THE CORTEX CEREBRI OF THE SHEEP.

The points of resemblance exhibited between the cortex of the Sheep and the Pig are so numerous that to describe the structure in each separate lobe of the Sheep's brain would be to a great extent a task of recapitulation. I shall therefore content myself with a description of the structure of the cortex in that great fundamental portion of the brain—the limbic lobe, and emphasise more especially those features where points of divergence are indicated.

Lamination of the great limbic lobe.—We have here in the anterior portions of the upper arc to deal with a five-laminated cortex over an area which, as will be pointed out further on, is coextensive with the same formation in the Pig.

First layer (Plate 6, A).—The average depth of this pale stratum is ·550 mm. It embraces the two forms of nucleated cells which are found in this layer in other animals, viz.: the perivascular and neuroglia corpuscles, measuring 5μ and 9μ in diameter. These cells contain large nuclei. Immediately beneath the pia mater great numbers of the spider-like bodies known as DEITER's cells are met with. They are distributed especially along the course of the blood-vessels which vertically traverse this superficial layer of the cortex. Closely packed beneath the pia mater, their long filaments spread in all directions embracing the nearest blood-vessel. They thin out in numbers below the upper third of this layer, becoming few, and scattered widely apart in the neighbourhood of the small pyramidal layer. It may here be stated that these peculiar bodies are found in *very* scant numbers throughout the various subjacent layers, but towards the lower realms of the spindle layer they again congregate in larger numbers, become especially abundant along the blood-vessels which course through the medullary strands at the confines of the cortex. It is evident from this distribution that they are more especially limited to the regions of the greater vascular channels, and do not intrude upon the regions of capillary distribution. This fact I regard as significant, when we also take into consideration their reappearance in Man as *morbid elements* in those diseases of the brain which are regarded as closely connected by a causal relationship with extensive vascular lesions. The pia mater is not only thickened, but is strongly attached to the cortex by means of these bodies, and over certain regions, as the two lower parietal convolutions, the adhesion is so strong that the pia cannot be separated without destroying the subjacent layer. In these regions of firm adhesions betwixt the pia mater and cortex the processes of these cells are extremely coarse, and often form a dense felt of fibres immediately beneath the pia. Such a condition I have not observed in Man except as the result of retrogressive changes in diseased regions. The depth of this characteristic formation is usually ·134 mm., and varies from $\frac{1}{3}$rd to $\frac{1}{5}$th the depth of the first layer. The other elements of this layer resemble in all essential features those of the corresponding formation in the Pig.

Second layer (Plate 6, B).—In the five-laminated region of the limbic lobe which we are now considering, this formation is peculiarly poor in cells and forms but an insignificant belt of small angular and pyramidal elements associated with larger pyramidal cells. It is often apparently absent, or forms by the closer approximation of its elements a limiting border to the important layer below it. This layer becomes deeper and more decided towards the posterior pole of the hemisphere. Again, in the anterior extremities of the lower parietal convolutions it is a more distinct formation. The cells are oval, angular, or small pyramids possessing a large spheroidal nucleus. The smaller cells average $13\mu \times 6\mu$ in dimensions, their nucleus having a diameter of 6μ.

Third layer (Plate 6, C).—The cells of this layer are remarkably uniform in size throughout, and in the anterior portion of the superior arc of the limbic lobe they are most densely aggregated in the upper half of the layer, their comparative paucity in its lower half gives to this region the appearance of a pale white belt. The whole layer is of fair depth, and presents but few of DEITER's corpuscles above alluded to. The cells vary much in contour, although the majority are pyramidal in form, their nucleus measures 6μ in diameter, and the cells average $18\mu \times 10\mu$. Their apical process is frequently contorted and twisted in its course upwards towards the first layer. Throughout this layer small pyramidal cells usually measuring 13μ in length by 9μ in breadth are found. These latter elements tend in *certain regions* of the brain to accumulate towards the lower half of the third layer; and the pale zone, poorly supplied by cells in the anterior extremity of the limbic lobe, becomes in its *posterior* half, and at other sites, occupied by these small pyramidal and angular elements which form the *fourth* layer of the *six-laminated* cortex. This layer of angular elements attains a depth of ·186 mm., the cells measuring from 6μ to 9μ in greatest length and breadth. Not only do we miss here the gradual increment in size of these cells with their greater depth which is so noticeable in Man and the Monkey, but we frequently find the largest elements of the series (27μ × 15μ) immediately beneath the second layer. This layer is as a rule richly supplied with cells which are densely aggregated.

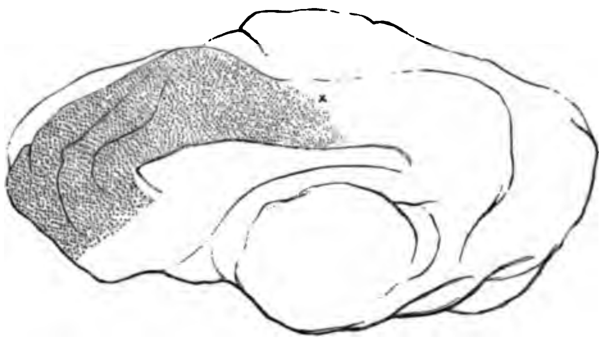
Ganglionic layer (Plate 6, D).—This important formation attains a depth of ·372 mm. below the superimposed stratum. The nerve-cells of this layer vary much in their conformation. The most frequent form assumed is that of an elongated pyramid, the apical process of which can be traced often as far as the first layer of the cortex. Other cells assume the form of elongated spindles, and a frequent form is that of a horned cell, which bears a striking resemblance to the egg case of the Skate. The apex process may bifurcate close to the cell, giving the latter the form of a horned pyramid, or it may divide into two secondary processes of equal diameter at any distance up the third layer. The apex process is particularly coarse and does not undergo rapid attenuation. The average dimensions of these cells is 46μ × 11μ, a measurement which reveals clearly the elongated conformation of these elements, in fact, the oval and plump rounded contour so frequent in the corresponding cells of Man, the Monkey, and the Carnivora is very rarely met with in these animals. Occasionally a larger cell than usual is met with, *e.g.*, 65μ × 23μ. They contain an oval nucleus measuring 13μ in length. These cells are disposed in oval or oblong nests, often covering an area 460μ deep by 110μ broad, and contain as many as twenty cells in some situations. The cells are frequently scattered somewhat irregularly in the anterior portion of the limbus, but they become progressively *larger and more distinctly nested* towards the *exposed portion of the limbic lobe.* The clusters are frequently so closely packed as to become confluent, and in many regions on this account the nests are badly differentiated. It is impossible after close examination of this formation to arrive at any other conclusion than that these cells not only resemble those larger pyramids of the third layer in Man in their general form, but also by their gradual increase in size from above downwards (the largest cell being invariably lowest in the series) claim further affinities to this formation. It is however distinctly differentiated from the third layer in the six-laminated regions by the interposition of the layer of small angular cells.

Spindle layer (Plate 6, E).—The lowest formation of the cortex in this region presents no special features demanding notice here. The spindle-cells are large in size and the stratum deep. Passing now to the subject of regional distribution of the different types of lamination, and the diverse arrangements of the ganglionic series, we shall rapidly glance at each individual lobe.

Regional Distribution.

Great limbic lobe.—The anterior half of the superior arc of this lobe, fig. 4, is distinctly five-laminated, whilst the ganglionic series of cells is arranged in clusters throughout the whole extent of this lobe as far back as the retro-limbic annectant. From the commencement of the crucial sulcus the nested arrangement of these cells becomes a more marked feature the further forward our examination extends, so that in the neighbourhood of the fronto-limbic sulcus the nests are large and richly crowded with ganglion cells. The groupings also in this situation are frequently confluent. Although the nested grouping of this formation is maintained throughout the superior arc of this lobe, a great dissimilarity is seen between sections taken near the posterior extremity of the corpus callosum and those from regions anterior to its genu. The posterior half of this upper arc differs however not only in the mode of grouping of its elementary constituents—a fundamental divergence is observed in its lamination. Beneath the whole extent of the sub-parietal sulcus the limbic lobe is found to be six-laminated (Plate 7). This change in the type of cortical stratification is assumed at fig. 4 (*), a little posterior to the point where the sub-parietal sulcus becomes superficial on the surface of the hemisphere, and is continued as the crucial sulcus. The intercalated series of small angular cells is first observed therefore where the limbic lobe comes close in contiguity with the sagittal gyrus of the parietal lobe at the origin of the crucial sulcus. A similar disposition is maintained in the lamination of the limbic lobe in the Pig, as we have already stated above. From this point the six-laminated cortex is spread downwards and backwards over the whole posterior portion of the superior limbic arc. The intercalated series of small angular elements becomes more and more richly developed towards the retro-limbic annectant, where it forms a dense and deep stratum, which readily characterises this region as wholly differing in important features from realms anterior to the crucial sulcus.

Fig. 4.



A most noteworthy feature presented by this layer is its tendency to take a higher position than is observed in the same region in the Pig; in other words, it appears to encroach upon the third layer, which becomes unusually shallow, whilst the ganglionic stratum is proportionately increased.

Parietal lobe.—Throughout the greater mass of the parietal lobe the six-laminated type prevails, and the solitary or laminar arrangement of the ganglionic series is found. Exceptions occur, however, near the crucial sulcus, in the ascending and lower parietal convolutions. Along the whole of the first parietal or Sylvian convolution the same structural peculiarities exist that were apparent in the Pig. The five-layered cortex and the clustered ganglion cells reappear here. The nests in this Sylvian convolution are dense in cells, and so closely approximated as frequently to become quite confluent, and especially is this the case towards the lower aspects of this convolution. They usually occupy areas ·325 mm. in depth by ·162 mm. in breadth, and the cells average $40\mu \times 13\mu$, with a nucleus having a diameter of 13μ. The largest cell observed measured $60\mu \times 27\mu$. It will be apparent from the foregoing statement that, as in the similarly stratified portion of the limbic lobe, the elements of this layer are peculiar from their elongated form and their rich dense clustering. Posterior to the fissure of SYLVIUS these nests become thinned out, the lamination assumes the transitional characters, passing eventually into the six-laminated type. The second parietal convolution anterior to the Sylvian fissure presents us with a realm transitional between a five and six-layered cortex. The ganglion cells assume a distinctly laminar arrangement, the angular cells increasing in number but not forming in this region a very distinct layer.

Frontal lobe.—Throughout the whole extent of this lobe the cortex is constituted of five layers. The cells of the ganglionic layer are small, well nested, and often confluent, but do not approach in their development the corresponding series in the exposed portion of the limbic lobe. The third layer in this lobe is constituted of cells maintaining uniform dimensions throughout the depth of this layer; but they approach close to the ganglionic series and fail to exhibit the pale zone betwixt these two layers, which is apparent in realms nearer the six-laminated cortex.

THE CORTEX CEREBRI OF THE CAT.

Taking the great limbic lobe as presenting in its most perfect development the cortical formation characteristic of the motor area, we proceed to note the essential features of its individual layers. For the present we restrict our examination to the anterior half of its superior arc, fig. 5 (*).

First layer.—A narrow, pale, white streak, averaging ·325 mm. in depth, constitutes this stratum as seen by the naked eye. It is found to consist of an extremely delicate meshwork of connective and nerve-fibrils, in which two forms of nucleated cells may be observed—the perivascular following the course of blood-vessels, and measuring 5μ, with a large spheroidal nucleus, and a true connective corpuscle, averaging 9μ in diameter, having one or more nuclei, and when not altered by reagents seen as a corpuscle of irregular contour with numerous delicate radiating processes. As I have stated elsewhere,* I regard the latter as the origin of DEITER's corpuscle, to which frequent allusion has been made. The coarser connective element recognised as DEITER's cell is present immediately beneath the pia mater, but far less abundantly than in the Pig and Sheep. The nuclei of these cells usually measure 4μ in diameter, whilst the nucleus of the angular cell, which in general conformation sometimes approaches them, measures fully 9μ. The superficial portion of this layer presents us with the usual belt of medullated fibres extending to a depth of ·162 mm. to ·192 mm., the individual fibres of which measure 2μ to 4μ in diameter.

Second layer.—This stratum, far better marked than in the Sheep, is constituted of small angular, ovoid, or pyramidal cells. The angular cell is frequently bifurcate or horned, and branches irregularly. All the cells contain a large oval or spheroidal nucleus, and are imbedded in a matrix identical with that of the first layer. Here we also recognise the large connective cell. The nerve-cells of this layer average $10\mu \times 6\mu$, with a nucleus 5μ in diameter; but amongst them are found larger pyramidal cells, identical in appearance with those of the upper part of the third layer, and which attain the dimensions of $13\mu \times 10\mu$. The depth of this layer varies between ·139 mm. and ·186 mm.

Third layer.—This layer and the one superimposed appears to the naked eye as a dark grey translucent band, and extends to the depths of ·744 mm. to ·790 mm. The pyramidal elements characteristic of this formation increase in size with their depth, as do the corresponding cells in the cortex of Man and Apes. The average dimensions of these cells is $16\mu \times 9\mu$, the larger cells at the lowest levels of this stratum attaining proportions of $23\mu \times 13\mu$. Seven processes are usually seen in vertical sections. In the *immediate* neighbourhood of the crucial sulcus the largest cells of this layer do not extend below the upper half of the stratum, leaving betwixt them and the ganglionic series a paler band, in which are very sparsely scattered a few smaller pyramidal cells and angular elements, the latter being found at all depths of this layer, although in very small numbers. On the *parietal* side of the sulcus, however, the pale band becomes a belt of small angular elements. Thus, although the angular cells are found at all depths from the second layer downwards, they do not constitute a distinct stratum betwixt the large pyramidal and ganglionic layers until the parietal boundary of the crucial sulcus is reached, and the limits of the anterior portion of the limbic arc overstepped. Perivascular and connective cells of the same character as those of superimposed layers appear here. The matrix consists of an extremely delicate connective with a meshwork of the minutest nerve-fibrils given off from the angular elements and the periphery of the small and large pyramidal and ganglionic cells. Through this layer course upwards the apex processes of its cells as also those of the great ganglionic series below.

---

* *Loc. cit.*

Ganglionic layer.—The cells of this formation are plump, oval, or pyriform bodies, differing notably from the elongated pyramidal cells of the Sheep and the Pig. Occasionally a fusiform cell occurs, and more frequently a well-formed pyramid. They attain the dimensions of $51\mu \times 23\mu$, with a large oval nucleus, measuring 18μ. The nests are usually large and uniform, and most frequently contain from three to four cells (as seen in vertical sections). One of the larger and richer nests measured ·340 mm. in depth by ·093 mm. in breadth, and showed nine cells. The ganglionic cells are decidedly largest in the gyri bounding the crucial sulcus. In a former investigation into the structure of the gyri in this neighbourhood * the following facts were elicited. The limbic boundary of this sulcus possessed ganglionic cells averaging $39\mu \times 17\mu$, whilst the parietal boundary of the sulcus contained cells averaging $46\mu \times 17\mu$, a large number even measuring $69\mu \times 27\mu$. At the external limit of the crucial sulcus, where the parietal gyrus bends round and unites with the limbic lobe, the largest cells were found ($83\mu \times 37\mu$), whilst an occasional gigantic cell occurred, attaining the dimensions of $106\mu \times 32\mu$.

The spindle layer calls for no special description here.

Parietal lobe.—On the parietal side of the fissure of Rolando we come upon a six-laminated type of cortex; and just as we shall find that the crucial sulcus presents us with a different type of structure on its limbic and parietal walls, so here the fissure of Rolando separates two typical stratifications of the cortex. In short, the structure of the ascending frontal differs materially from that of the ascending parietal. The difference consists essentially in the reappearance of a distinct layer of angular cells below the large pyramidal series, in the larger dimensions of its ganglionic cells, and the fairly rich and well-formed nests of these elements. We find the average depth of the first cortical layer has decreased to ·232 mm., and that of the next two subjacent layers to ·697 mm. The cells of the ganglionic series attain an average of $37\mu \times 19\mu$, with a nucleus 13μ in diameter. Passing from the ascending parietal gyrus to the four tiers of parietal gyri, we find the laminar conformation and structural peculiarities of this great extra limbic mass to be as follows. The first and second parietal convolutions exhibit various stages of transition betwixt a five- and a six-laminated conformation. The small pyramidal and angular layer above the ganglionic series is but poorly represented, and in the anterior extremities of these convolutions the cells are so few and so thinly scattered that the formation scarcely deserves the name of a distinct layer, whilst in the posterior extremities of the same gyri the layer is more richly developed. The arrangement of the ganglionic cells varies at different sites, but the nested is that which predominates in the first or Sylvian convolution, especially in the immediate neighbourhood of the Sylvian fissure. Towards the inter-parietal sulcus these clusters become more distant, and constituted of fewer cells, until a purely laminar arrangement is maintained. The third parietal convolution is six-laminated throughout, its fourth layer of small pyramidal elements being very distinct and richly developed. The ganglionic cells are arranged in clusters towards the anterior half of this convolution, becoming larger and denser nests the nearer they approach the sigmoid gyrus. In this latter region the cells are of *large size.* Towards the posterior extremity of the third parietal convolution the clustered arrangement gives place to the laminar. Passing now to the fourth parietal or sagittal gyrus, we observe at the anterior extremity, where it impinges upon the crucial sulcus, the small pyramidal cells below the third layer are so scanty that nothing beyond a mere tendency to the formation of a separate stratum can here be discerned. The ganglionic cells, however, are very *large* and *globose*, and the nests rich and extensive. Except at the very extreme anterior portion of this convolution, the cortex is distinctly *six-laminated* throughout, and the cell-clusters become thinned posteriorly to a solitary or laminar formation, as far as the posterior pole of the hemisphere.

---

* Vide 'Brain,' part 1, April, 1878.

Frontal lobe.—This lobe forms the extreme anterior pole of the hemisphere, and presents a well-marked fissure directed obliquely downwards and outwards—the fissure of ROLANDO, bounded externally by the ascending parietal, and internally by the ascending frontal convolution. The cortex of this lobe is five-laminated, except in the immediate neighbourhood of the fissure of ROLANDO, where a transition lamination presents itself.

Taking the structure of this lobe in the order of its layers, we notice that the first layer attains a depth of ·325 mm., and differs in no essential points from that of the limbic lobe. The second layer is here scarcely appreciable, always poorly represented, and often absent over limited areas. It becomes distinctly defined towards the ascending parietal gyrus. The second and third layers have a depth of ·930 mm. The pyramidal elements of the third layer increase in size with their depth, are most densely congregated in the upper half of the layer, whilst the lower half is left as a paler zone, in which towards the fissure of ROLANDO we can readily trace the gradual increase of angular and small pyramidal elements, which, as above stated, form in the ascending parietal a distinct belt dividing the cortex into six layers.

Regional Distribution.

It will be seen that we have in the foregoing description anticipated many of the facts regarding relative areas of different laminations and distribution of the ganglionic series. The whole superior arc of the great limbic lobe, from its origin near the olfactory lobe backwards as far as the retro-limbic annectant, presents us with the best example of the clustered arrangement of the ganglionic cells. These cell clusters are most thickly grouped, and most rich in cells in the *anterior* regions of this upper arc, attaining their *maximum development* in the vicinity of the *crucial sulcus.* The elements of this layer in the latter region differ much amongst themselves in contour and size according to their local distribution. Thus the cells found in that portion of the parietal lobe dipping into the crucial sulcus are peculiarly *swollen, globose, and of great size—frequently solitary*, or arranged in twos or threes. The cells of the limbic portion of this sulcus are, on the other hand, *smaller, elongated, and thickly grouped.* In front of the crucial sulcus the whole cortex of the limbic lobe is distinctly modelled upon the five-laminated plan, fig. 5. Posterior to the crucial sulcus the small pyramidal and angular cells begin to congregate towards the lower confines of the third layer, and this tendency increases from before backwards, so that towards the posterior end of the corpus callosum these small elements form a distinct layer above the ganglionic series. This six-laminated structure is maintained backwards as far as the retro-limbic annectant, which is also modelled upon the same type. Tracing the five-laminated type beyond the confines of the limbic lobe, we find it extends through the whole of the frontal lobe, and that it is limited externally by the fissure of Rolando. In the anterior part of the Sylvian convolution we meet with a transitional form of lamination, whilst a more purely five-laminated cortex occupies the anterior extremities of the third and fourth parietal convolutions. The latter or sagittal convolution in the neighbourhood of the crucial sulcus shows an especially rich development of the five-laminated cortex and its clusters of ganglionic cells.

Fig. 5.

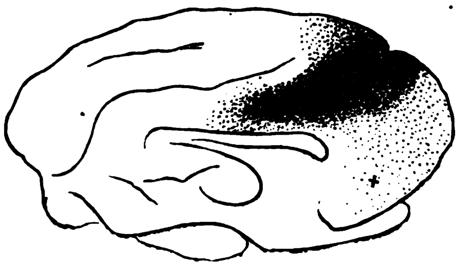


Fig. 6.

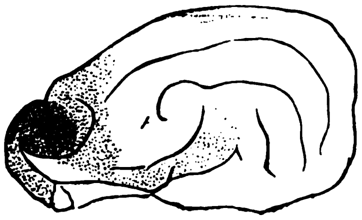
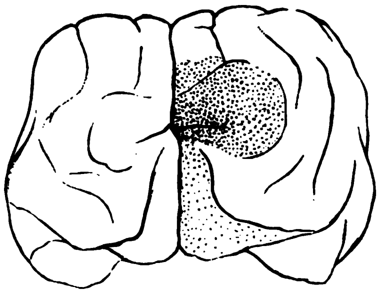


Fig. 7.



Relative Area of Five-laminated Cortex in different Animals.

When inquiring into the architecture of the human cortex cerebri, it was stated that the five-laminated cortex, conjoined to the clustered arrangement of ganglion cells, was most highly characteristic of motor areas.[*] We have acquired from the foregoing examination sufficient data for mapping out approximately the relative areas of the five- and six-laminated cortex in the Pig, Sheep, and Cat; and the subjoined table gives the information necessary for the comparison of such results :—

Area of five-laminated cortex.

The Pig. (Figs. 1, 2, 3.)

1. Anterior portion of superior limbic arc in front of the crucial sulcus. Fig. 1 (7–19).
2. Frontal lobe.
3. Ascending parietal convolution.
4. First and second parietal convolutions united in front of Sylvian fissure.

The Sheep. (Fig. 4.)

1. Anterior portion of superior limbic arc in front of the crucial sulcus.
2. Frontal lobe.
3. Ascending parietal convolution.
4. First parietal convolution (Sylvian).

The Cat. (Figs. 5, 6, 7.)

1. Anterior portion of superior limbic arc in front of the crucial sulcus.
2. Frontal lobe.
3. First parietal or Sylvian convolution.
4. Anterior extremity of second parietal convolution.
5. Anterior extremity of fourth parietal or sagittal convolution.

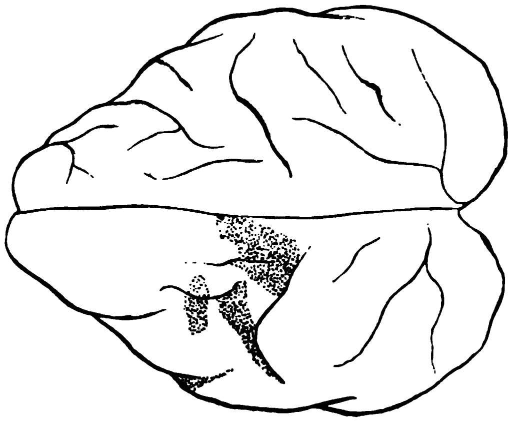
From this summary it is evident that, whilst in all these animals corresponding portions of the limbic and the whole of the frontal lobe are framed upon the same plan, the ascending parietal convolution partakes of this formation in the Pig and Sheep alone; the first and partly the second parietal convolution is (in front of the Sylvian fissure) so constructed in all these animals, whilst a well-developed area of the same formation occupies the anterior extremity of the fourth or sagittal convolution in the Cat. It is interesting to note here the fact that the crucial sulcus forms in all these animals alike a boundary line betwixt the two typical laminar arrangements of the vertex, and marks at its commencement the transition of the one formation into that of the other in the limbic lobe. In summarising, as above, the areas of the five-laminated cortex, with its coextensive nested series of ganglionic cells, due regard must be paid to the varying richness in development of the latter formation. In the diagram accompanying this memoir attention has been paid to this point, the dark shading indicating the rich developments of this formation; the paler shading covers regions where the same formation is poorly developed. Guided by these diagrams, let us summarise for comparison the limits of the richly-developed motor cortex in each animal under consideration.

---

[*] *Loc. cit.*

Barbary Ape (fig. 8.)—The most important grouping occupies the posterior extremity of the frontal convolutions, and the upper end of the ascending frontal gyrus.

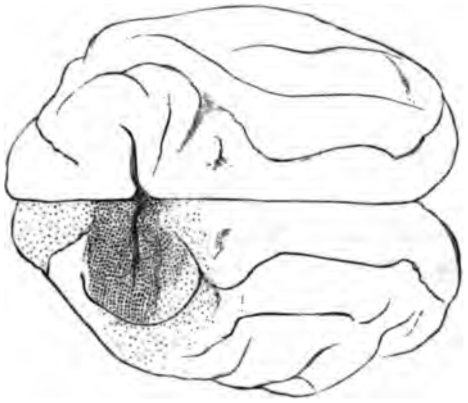
Fig. 8.



At the middle third of the latter convolution two other small areas are represented. The upper and anterior end of the larger group is continuous with a large formation extending over the anterior half of the paracentral lobule. Over all these areas the cells are large and richly clustered. The greater part of the unshaded portion of the figure consists of the transitional and six-laminated realms.

The Ocelot (fig. 9).—Here the great ganglionic nests centre around the crucial sulcus, entering into the structure of its limbic and parietal boundary, and exhibiting cells of large size congregated in rich clusters, and so closely resembling those found in

Fig. 9.



the Cat that a full description would here be out of place. I subjoin, however, a table of measurements of these great cells at different points of the limbic and parietal boundaries of the crucial sulcus :—

DIMENSIONS of ganglionic cells in the Ocelot, from sections taken across the crucial sulcus.

		Limbic boundary.	Parietal boundary.
		μ μ	μ μ
Upper third.	Average . . .	46 × 20	48 × 27
	Largest . . .	88 × 27	93 × 32
Middle third.	Average . . .	51 × 23	48 × 22
	Largest . . .	92 × 27	93 × 41
Lower third.	Average . . .	64 × 29	49 × 27
	Largest . . .	92 × 46	92 × 41

The Cat (figs. 5, 6, 7).—The nested cells extending through the faintly-shaded portion of the Sylvian convolution can bear no comparison, either in size or number, to those seen in the limbic lobe. Throughout the whole limbic boundary of the crucial sulcus the ganglionic cells attain such great dimensions, and are so densely grouped, that they form quite a characteristic area; whilst still greater cells of comparatively gigantic dimensions, although in smaller numbers, occupy the parietal boundary of the same sulcus. The obliquity which the crucial sulcus presents on its deep aspects affords a larger area for distribution than would be imagined from a superficial view of the hemisphere. The faintly-shaded parietal and frontal convolution represents the less richly-developed areas of this formation, but the regions above described we should regard as endowed with the higher manifestations of functional activity.

The Sheep and Pig (figs. 3, 4).—The distribution is so similar in these two animals that a separate description is not required. In the diagram it will be seen that all portions of the five-laminated cortex are equally shaded, to indicate that, in size of cell and complex relationships, the nested ganglionic layer differs but slightly through this wide area. That slight differences in size between the cells do occur has already been indicated, but we fail to discover over this extensive field the extraordinary divergence in these particulars which is presented by the five-laminated cortex of Man, the Ape, and members of the Carnivora. The chief features of interest connected with regional distribution in the brains of these animals appear to be—

1. The wide area of distribution occupied by the five-laminated cortex.
2. The great uniformity observed in the dimensions and complex relationships of the ganglionic cells.

DIVERGENCE IN TYPE OF GANGLIONIC CELL.

In close connexion with the subject just considered are the following points, to which I am anxious to draw attention. It has been shown that the cells of the ganglionic series in the higher development of the motor cortex are *larger* in Man, the

Ape, the Cat, and the Ocelot than in the corresponding series in the Sheep and Pig. This increase in size is chiefly assumed in their short diameter or width, the cell being swollen, ovoid, and often approaching a globular contour. The protoplasmic mass forming these cells is therefore much greater in these higher animals. The important consideration attached to this form is dependent upon the fact that the swollen form is invariably associated with a far more complex branching from all parts of the periphery of the cell, whilst in the pyramidal cell the lateral angles give rise to the greater number of processes. In fact, the oval swollen cell shows numerous angular projections spread over its surface, from which delicate processes arise, and it is presumably the situation and number of these branches which form the chief elements in moulding the contour of the cell. Thus, the simplest cell is elongated, spindle-shaped, and bi-polar; the acquirement of a third process thrown off the centre of the body of the cell gives the spindle an angularity frequently observed in the last layer of the cortex. This appears to be the origin of the pyramidal form of cell and its various modifications and multipolar varieties. Two elementary forms of cell constantly occur in the cortex cerebri, which develop the one into the small pyramidal and angular, and the other into the large pyramidal and ganglionic series. It appears that when the processes arising from the *larger cells are most numerous,* we get the plump ganglionic cell—when *less numerous,* the large pyramidal cells of the third layer; whilst, if the complexity in branching of the smaller cell is great, we have an ovoid, irregular angular cell, or if few branches arise, we obtain the small pyramidal cell. I have stated elsewhere* that the absolute number of branches of these cells could never be determined with any degree of certainty; yet it is important to bear in mind the fact that the irregular contour of the large swollen ganglionic cell depends upon multiplicity of cell branchings, and that therefore this contour in the normal cell may be regarded generally as indicative of greater complexity. We therefore infer what we may approximately ascertain by examination of vertical and horizontal sections—that the cell groups of the ganglionic series have far more branches than those of the third layer. In Man, the Ape, Cat, and Ocelot we found the elements of the ganglionic series possessed this swollen contour, whilst in the Sheep and Pig they were almost universally *elongated pyramids.* The first glance at these cells in the latter animals suffices to establish their peculiar resemblance to the large pyramids of the third layer in higher animals. We therefore may with justice assume that—

1st. The large ganglionic cells of the Pig and Sheep are far less complex in their relationships and connexions than the corresponding cells in the Cat, Ocelot, Ape, and Man.

2nd. The same cells in the Pig and Sheep resemble closely in their general conformation and complexity the large pyramids of the third layer in higher animals.

---

* *Loc. cit.*

Relative Increase of Connective Matrix.

Examination of the brain of animals below Man in the scale of organization shows in a striking manner that the comparative amount of connective to nerve elements is greatly increased. This fact I have already drawn attention to in discussing the relative and absolute depth of the first layer of the cortex in different animals—a layer which in all cases essentially consists of a larger amount of connective than the subjacent strata.[*] Meynert also gives a table illustrative of this point.[†] Independent of the increase in relative and absolute depth of this layer, we have by these investigations been taught to recognise a great increase in the quantity of the cellular constituent of this matrix in the form of Deiter's corpuscles, which crowd the upper regions of this layer. In Man they appear in scanty numbers; in the Barbary Ape they become more frequent; in the Cat and Ocelot they are still more abundant; in the Pig and Sheep so profusely scattered are they that they form a most characteristic stratum immediately below the pia mater, and the meshwork formed by their fibres is dense and coarse, binding the blood-vessels to the cortex and rendering the pia mater strongly adherent (Plate 6). We find these corpuscles more freely in human brain which has undergone senile degeneration and in other diseases attended by reductions in functional activity and vascular affections resulting in retrogressive changes and a reversion to a low type of structure.

The Globose Cell and its Affinities.

It has been already mentioned, when referring to the nerve-cells of the second layer of the Pig, that peculiar *globose* elements occur here with few processes, and no angular projections as in the angular cells which enter into the constitution of the fourth layer. These cells, which are more numerous in some regions than others, are peculiar in that they resemble none of the usual elements described as forming the various layers of the cortex. They look like small pyramidal cells whose angles have been rounded off by the uniform swelling of the cell, and in most of these cells the apex process can alone be seen. It is interesting to note that these cells are found in the third and second layers of the cortex in the *Ape*, and that I have never recognised them in any human brain except in the brain of *idiots* and *imbeciles*, where they appear in abundance.

The most characteristic feature presented in the cortex of idiots which I have had the opportunity of examining, has been the presence of these peculiar elements, which to a great extent take the place of the ordinary pyramidal cell of the second and third layer. Their essential characters consisting in their swollen, globose contour, and great paucity of branches.

---

[*] 'Brain,' part 3, October, 1878.

[†] "Brain of Mammals," by Th. Meynert, in Stricker's 'Human and Comparative Histology,' Sydenham Society, vol. 2, p. 383.

SUMMARY.

In conclusion it will be well to summarise briefly the chiefs facts of interest which have resulted from these investigations. These may be stated as follows :—

1. A five and a six-laminated cortex is found in all the animals examined.

2. The fundamental structure of the layers is very similar in all.

3. Divergence in type is induced through varied character and distribution of the elementary units of these layers.

4. As in Man so in other animals variations in laminar type centre about the mid-regions of the cortex.

5. The ganglionic series is arranged upon *grouped* and *laminar* type.

6. The five-laminated cortex and nested cells are characteristic of motor areas.

7. Transition realms are readily recognisable in the Pig.

8. The first layer of the cortex in the Pig is deeper than in animals higher in the series and is crowded with DEITER'S corpuscles.

9. There is great uniformity in size of the cells of the third layer in the Pig.

10. The six-laminated type is formed by the intercalation of a belt of small angular and pyramidal elements.

11. Great uniformity in contour and complexity of cells is observed throughout the ganglionic series in the Pig and Sheep.

12. The contour of these cells is almost invariably that of an elongated pyramid in these two animals.

13. The ganglionic series is distinguished from the third layer by the interposition in six-laminated realms of a belt of angular cells.

14. The area of five-laminated cortex in the Pig extends over the great limbic and frontal lobes and the first and second parietal gyri.

15. The intimate structure of the brain of the Sheep closely resembles that of the Pig.

16. The crucial sulcus at its origin indicates the transition from the one form of lamination into that of the other.

17. The cells of the third layer in the Cat increase in size with their depth.

18. The ganglionic cells of the Cat are of great size, and closely crowded around the crucial sulcus.

19. Their arrangement in the limbic and parietal boundaries of the sulcus differ.

20. Whilst the limbic and frontal lobes in the Pig, Sheep, Cat, and Ocelot have areas of the five-laminated cortex, greater variation is found in its disposition over the parietal regions.

21. The main feature of importance observed in the regional distribution of the ganglionic series in the *Carnivora* is its concentration around a limited area embracing the crucial sulcus; in *Man* and the *Ape* its wide-spread area conjoined to great variations in developmental complexity; and in the *Pig* and the *Sheep* its wide-spread distribution conjoined to a notable uniformity throughout in complex relationships.

22. The ganglion cells of the *Pig* and *Sheep* differ wholly in type from those of higher Mammals, and approach closely in appearance the large pyramidal cells of the third layer in Man and the Ape.

23. The appearance presented by the first layer in all this series of animals indicates very strongly the proportionate increase of the connective element over the nervous in animals lower in the scale of organisation.

24. The globose cell characterised by its contour and great deficiency of organic connexions, and which is found in large numbers in the brain of idiots, reappears as a normal element in the cortex of the Pig and Sheep.

DIMENSIONS of Ganglionic Cells in Cortex of Pig. (See fig. 2.)

Site of section.	**Limbic lobe.**	**Fourth parietal gyrus.**	**Third parietal gyrus.**	**First and second parietal gyrus.**
	μ μ	μ μ	μ μ	μ μ
A. . . .	47 × 17*	42 × 20	..	37 × 21
B. . . .	48 × 17†	37 × 21	..	37 × 18
C. . . .	41 × 11	36 × 19	26 × 18	28 × 18
D. . . .	38 × 18	39 × 24	31 × 18	37 × 18
E. . . .	28 × 17	40 × 17	26 × 20	37 × 27
F. . . .	25 × 19	25 × 19	33 × 19	21 × 19
G. . . .	..	29 × 17	29 × 17	29 × 17

Note.—μ, or micromillimetre = ·001 millim.

DEPTH of Cortical Layers from before backward.

LIMBIC LOBE.

	mm.	mm.	mm.	mm.	mm.	mm.		mm.
First layer	·790	·558	·558	·511	·651	·651	Average =	·619
Second and third layers .	·930	·837	·799	1·488	·697	·744	,,	·897
Ganglionic layer	·650	·465	·604	·372	·697	·558	,,	·557
Spindle layer	2·604	2·617	1·739	2·604	1·488	1·813	,,	2·144

UPPER PARIETAL CONVOLUTIONS.

	mm.	mm.	mm.	mm.	mm.	mm.		mm.
First layer	·372	·372	·372	·465	·744	·558	Average =	·547
Second and third layers .	·937	·744	·744	1·116	1·116	1·116	,,	·962
Ganglionic layer	·372	·279	·232	·186	·372	·372	,,	·302
Spindle layer	1·600	1·255	1·730	2·139	1·999	1·395	,,	1·689

LOWER PARIETAL CONVOLUTIONS.

	mm.	mm.	mm.	mm.	mm.	mm.		mm.
First layer	·418	·372	·372	·558	·372	..	Average =	·418
Second and third layers .	·930	1·300	1·116	·744	·930	1·116	,,	1·022
Ganglionic layer	·232	·373	·232	·558	·232	·325	,,	·344
Spindle layer	1·860	1·860	2·232	1·488	1·488	2·232	,,	1·860

---

* Largest cell observed = 69μ × 32μ.

† Largest cell observed = 70μ × 27μ.

REFERENCES TO WOODCUTS IN TEXT.

Fig. 1. Right hemisphere of Pig. Internal aspect.
  1. Superior arc of the great limbic lobe.
  2. Inferior arc of the great limbic lobe.
  3. Retro-limbic annectant.
  4. Superficial portion of the superior arc (great limbic lobe).
  5. Sub-frontal sulcus.
  6. Sub-parietal sulcus.
  7. Commencement of crucial sulcus.
  8. Inferior portion of the great limbic fissure.
  10. Inner aspect of the frontal lobe.

Fig. 2. Left hemisphere of Pig. External aspect.
  9. Fissure of ROLANDO.
  10. Frontal lobe: portion corresponding to ascending frontal convolution.
  11. Anterior part of parietal lobe, corresponding to ascending parietal convolution. The tiers of parietal gyri terminate here (gyrus post-Rolandique).
  12. Inter-parietal sulcus.
  13. Sylvian fissure.
  14. Temporal lobule of the parietal lobe.
  15. First parietal or Sylvian convolution.
  15A. Anterior limb of first and second parietal convolutions united.
  16. Posterior limb of second parietal gyrus.
  17. Third parietal convolution.
  18. Fourth parietal or sagittal convolution.
  19. Olfactory lobe.

Fig. 3. Brain of Pig. Illustrative of the distribution of the five-laminated cortex over the limbic, frontal, and parietal lobes.

Fig. 4. Brain of Sheep (internal aspect of right hemisphere). Illustrative of the distribution of the five-laminated cortex over the frontal and great limbic lobes.

Fig. 5. Brain of Cat (inner aspect of left hemisphere). Illustrative of the distribution of the five-laminated cortex over the frontal and great limbic lobes.

Fig. 6. Brain of Cat (outer aspect of left hemisphere). Illustrative of the distribution of the five-laminated cortex over the frontal, limbic, and parietal lobes.

Fig. 7. Brain of Cat (frontal aspect). Illustrative of the distribution of the five-laminated cortex over the frontal, limbic, and parietal lobes.

Fig. 8. Brain of Barbary Ape. Illustrative of the distribution of the five-laminated cortex over the upper surface of the hemisphere.

Fig. 9. Brain of Ocelot (*Felis pardalis*). Illustrative of the distribution of the five-laminated cortex, as seen from the vertex.

REFERENCES TO PLATES.

PLATE 6.

The five-laminated cortex cerebri of the Sheep and Pig, as seen in vertical sections taken from the limbic lobe.

A. First layer exhibiting large numbers of DEITER's cells beneath pia mater.
B. Second layer of small pyramidal and angular elements which in the Pig form by their branches a dense meshwork in the lower regions of the first layer.
C. Third layer of large pyramidal cells.
D. Fourth layer or ganglionic series.
E. Spindle cells of fifth layer.

PLATE 7.

From the six-laminated cortex of the Sheep. The five upper layers are alone seen, the spindle or sixth layer being excluded. A belt of small angular elements is seen at D, interposed between the large pyramidal (C) and the ganglionic layers (E). × 55.

A group of ganglionic cells from the five-laminated cortex of the Pig. × 270.

Section through the cortex cerebri of the Pig. This section having been taken from a sulcus in the limbic lobe exhibits the laminar or solitary arrangement of the ganglionic cells, as contrasted with the appearance of the "nests" of BETZ seen in the group above. × 98.

Note.—All the above have been sketched under high powers, and reduced subsequently by photography.

IV. *Experimental Researches on the Electric Discharge with the Chloride of Silver Battery.*

By WARREN DE LA RUE, *M.A., D.C.L., F.R.S., and* HUGO W. MÜLLER, *Ph.D., F.R.S.*

Received August 7,—Read November 20, 1879.

[PLATES 8–10.]

PART III.—TUBE-POTENTIAL; POTENTIAL AT A CONSTANT DISTANCE AND VARIOUS PRESSURES; NATURE AND PHENOMENA OF THE ELECTRIC ARC.

Tube-potential.

DURING the course of the experiments described in Part II.* it could not fail to be noticed that the potential necessary to produce a discharge in partially exhausted tubes diminished with the pressure until a certain minimum pressure had been attained; and that after this minimum had been reached, the potential had to be increased as the rarefaction was carried on, until at last 11,000 cells could no longer produce a discharge. Although Part II. contains many measurements from which it would be possible to calculate the tube-potential (the potential necessary to produce a discharge in a tube) for certain pressures, yet as they would not form a continuous series it was deemed desirable to make a special set of experiments with a constant number of cells, 11,000. This we have done in the case of hydrogen with a new tube, 162, 33 inches long and 2 inches in diameter, the distance between the ring and straight-wire terminals being 29·75 inches. In commencing each set of experiments the deflection of a tangent-galvanometer was observed when the battery was short-circuited; by a table previously calculated, the value of the deflection in ohms of resistance per cell could be read off: this multiplied by 11,000 gave the total resistance of the battery; the tube was then connected with the terminals and the galvanometer again observed, this gave a less deflection and indicated a greater resistance, which, multiplied by 11,000, gave the total resistance of the tube and battery; by subtracting the resistance of the battery the resistance of the tube was ascertained. Calling the total resistance R, the tube-resistance r, the tube-potential V, $V=\frac{r\times 11,000}{R}$. Thus in the first experiment cited, the battery, short-circuited, gave a deflection of 61°, indicating a total battery-resistance of 230,000 ohms; when the tube was connected a deflection of 7°, indicating a total resistance, tube and battery, of 3,383,000 ohms $=R$, $3,383,000-230,000=3,153,000=r$ the tube resistance, $\frac{3,153,000\times 11,000}{3,383,000}=10,250=V$. The deflection observed was found to be different when the ring was made positive and negative respectively; the following results were obtained in each particular case:—

---

* Phil. Trans. for 1878, Part I. (Vol. 169).

Observations July 3rd, 1878.—11,000 cells, short-circuit, deflection 61°, total battery-resistance = 230,000 ohms; deflections afterwards taken with the tube connected, the ring being positive.

Pressure.		Deflection with tube.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M.	ring +			cells.
35·5	46,710	7°·0	3,383,000	3,153,000	10,250
32·8	43,158	4·5	5,314,000	5,084,000	10,520
31·3	41,184	5·0	4,760,000	4,530,000	10,470
29·2	38,421	5·0	4,760,000	4,530,000	10,470
28·0	36,842	6·5	3,654,000	3,424,000	10,300
26·0	34,211	14·0	1,664,000	1,434,000	9479
25·0	32,895	14·0	1,664,000	1,434,000	9479
23·8	31,316	15·5	1,502,000	1,272,000	9340
21·7	28,553	18·5	1,242,000	1,012,000	8950
20·8	27,368	20·0	1,141,000	911,000	8782
19·8	26,053	21·5	1,054,000	824,000	8592
18·7*	24,605	21·0	1,082,000	852,000	8660
17·8†	23,026	23·0	976,600	746,600	8410
16·0	21,053	25·0	890,600	660,600	8157

July 4th.—11,000 cells, short-circuit, deflection 57°, total battery-resistance = 263,600 ohms.

Pressure.		Deflection with tube.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M.	ring +			cells.
13·5	17,763	26°·0	851,100	587,500	7593
12·1	15,921	27·0	815,100	551,500	7446
10·7	14,079	29·0	749,200	485,600	7129
9·0‡	11,842	38·0	531,600	268,000	5546
7·9	10,395	37·0	556,200	292,600	5788
6·8	8947	43·0	445,300	181,700	4487
5·3	6974	43·0	445,300	181,700	4487
4·6	6053	48·0	374,000	110,400	3247

* One luminosity like that in Plate 15, fig. 1, Part II.

† Two luminosities like those in Plate 15, fig. 2, Part II.

‡ A phase as 4, fig. 50, Part II., p. 192.

July 4th, 4 P.M.—11,000 cells, short-circuit, deflection 58°, total battery-resistance = 253,300 ohms.

Pressure.		Deflection with tube.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M.	ring + °			cells.
4·3	5658	46·0	400,300	147,000	4039
3·8	5000	45·0	415,000	161,700	4285
3·41	4487	46·0	400,300	147,000	4039
2·789	3670	51·0	336,200	82,900	2713
2·488	3273	50·0	348,500	95,200	3004
2·327	3062	51·0	336,200	82,900	2713
1·966	2587	51·0	336,200	82,900	2713
1·745	2296	53·0	313,000	59,700	2098

July 5th.—11,000 cells, short-circuit, deflection 58°, total battery-resistance = 253,300 ohms.

Pressure.		Deflection with tube.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M.	ring + °			cells.
1·023	1346	54·0	302,100	48,800	1776
0·822	1082	51·0	336,200	82,900	2713
0·642	845	57·0	263,600	10,300	430
0·421	554	49·0	360,800	107,500	3278
0·341	449	49·0	360,800	107,500	3278
0·265	349	45·0	413,000	159,700	4254
0·169	222	48·0	374,000	120,700	3550
0·090	126	43·0	445,300	192,000	4742
0·076	100	42·0	461,500	208,200	4963
0·046	61	38·0	531,600	278,300	5758

July 6th.—11,000 cells, short-circuit, deflection 60°, total battery-resistance = 239,500 ohms.

Pressure.		Deflection with tube.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M.	ring + °			cells.
0·023	30·0	34·0	615,800	376,300	6720
0·013	17·0	32·0	664,600	425,100	7036
0·008	10·5	31·0	691,100	451,600	7171
0·0074	9·7	27·0	815,100	575,600	7769
0·0070	9·2	22·0	1,028,000	788,500	8431
0·0065	8·6	18·0	1,277,000	1,037,500	8937

The following observations were made when the straight wire was made positive; when the contact-key (figs. 2 and 3, Part I., page 59) was in the position fig. 2, this was effected. It was noticed that when in this position the current of the battery itself, when short-circuited, was less than when the key was reversed as for the first series, when the ring was positive, the contact being more perfect in one or other case.

JULY 4th.—11,000 cells, short-circuit, deflection 54°, total battery-resistance = 302,100 ohms.

Pressure.		Deflection with tube.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M.	ring – °	ohms.	ohms.	cells.
18·7	24,605	17·0	1,358,000	1,055,900	8531
17·8	23,421	15·0	1,550,000	1,247,900	8837
16·0	21,053	29·0	749,200	447,100	6564

11,000 cells, short-circuit, deflection 50°, total battery-resistance = 348,500 ohms.

Pressure.		Deflection with tube.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M.	ring – °	ohms.	ohms.	cells.
13·5	17,763	24·0	932,600	584,100	6890
12·1	15,921	25·0	890,900	542,100	6695
10·7	14,079	29·0	749,200	400,700	5884
9·0	11,842	31·0	691,100	342,600	5453
7·9	10,395	34·0	615,800	267,300	4775
6·8	8947	37·0	556,200	207,700	4109
5·3	6974	42·0	461,500	113,000	2694
4·6	6053	42·0	461,500	113,000	2694

11,000 cells, short-circuit, deflection 51°, total battery-resistance = 336,200 ohms.

Pressure.		Deflection with tube.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M.	ring – °	ohms.	ohms.	cells.
4·3	5658	41·0	478,000	141,800	3062
3·8	5000	42·0	461,500	125,300	2986
3·410	4487	43·0	445,300	109,100	2695
2·789	3670	46·0	400,300	64,100	1762
2·488	3273	45·0	415,000	78,800	2088
2·327	3062	46·0	400,300	64,100	1762
1·966	2587	45·0	415,000	78,800	2088
1·745	2296	50·0	348,500	12,300	388

JULY 5th.—11,000 cells, short-circuit, deflection 51°, total battery-resistance = 336,200 ohms.

Pressure.		Deflection with tube.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M.	ring − °	ohms.	ohms.	cells.
1·023	1346	47·0	386,700	50,500	1436
0·822	1082	50·0	348,500	12,300	388
0·642	845	47·0	386,700	50,500	1436
0·421	554	47·0	386,700	50,500	1436
0·341	449	46·0	400,300	64,100	1762
0·265	349	47·0	386,700	50,500	1436
0·169	222	49·0	360,800	24,600	750
0·096	126	47·0	386,700	50,500	1436
0·076	100	45·0	415,000	78,800	2088
0·046	61	44·0	430,700	94,500	2415

The observations point clearly to the resistance of a tube diminishing as the pressure decreases up to a certain point, after which it rapidly increases; the pressure of least resistance in the case of hydrogen, as shown by the experiments when the ring was positive, being about 0·642 m.m., 845 M. The resistance increases very rapidly when a pressure of 0·0076 m.m., 10 M, is reached, and the curve then becomes nearly a vertical to the abscissæ. At a pressure of 0·002 m.m., 3 M, the discharge of 11,000 cells only just passed. (Part II., page 187, footnote.)

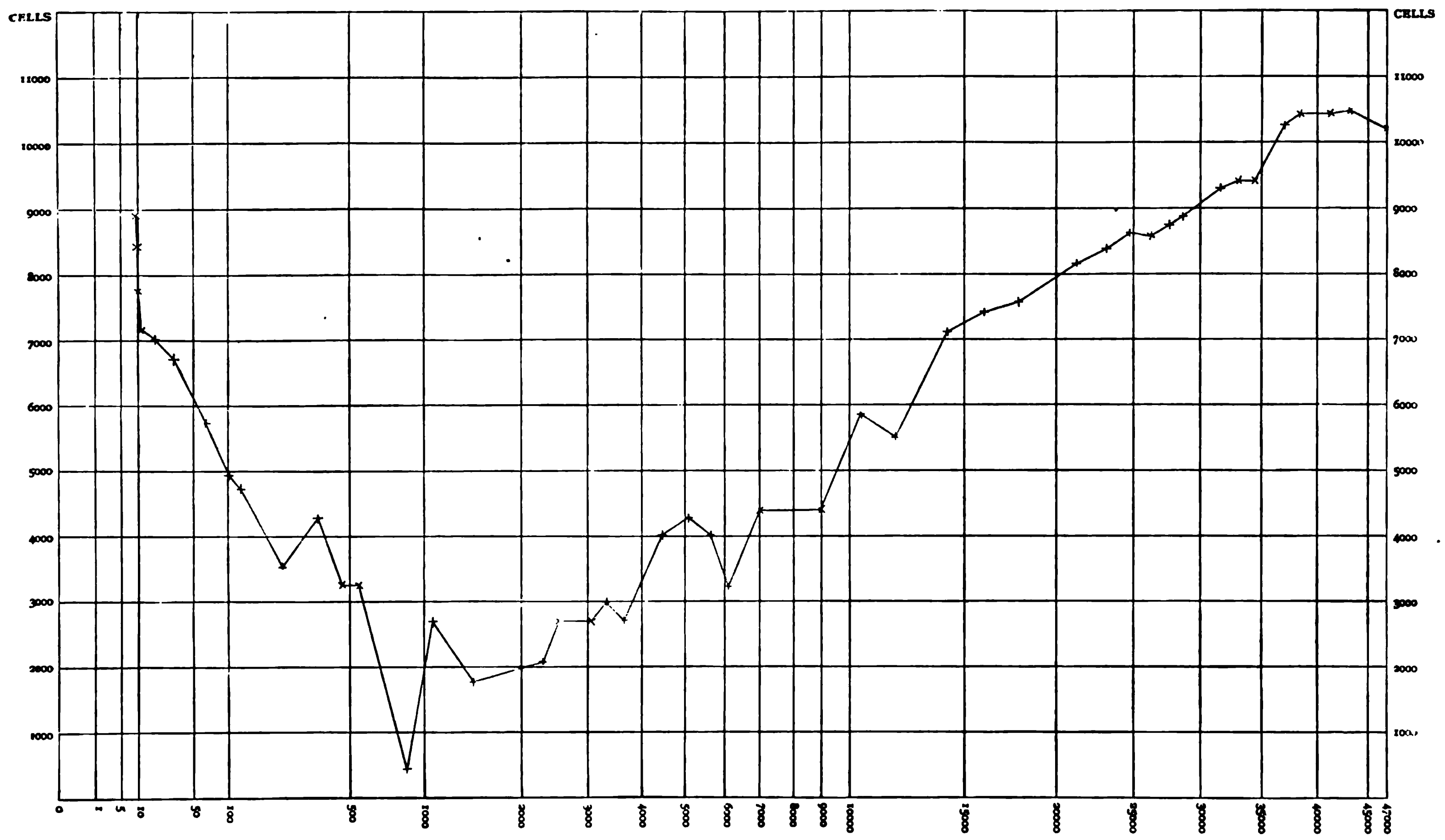
With a pressure of 0·00137 m.m., 1·8 M, 11,000 cells would not pass, and with a pressure of 0·000055 m.m., 0·066 M, even a 1-inch spark from an induction coil would not pass. (Part II., page 215.)

The diagram (fig. 71) laid down from the results when the ring was made positive, shows the curve of the observations as actually obtained without being smoothed. The figure is a reduction to $\frac{1}{3\cdot5}$ of the original; the abscissæ are as the cube-roots of the various pressures in millionths of an atmosphere, and show relatively the number of molecules in a given linear space, the ordinates are as the number of cells.

The life of the battery is so much exhausted by such a series of experiments, in consequence of the length of time the current has to remain on while the galvanometer becomes steady, that it was not deemed desirable to extend them to other gases or to repeat the experiments in order to obtain an average smoother curve for hydrogen. There is, however, no reason to doubt the accuracy of the results as recorded at the moment of each experiment, for the character of a tube is continually altering during a continuous series of experiments, and no precautions can be taken which will render a tube absolutely constant for any length of time. In order to avoid running down the battery, other methods for determining the tube-potential were subsequently adopted; these are described in notes A and B in the Appendix.

When the observations with the ring positive were laid down with the abscissæ representing millionths (not their cube-roots) in order to extend the scale, and a smooth

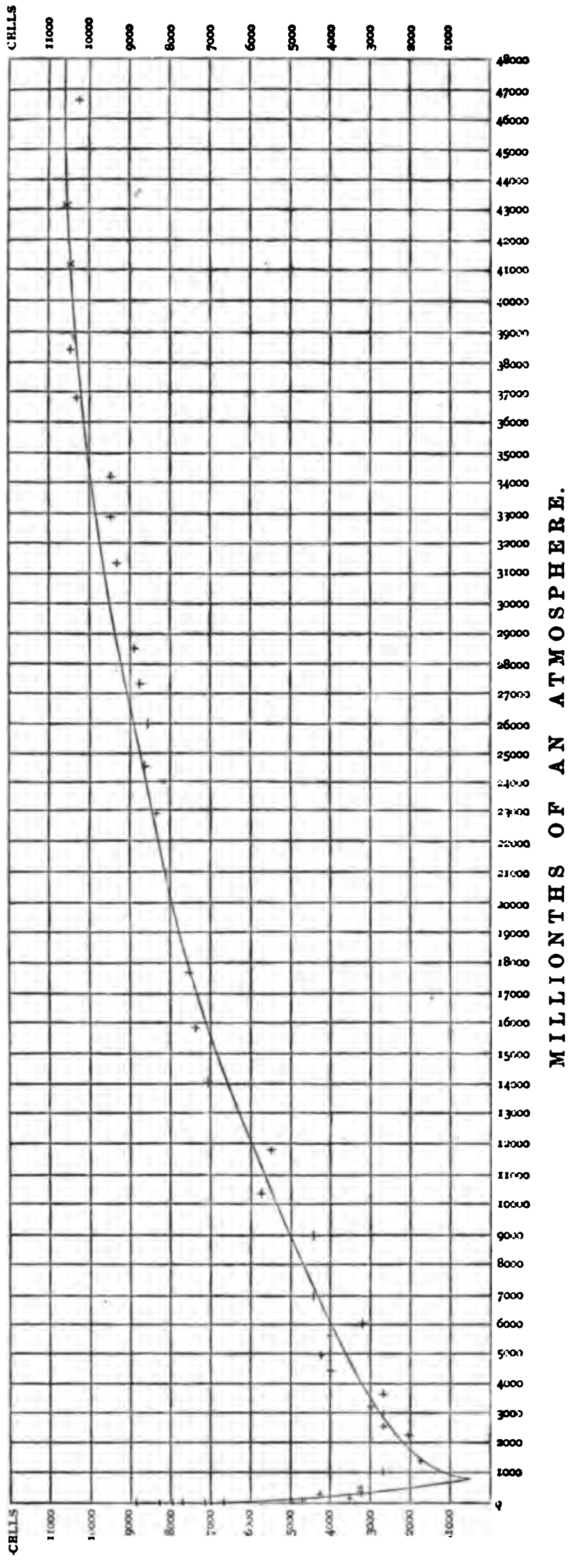
Fig. 71.



ABSCISSÆ AS THE CUBE ROOT OF THE PRESSURE IN MILLIONTHS OF AN ATMOSPHERE.

THE DISTANCES 0 TO 1, 0 TO 100, 0 TO 1000, &c., SHOW RELATIVELY THE NUMBER OF MOLECULES IN A GIVEN LINEAR SPACE.

Fig. 72.


CELLS
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
48000
47000
46000
45000
44000
43000
42000
41000
40000
39000
38000
37000
36000
35000
34000
33000
32000
31000
30000
29000
28000
27000
26000
25000
24000
23000
22000
21000
20000
19000
18000
17000
16000
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
MILLIONTHS OF AN ATMOSPHERE.

curve was drawn through them, as in fig. 72, which is a reduction to one-sixth, the following numbers were read off from measurements on the diagram :—

Pressure.	V.		Pressure.	V.	
	Cells.	Increase per 1000 M.		Cells.	Increase per 1000 M.
M		cells.	M		cells.
845	430		23,000	8490	
1000	1000		24,000	8630	140
1500	1780	1190	25,000	8800	170
2000	2190		26,000	8960	160
3000	2780	590	27,000	9100	140
4000	3230	475	28,000	9250	150
5000	3660	430	29,000	9390	140
6000	4030	370	30,000	9530	140
7000	4380	350			
8000	4750	370	31,000	9650	120
9000	5070	320	32,000	9770	120
10,000	5380	310	33,000	9880	110
			34,000	9980	100
11,000	5710	330	35,000	10,070	90
12,000	6030	320	36,000	10,150	80
13,000	6350	320	37,000	10,230	80
14,000	6630	280	38,000	10,300	70
15,000	6900	270	39,000	10,360	60
16,000	7160	260	40,000	10,420	60
17,000	7400	240			
18,000	7630	230	41,000	10,475	55
19,000	7840	210	42,000	10,520	45
20,000	8000	180	43,000	10,550	30
			44,000	10,580	30
21,000	8180	180	45,000	10,590	10
22,000	8340	160	46,000	10,600	10
23,000	8490	150	47,000	10,600	0

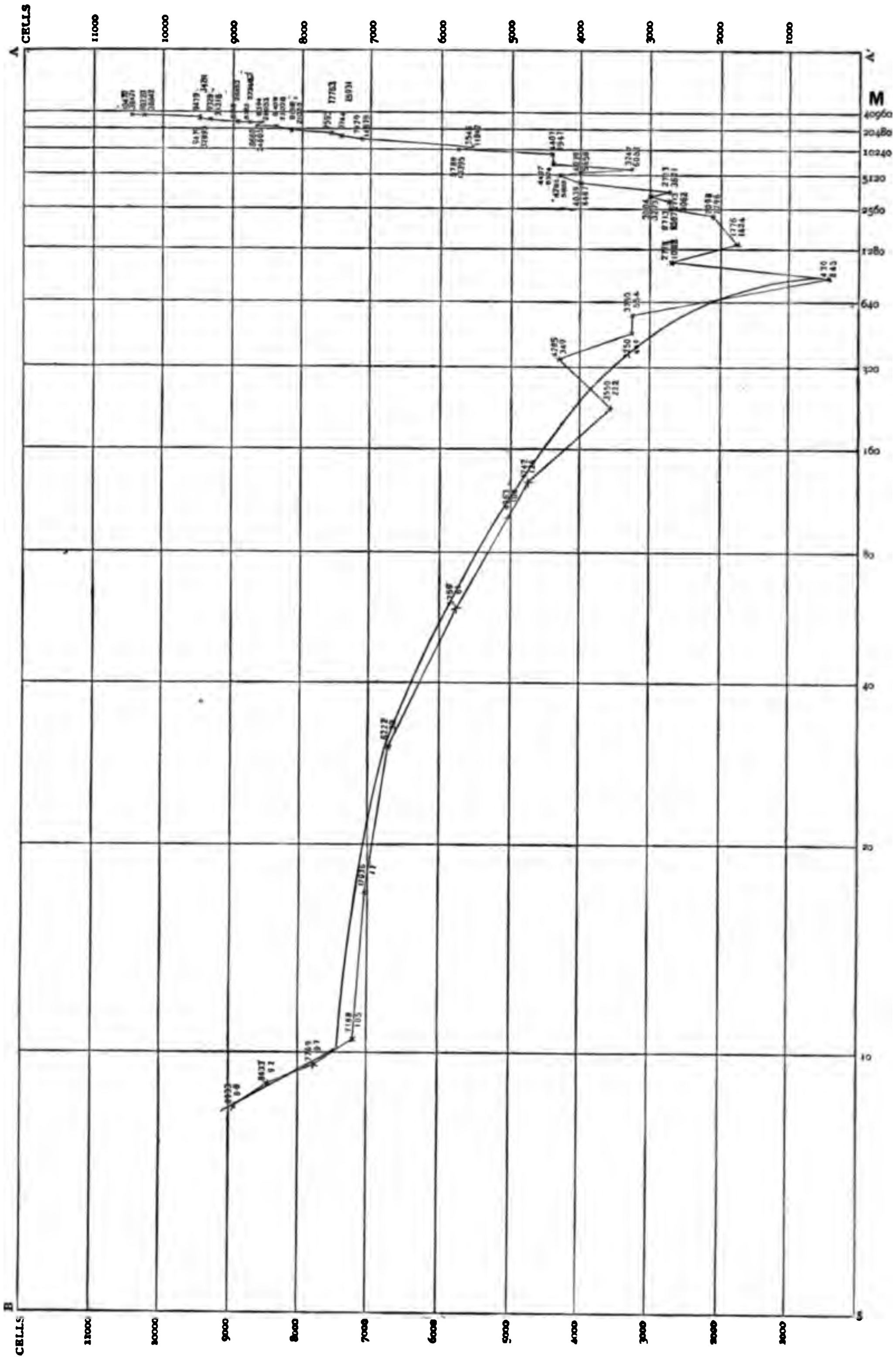
In order to obtain a smooth curve for the observations beyond the minimum of resistance of the tube, they were again plotted down so as to make the abscissæ represent relatively the mean distance of the molecules at the various pressures, as in fig. 73, which is a reduction of the original diagram to one-third; this has the effect of extending the scale for this portion of the curve, while it compresses the scale on the opposite side for increasing pressures.

The following were read off:—

Pressure.	V.		Pressure.	V.	
	Cells.	Decrease per 10 M increase.		Cells.	Decrease per 10 M increase.
M.		cells.	M.		cells.
8	9600		90	5280	
		11,400			135
9	8460		100	5145	
		9600			94·5
10	7500		200	4200	
		420			60
20	7080		300	3600	
		358			48
30	6722		400	3120	
		332			45
40	6390		500	2670	
		300			39
50	6090		600	2280	
		270			45
60	5820		700	1830	
		195			51
70	5625		800	1320	
		180			16
80	5445		900		
		165			
90	5280		1000	1000	

The various phenomena of stratification observed in tube 162 during the course of the preceding series of measurements of tube-potential were very similar to those already described under like conditions in tubes 129 and 139 containing residual hydrogen, in Part II., and represented in plates 15 and 16 of that part. Sketches were made of the various forms of strata, but it would not have been possible to take photographs of these phenomena, as this would have seriously interfered with the continuity of the measurements and have prolonged their duration disadvantageously for the battery. A particular phase worthy of record was, however, observed which had been seen before with tube 145, and which has been described in Part II., experiment 218, page 215. Of this and two preceding phases, photographs were taken and are shown in Plate 9, figs. 1, 2, and 3. The pressure in all three cases was 0·747 m.m., 983 M; the current was not measured, but the potential used was 4800 cells. On making the straight wire negative, the appearance shown in fig. 1 was seen: this is copied from a photograph obtained in 5 seconds.

Fig. 73.



A—B REPRESENTS AN ASSUMED MEAN DISTANCE OF THE MOLECULES AT A PRESSURE OF 5 MILLIONTHS OF AN ATMOSPHERE.

A' TO 10, A' TO 20, ... A' TO 40960 THE CORRESPONDING DISTANCES AT PRESSURES 10, 20, 40960 MILLIONTHS.

On making the straight wire positive the phase was produced which is represented in Plate 9, fig. 2; this is copied from a photograph obtained in 7 seconds. It will be noticed that several of the cup-shaped strata after the first three, counting from the negative terminal, are followed by a secondary series which are tongue-shaped; these latter kept continually disappearing and reappearing precisely in the same position, so that the photograph is in no way indistinct or blurred.

On introducing a resistance of 230,000 ohms, the particular phase to which it is desired to call attention was obtained; it is represented in Plate 9, fig. 3, copied from a photograph obtained in 7 seconds. The somewhat confused discharge near the positive shown in the previous figure has disappeared, and tongue-shaped strata cross each other like the component lines of the letter X: fig. 3 is an exact copy of the perfectly distinct photograph. Such complex phenomena present many difficulties in the way of a theoretical explanation of the forces which concur in their production.

Potential necessary to produce a discharge between discs at a constant distance and at various pressures.

In the first instance, at the suggestion of Professor MASCART, an experiment was made in order to ascertain whether there was either any condensation or dilatation of the gas about the terminals before the actual passage of the discharge. In order to do this an apparatus was constructed as shown in fig. 74.

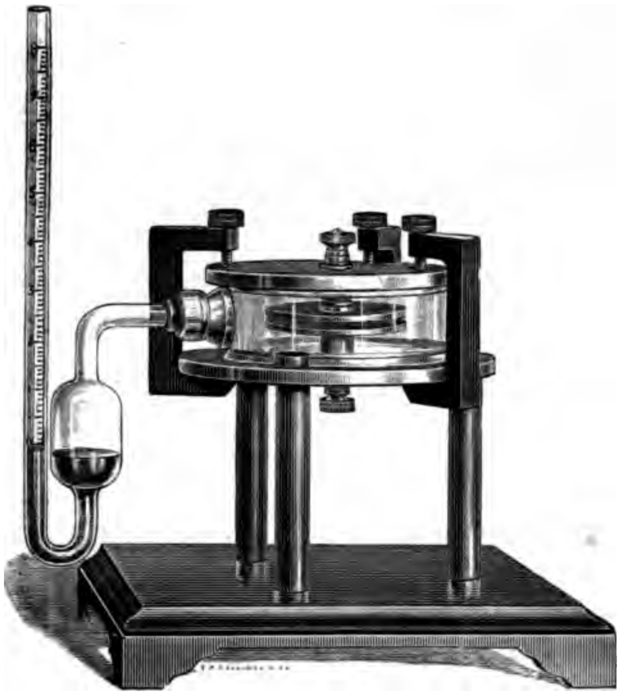
It consists of a glass cylinder, 4·35 inches inside diameter, the depth of which is accurately the same in every part, 1·6 inch, so as to ensure the parallelism of two glass discs which close its ends. Its cubical content, exclusive of the terminals, was found to be 385 cub. centims.

These are held in contact with the ends of the cylinder by means of screw-clamps made of ebonite, and the whole apparatus is supported on a tripod ebonite stand, which is fastened to a square wooden foot. Attached parallel to the top and bottom glass discs, by means of flanged-screw rods, are two brass discs with rounded edges 3·1 inches diameter; these are maintained at a distance of 0·13 inch, at which the discharge of 11,000 cells would only just take place.

The ends which project through the glass discs are furnished with binding-screws for attaching wires from the battery.

On the side of the cylinder is a tubulure in which is fitted a gauge containing strong sulphuric acid, so as to dry the inside of the apparatus, and to indicate whether any condensation or dilatation of the gas contained in the cylinder occurs on connecting the metallic discs with the battery by means of the contact-key. The edges of the cylinder were rubbed with grease, and care was taken to prove that the apparatus was perfectly tight, by causing the fluid in the limb of the gauge to stand for some time higher than that in the bulb. When contact was made with a battery of 9800 cells there was not the slightest indication of any alteration of volume of the contained gas, so that there was neither condensation about the discs which would have caused a contraction, nor repulsion from the discs which would have caused an expansion of volume. The fluid in the stem, whose internal diameter is 0·185 inch, was observed with a lens, but not the slightest motion of it took place. The same result was noticed even when water was substituted for sulphuric acid. So far, then, as this apparatus, in which the area of the gauge is $\frac{1}{280}$th of that of each disc, would indicate it, the result is entirely negative.

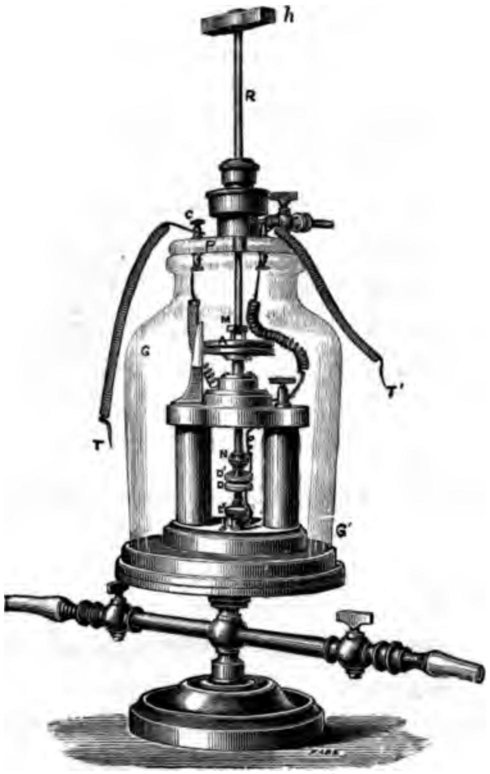
Fig. 74.



The discharge between two discs 1·5 *inch in diameter.*

The following series of experiments were made by placing the micrometer-discharger under a bell-jar, and in the first instance adjusting the discs to the striking distance at atmospheric pressure for the battery of 11,000 cells. Afterwards a less number of cells was connected with the discs and the bell-jar gradually exhausted until the discharge occurred; the height of the gauge was then read off. Then a less and lesser number of cells was connected with the discs and the operation was repeated. Fig. 75 shows the arrangement of the micrometer discharger under the bell-jar. The gauge used to indicate the pressure beyond the range of the gauges attached to the pumps is not shown in this figure, but is in fig. 81 to be hereinafter described.

Fig. 75.



AIR.—The discs distant 0·13 inch = 3·3 m.m.

I.			II. Temp. 15°·2 C.				III. Temp. 11°·8 C.			
Cells.	Pressure.		Cells.	Pressure.		Current.	Cells.	Pressure.		Current.
	m.m.	M.		m m.	M.	W.		m.m.	M.	W.
10,940	761·6	1,002,105	10,980	709·6	933,684	0·03071	10,980	700·3	921,447	0·03259
9900	621·0	817,105	9920	636·5	837,500	0·03259	9780	620·9	816,974	0·03071
8840	537·7	707,500	8840	548·7	721,974	0·03071	8580	537·7	707,500	0·02881
7760	446·4	587,368	7760	456·0	600,000	0·03657	7380	440·8	580,000	0·02698
6300	355·6	467,894	6300	358·7	471,974	0·15470	6180	349·5	459,868	0·02881
4800	254·9	335,395	4500	244·6	321,842	0·16080	4680	243·7	320,658	0·01412
3600	177·6	233,684	3600	188·5	248,026	0·14350	3220	158·9	202,500	0·03071
2400	105·3	138,552	2400	109·1	143,552	0·16080	2140	82·1	108,026	0·02881
1200	38·0	50,000	1200	38·3	50,394	0·13340	1060	31·1	40,921	
600	10·6	13,947	600	9·7	12,763	0·07036				
300*										

It will be observed that although the discharge took place at a pressure beyond 760 m.m. in Series I., it would not do so in II. and III. The greater current in some instances is in a great measure to be attributed to more rod-cells being employed in some of the experiments, as these offer less internal resistance than powder-cells.

---

* Would not pass at 2·2 m.m., 2631 M.

HYDROGEN.—Discs distant 0·22 inch = 5·588 m.m.

IV. Temp. 15°·6 C.				V. Temp. 12°·7 C.			VI. Temp. 10°·2 C.		
Cells.	Pressure.		Current.	Cells.	Pressure.		Cells.	Pressure.	
	m.m.	M.	W.		m.m.	M.		m.m.	M.
10,980	760·6	1,000,789	0·03071	1·0980	759·7	999,605	10,980	751·0	988,158
9780	671·8	883,947	0·02881	9780	658·9	866,974	9780	661·0	869,737
8580	564·6	742,895	0·02881	8580	559·6	736,316	8580	576·5	758,553
7380	472·3	621,447	0·02881	7380	483·8	636,579	7380	466·5	613,816
6180	400·6	527,105	0·02698	6180	395·0	519,737	6180	387·6	510,000
4680	271·9	357,763	0·01772	4680	272·8	358,947	4680	258·5	340,132
3220	160·1	210,658	0·03259	3220	178·5	228,289	3220	164·6	216,579
2140	97·4	128,158	0·02505	2140	98·1	129,079	2140	93·6	123,158
1060	37·0	48,684	0·02136	1060	37·8	49,737	1080	38·6	50,789
600	13·9	18,289	0·05801	600	15·4	20,263	600	12·5	16,447

CARBONIC ACID.—Discs distant 0·122 inch = 3·096 m.m.

VII. Temp. 12°·2 C.			VIII. Temp. 14°·5 C.			IX. Temp. 11°·6 C.		
Cells.	Pressure.		Cells.	Pressure.		Cells.	Pressure.	
	m.m.	M.		m.m.	M.		m.m.	M.
10,960	757·9	997,237	10,960	747·2	983,158	10,960	747·6	983,684
9880	693·0	911,842	9880	647·5	851,974	9880	676·7	890,395
8800	602·2	792,368	8800	576·7	758,816	8800	588·9	774,868
7720	492·4	647,894	7720	485·9	639,342	7720	505·1	664,605
6300	370·7	487,763	6300	383·2	504,211	6300	393·3	517,500
4800	277·9	365,658	4800	249·4	328,158	4800	258·6	340,263
3600	186·0	244,737	3600	177·7	233,816	3600	183·8	241,842
2400	104·2	137,105	2400	95·8	126,052	2400	94·9	124,868
1200	31·3	41,184	1200	30·0	39,474	1200	30·0	39,474
			600	5·1	6,710	600	5·3	6974

The numbers obtained for air, hydrogen, and carbonic acid respectively were plotted down on millimetre scale paper, the abscissæ being 1 m.m. = 2500 **M**, the ordinates 1 m.m. = 25 cells, and curves drawn to give a mean of the several observations. These appeared to resemble hyperbolic curves so closely that true hyperbolic curves were found (partly by a geometric construction,[*] partly by computation) which would intersect the mean experimental curves in two points. The results of experiment were again laid down on these new curves, and it will be seen from fig. 76 that they do not differ more from the experimental results than the experimental numbers from each other.

---

[*] The following, suggested to us by Professor STOKES, is the convenient method employed. The experimental curve is laid down on squared paper, as shown by the dotted line in fig. 75*, and two points, g' and h', for the intersection of the proposed hyperbola are selected, and two corresponding points g, h at the same distances respectively on the opposite side of the vertical axis are marked; through g' and h and through g and h' lines are drawn intersecting in A, lines are also drawn through h' and g', and h and g intersecting in B; a line is drawn through A and B and prolonged on each side in the direction of the axis. The distances of VB and VA are ascertained in terms of the squared paper, their reciprocals taken

On the diagram the results for each series are distinguished by no dot, one dot or two dots on the crosses marking the position of the observations (fig. 77). It would appear, therefore, that the hyperbola fairly represents the law of discharge between planes.*

and the difference of these reciprocals obtained; the reciprocal of this difference gives the distance VC the semi-axis major $=a$, C being the centre; the ordinates $y, y^1 \ldots y^{15} \ldots$, are obtained by the formula

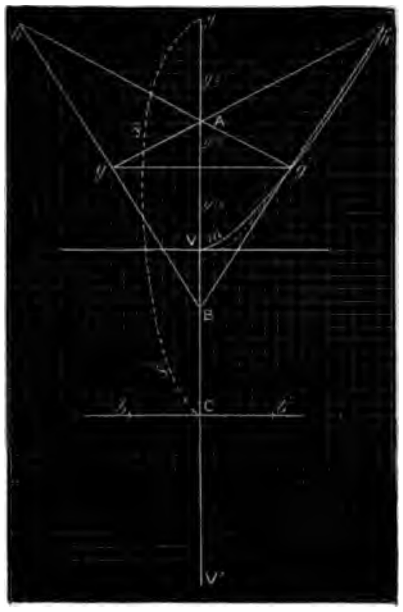
$$y=\frac{b\sqrt{(x+a)(x-a)}}{a};$$

the value of $b=Cb$, the semi-axis minor, is calculated from the measured ordinate y for both the points of intersection g' and h' of the experimental curve and the proposed hyperbola, and their mean taken for the value of b.

$$b=\frac{ya}{\sqrt{(x+a)(x-a)}}.$$

The calculations are simplified by taking $2a=VV'$ and adding it to Vy to give $=x+a=\alpha$, taking Vy as $=x-a=\beta$ and making $\frac{b}{a}=\delta$, the formula then is $y=\sqrt{\alpha\beta}\,.\,\delta$.

Fig. 75ᵃ.



---

* Dr. ALEXANDER MACFARLANE has published in the Transactions of the Royal Society of Edinburgh, 1878, vol. xxvii., an elaborate and careful research of the "Disruptive Discharge of Electricity" in air and different gases, and between terminals of various forms. An abstract of this paper will be found in 'Nature,' December 26, 1878, pp. 184, 185. Dr. MACFARLANE used a HOLTZ machine and employed higher potentials than those used by us; he found that the results for the discharge between two discs 4 inches in diameter at various distances up to 1·2 centims. and with various pressures were satisfactorily represented by the hyperbola.

Fig. 76.

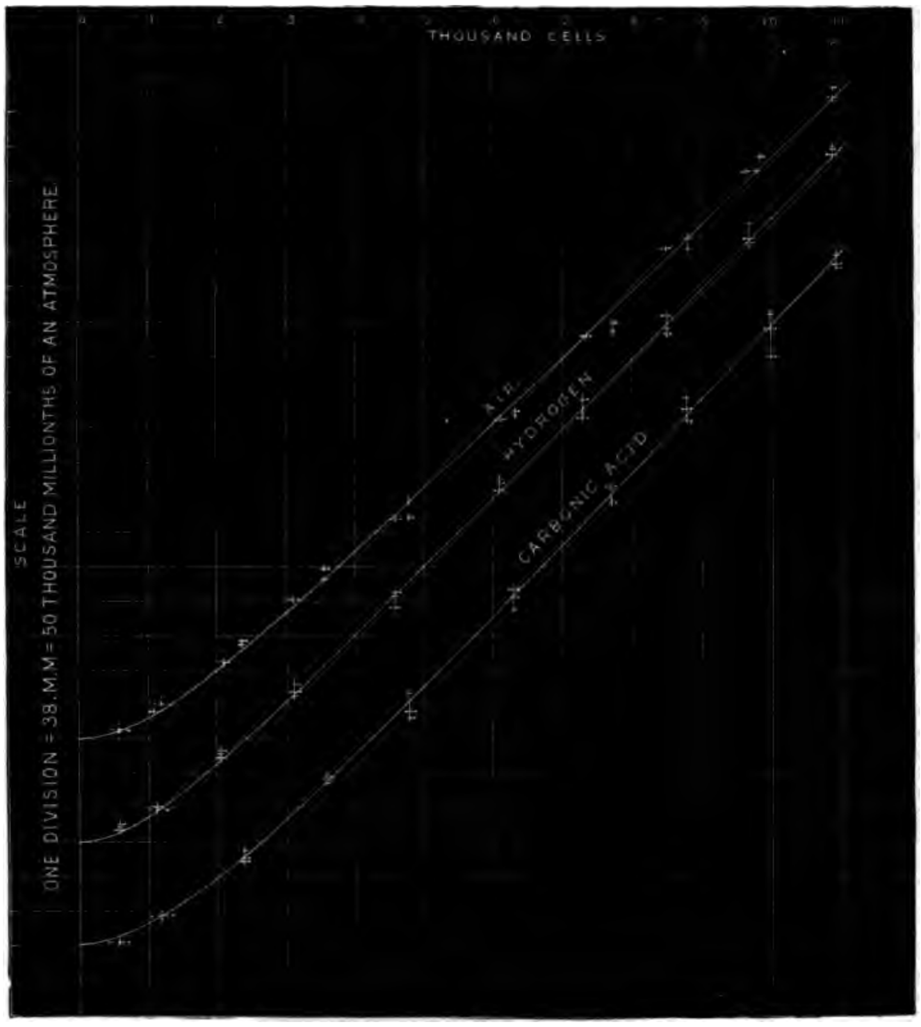

0
1
2
3
4
5
6
7
8
9
10
11
THOUSAND CELLS
SCALE
ONE DIVISION = 38.M.M = 50 THOUSAND MILLIONTHS OF AN ATMOSPHERE
AIR
HYDROGEN
CARBONIC ACID

Fig. 77.

The ratios of the sides of the parallelogram between the assymptotes of these hyperbolas differ somewhat, but not much, from each other, thus—calling CV the transverse axis, CB the conjugate axis.

$$\left.\begin{array}{lll} \text{For air} \ldots\ldots & \dfrac{CV=28{\cdot}16}{CB=29{\cdot}13}=0{\cdot}9665 \\ \quad\text{,, hydrogen} \;. & \dfrac{CV=25{\cdot}56}{CB=25{\cdot}13}=1{\cdot}017 \\ \quad\text{,, carbonic acid} & \dfrac{CV=38{\cdot}6}{CB=36{\cdot}11}=1{\cdot}069 \end{array}\right\}\ \textbf{Mean } 1{\cdot}0175.$$

Measurements of the hyperbolic curve gave the results shown in the following tables. The greatest discordances between the numbers derived from the mean experimental curves and those from the hyperbolic curves are in the lowest pressures up to about 19 m.m., 25,000 M; the potential necessary for a discharge to take place being less than that required by the law of the hyperbola.

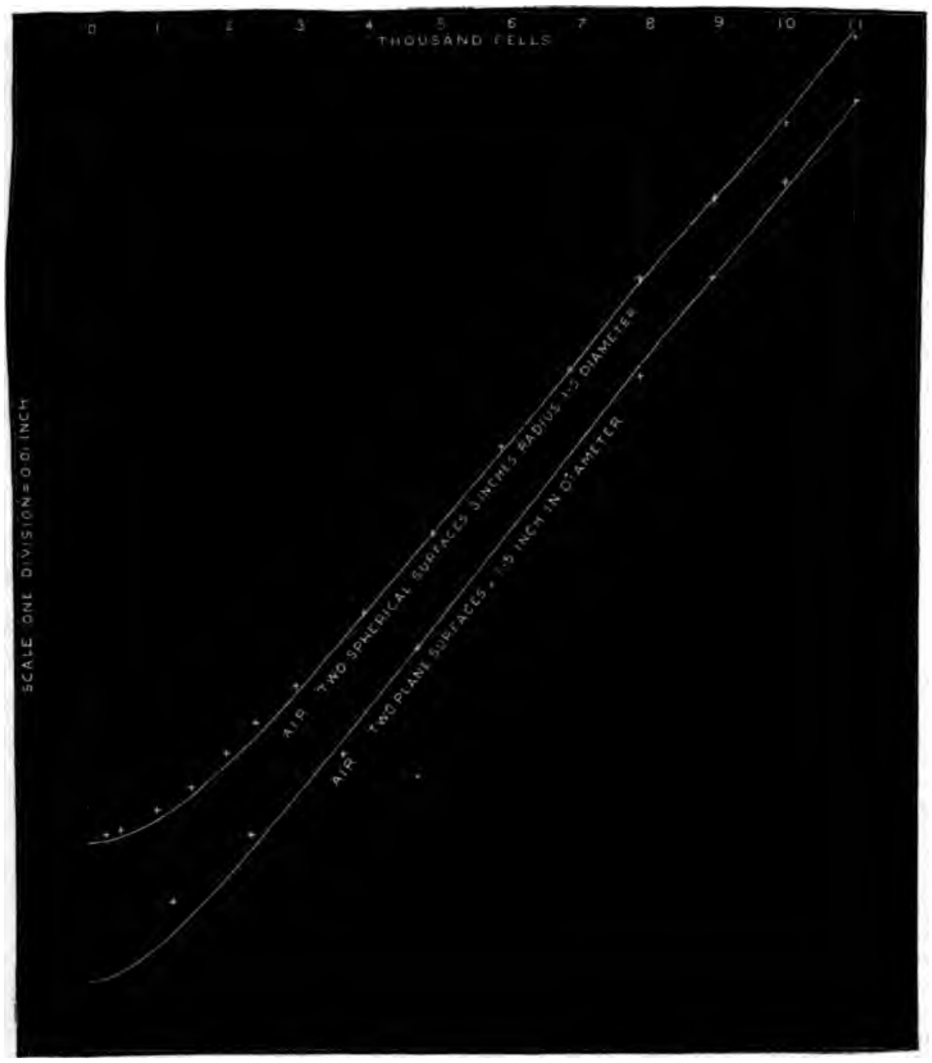
NUMBERS obtained from curves.

Pressure.	Air.		Hydrogen.		Carbonic acid.	
	Hyperbolic curve.	Experimental curve.	Hyperbolic curve.	Experimental curve.	Hyperbolic curve.	Experimental curve.
M.	cells.	cells.	cells.	cells.	cells.	cells.
5000	391·5	250	355	250	413·5	275
15,000	690	575	626·5	525	722·5	625
25,000	905·5	825	823·5	750	949	850
50,000	1332	1275	1215·5	1200	1381	1300
75,000	1692	1625	1549	1575	1739·5	1700
100,000	2021	2000	1854	1925	2062·5	2025
150,000	2632·5	2650	2425	2525	2652·5	2650
200,000	3211	3250	2967·5	3075	3202·5	3200
250,000	3771·5	3825	3495·5	3625	3730	3725
300,000	4321·5	4400	4014	4150	4244	4250
400,000	5402	5475	5035	5175	5246	5275
500,000	6467·5	6550	6043	6125	6226·5	6275
600,000	7523	7600	7044·5	7100	7195	7225
700,000	8575·5	8625	8041	8050	8156·5	8200
800,000	9623·5	9625	9035	9000	9112	9150
900,000	10,670	10,600	10,026·5	9975	10,064·5	10,100
1,000,000	11,712	11,600	11,017	10,925	11,013·5	11,100

The striking distances at atmospheric pressure and various potentials for spherical surfaces 3 inches radius and 1·5 inch diameter, as given in page 68, curve VIII., and at page 118, also those for nearly flat surfaces in pages 73 and 118, Part I., were reduced to millimetres distance and plotted down in the same way, but not on precisely the same scale, as the preceding curves for constant distance and various pressures. Hyperbolic curves were also found which intersected the experimental curves in two points. A reduction of the original diagram is given in fig. 78.

It will be seen that in the case of spherical surfaces, the result having been obtained as the average of a great number of experiments, the hyperbola coincides closely with the observations, while in that of plane surfaces, for which only a few experiments were made, the coincidences are not so perfect. Nevertheless, it would appear that the law of the hyperbola holds equally well for a constant pressure and varying distance as it does for a constant distance and varying pressure; the obstacle in the way of a discharge being as the number of molecules intervening between the terminals up to a certain point.

Fig. 78.



In the two cases of spherical and plane surfaces the ratio between the transverse and conjugate axes of the respective hyperbolas laid down on the scale adopted—

$$\left.\begin{array}{lll}\text{For spherical surfaces} & \dfrac{CV=38{\cdot}87}{CB=32{\cdot}16}=1{\cdot}240 \\ \text{,, discs. . . . .} & \dfrac{CV=28{\cdot}4}{CB=22{\cdot}1}=1{\cdot}285\end{array}\right\}\text{Mean } 1{\cdot}267.$$

Striking distance between two flat discs.

Since the publication of Part I. we have laid down a fresh smooth curve of the striking distance between two flat discs, on the scale of 5 centims. to 1000 cells and 10 centims. for a millimetre, using the following numbers:—

Cells.	Inch.	Centimetres.	
1200	0·012	0·0305	Taken from Part I., page 73, Curve X.
2400	0·021	0·0533	
3600	0·033	0·0838	
4800	0·049	0·1245	
5880	0·058	0·1473	
6960	0·073	0·1854	
8040	0·088	0·2236	
9540	0·110	0·2794	Actual observation.
11,000	0·133	0·3378	

It is not necessary to give a diagram of the curve, which agrees fairly well with the observations; it runs—

Below	that for	1200	cells	by 0·0045 centim.
Through	„	2400	„	
„	„	3600	„	
Below	„	4800	„	by 0·0085 centim.
Above	„	5880	„	„ 0·0015 „
Through	„	6960	„	
Above	„	8040	„	by 0·0010 centim.
„	„	9540	„	„ 0·0010 „
Through	„	11,000	„	

From the curve thus laid down the following numbers were deduced:—*

EMF in volts.	Striking distance in centimetres.	Difference of potential per centimetre.	Intensity of force.	
			Electromagnetic.	Electrostatic.
		volts.		
1000	0·0205	48,770	$4·88 \times 10^{12}$	163
2000	0·0430	46,500	4·65 „	155
3000	0·0660	45,450	4·55 „	152
4000	0·0914	43,770	4·38 „	146
5000	0·1176	42,510	4·25 „	142
6000	0·1473	40,740	4·07 „	136
7000	0·1800	38,890	3·89 „	130
8000	0·2146	37,280	3·73 „	124
9000	0·2495	36,070	3·61 „	120
10,000	0·2863	34,920	3·49 „	116
11,000	0·3245	33,900	3·39 „	113
11,309	0·3378	33,460	3·35 „	112

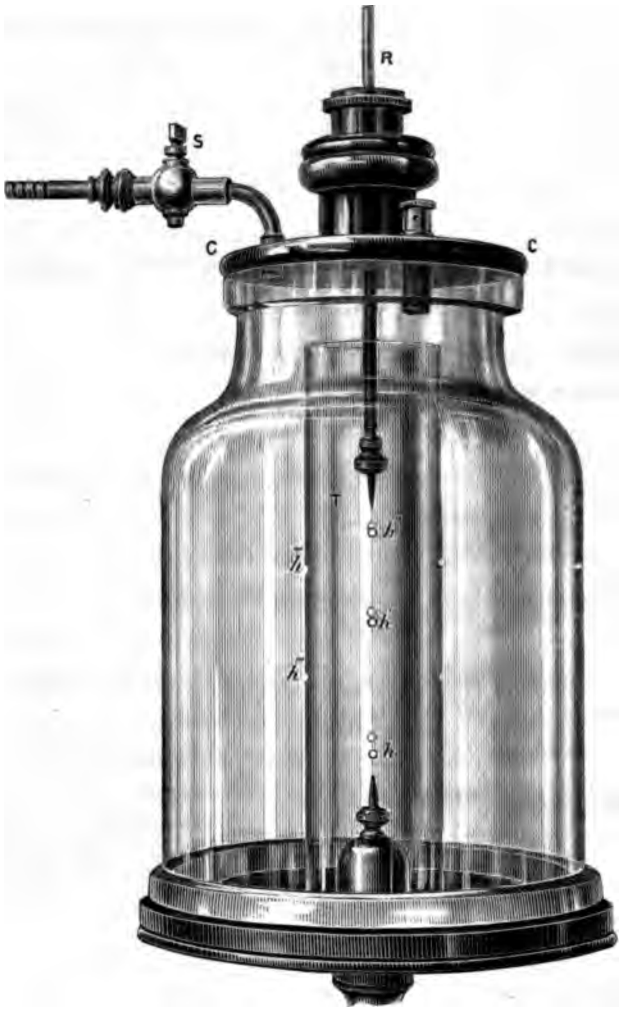
---

* These were communicated, in April, 1878, to Professor EVERETT, for his 'Units and Physical Constants,' 1879, pp. 141, 142.

The electric arc.

We have already stated[*] that "*the discharge in a vacuum tube does not differ essentially from that in air and other gases at ordinary atmospheric pressures; it cannot be considered as a current in the ordinary acceptation of the term.*" . . . We had a strong conviction while writing the above that the stratified discharge in a vacuum tube was simply a magnified form of arc, and in consequence we planned out a series of experiments to test the correctness of this supposition; these were commenced in October, 1878, and although not yet concluded, we venture to lay before the Society an account of the results hitherto obtained, as they seem to support the view that the arc and the stratified discharge are merely modifications of the same phenomenon.

Fig. 79.



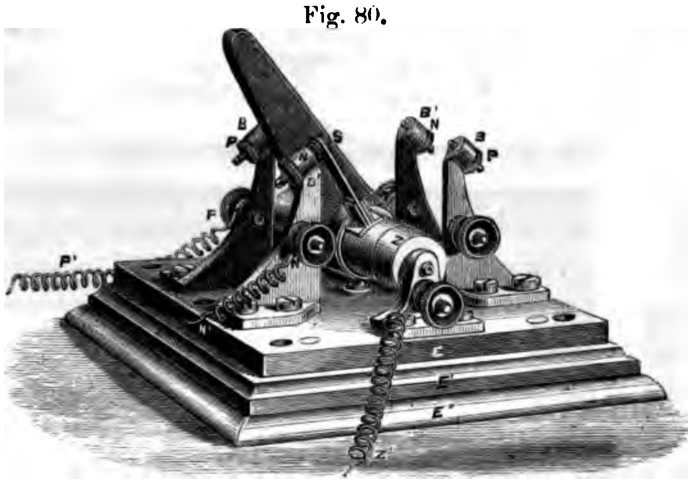
The experiments were made in a bell-jar, containing the terminals, which could be gradually exhausted after having been filled with air or other gas. One of the terminals was fixed to the bottom plate, the other could be adjusted to any distance from it by a rod sliding through a stuffing-box in the glass cover. The foot of the stand was insulated by a disc of ebonite, on which it stands. One such bell-jar is represented in fig. 79. It is 9¼ inches (23·4 centims.) high and 5⅞ inches (14·9 centims.) in diameter; its cubical content, obtained by covering the open ends with glass plates and filling with water from a graduated measure, was found to be 3787 cubic centims. This jar was destroyed in the course of the experiments, and was replaced by others. In the figure an inner tube is shown with a series of holes in it, which was not used on this occasion, but will be spoken of hereafter.

---

[*] Part II., page 230.

A remarkable phenomenon was observed on making connexion between the terminals and the battery by means of the discharging key, fig. 80 (already described in Part I., page 58), namely, that within certain limits of pressure in the bell-jar a sudden expansion of the gas took place, and that as soon as the connexion was broken the gas then as suddenly resumed almost exactly its original volume, showing only a small increase due to a slight elevation of temperature. The effect was similar to that which would have been produced if an empty bladder suspended between the terminals had been suddenly inflated and as suddenly emptied.* We communicated this observation unofficially to our friend Professor STOKES in October, 1878.

Fig. 80.



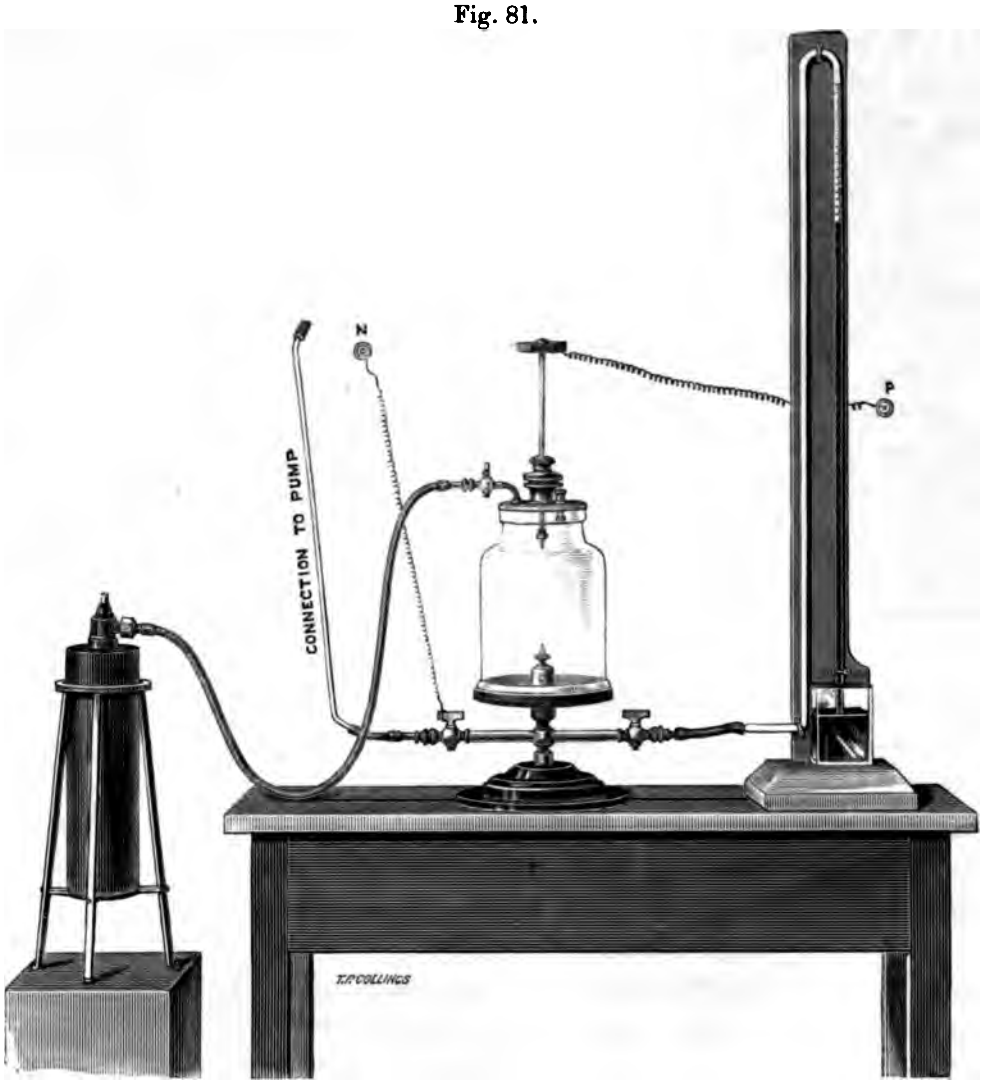
This phenomenon was quite unexpected, and being difficult to account for, was made the object of special study by means of an arrangement of apparatus as shown in fig. 81.

The glass cover of the bell-jar is furnished with a stop-cock which communicates with a gas-holder, and in the bottom plate there is a **T**-tube connected to two other stop-cocks, one of which communicates with the pumps, the other with a gauge about 3 feet high. The right hand limb of the gauge dips into a rectangular glass cistern, the front face of which is polished to facilitate the reading; this limb is graduated from a zero well below the level of the mercury in the cistern, so that the height of the column of mercury may be determined with facility. P and N are screw terminals connected with the battery.

---

* This was first noticed by my former assistant, Mr. SEATON.

Fig. 81.



The following experiment in rarefied air will give an idea of the amount of instantaneous expansion which occurs when the terminals are connected with the poles of the battery.

	Pressure. m.m.	Pressure. M.
Distance of the terminals, 6 inches, the top one a point, the lower a disc	56	73,684
On making contact the arc passed and the column of mercury was depressed	15·8	20,789
Pressure on connexion	71·8 =	94,473

11,000 cells, current 0·01102 W; the resistance of the bell-jar was reproduced by substituting 600,000 ohms wire resistance.

The volume of the bell-jar is 3787 cub. centims., the temperature at the time of the experiment 17°·5 C., and if the gas had been allowed to expand, its volume would have been $\frac{3787 \times 71{\cdot}8}{56}=4855$ cub. centims., or an increase of 1068 cub. centims.; as the gas was kept at a constant volume the pressure increased in the ratio 1·282 to 1. Supposing the expansion to be due to an increase of temperature, it would follow that as the volume was kept constant the pressure would vary as the absolute temperature,* therefore $\frac{T'}{T}=\frac{71{\cdot}8}{56}=1{\cdot}282$, whence $T'=1{\cdot}282 \times 291{\cdot}2=373^{\circ}{\cdot}3$ C. The temperature while the discharge was passing would be $(373{\cdot}3-273{\cdot}7)=99^{\circ}{\cdot}6$ C., consequently the rise of temperature $(99{\cdot}6-17{\cdot}5)=82^{\circ}{\cdot}1$. The temperature of the bell-jar as determined by a thermometer enclosed in it with its bulb uppermost only rose 0°·64 C. per second, taking into account the rate of cooling. It is evident, therefore, that the increase of pressure cannot be ascribed to the instantaneous heating of the bell-jar 82° C.

A photograph of the discharge obtained in 60 seconds, and copied in Plate 9, fig. 8, shows that the central spindle or arc-proper could not have been so much as half-an-inch, 1·27 centims., in diameter, and its length we know was 6 inches, 15·24 centims.; the cubical content of a cylinder of these dimensions is 19·3 cub. centims. If we assume that this volume increased 1068 cub. centims., or to $19{\cdot}3+1068=1087{\cdot}3$, then $\frac{1087{\cdot}3}{19{\cdot}3}=56{\cdot}337$ represents the number of times it increased, and accordingly $T'=56{\cdot}337\,T=56{\cdot}337 \times 291{\cdot}2=16{,}405^{\circ}$ C., and $16{,}405-291{\cdot}2=16{,}114^{\circ}$ C. would therefore be the temperature of the arc.

Experiments were made to ascertain roughly the temperature of different parts of the arc, and for this purpose wires were supported by one end being twisted round a vertical glass rod, the other end being made to project into the arc at different heights, or else wires were strained through opposite holes h, h', h'', &c., drilled at different heights in the side of a glass tube 1·75 inch diameter, which was placed inside the bell-jar, as shown in fig. 81. Platinum wires 0·0125 inch diameter were not fused in any part of the arc; they were heated to a white heat in the luminous but not in the non-luminous part. But platinum wires 0·001 inch in diameter, supported by wire 0·0125 inch, in various parts of the arc were immediately fused; the temperature of the arc was therefore as high as the fusion-point of platinum, and possibly considerably higher (Plate 9, figs. 9, 10).

The whole of the heat evolved by a current of 0·01102 W through a resistance of 600,000 ohms would raise 73·1 grammes of air 1° C. per second; and if communicated to the air in the bell-jar, weighing 0·339 gramme, would have raised its temperature to $\frac{73{\cdot}1}{0{\cdot}339}=215^{\circ}{\cdot}6$ C. in that time. We know from direct experiment that such an enormous evolution of heat was not communicated to the air in the bell-jar, because its temperature only increased about 0°·64 C. per second; and it would have to be assumed that the rest of the heat escaped almost instantaneously by radiation. It is difficult consequently to realise the conjecture that the enormous dilatation which occurred instantaneously could have been caused by increase of temperature. On the other hand the ascertained facts appear rather to point to the phenomenon being caused by the projection of the gas-molecules by electrification against the walls of the glass vessel, producing thereby effects of pressure, which, however, are distinct from the molecular motion induced by heat.

---

* Absolute zero=273·7 C., 273·7+17·5=291·2.

Recently (July 5th, 1879), Professor DEWAR called our attention to a similar phenomenon, which confirms this view. He was working in the chemical laboratory of the Royal Institution on the Electric Arc, the source of electricity being a SIEMENS' dynamo-machine driven by a gas engine. The carbon electrodes were, for the purposes of his experiments, enclosed in a metallic cylinder open at the bottom but closed at the top; the bottom was kept air-tight by immersion in mercury.

The total radiant force was being accurately measured by carrying a stream of water through the cylinder into a separate vessel at a known rate, and measuring its increase of temperature by very sensitive thermometers, which could be read to a hundredth of a degree centigrade; their rapid pulsations were easily seen as the radiant force varied. Suddenly, from time to time, there was an increase of current and a great expansion of the volume of the air in the cylinder took place, and, although there were apertures in this vessel, the mercury joint was forced by the sudden expansion, and yet no sudden rise of the thermometer was noticed when these expansions occurred.*

The arc under various circumstances.

We now proceed to describe the appearance of the arc with terminals of various forms at different distances and with various pressures. It will be seen that the light emitted by different parts of the arc is not of the same intensity throughout, and that under most circumstances there is a tendency to break up into distinct entities, and ultimately to take a stratified appearance.

The appearance of the arc, between discs in hydrogen at the various pressures used in determining the potential necessary to produce a discharge, is represented in Plate 8, figs. 11–19.

		M.		cells.			seconds.
Fig. 11,	at pressure of	18,684,	with	600,	from a photograph	taken in	50
,, 12	,,	58,684	,,	1200	,,	,,	50
,, 13	,,	141,974	,,	2400	,,	,,	50
,, 14	,,	252,368	,,	3600	,,	,,	50
,, 15	,,	386,316	,,	4800	,,	,,	20
,, 16	,,	558,816	,,	6300	,,	,,	4
,, 17	,,	558,816	,,	6300	,,	,,	1
,, 18	,,	651,316	,,	7760	,,	,,	5
,, 19	,,	1,008,421	,,	10,920	,,	,,	5

---

* DE LA RIVE noticed that oscillations occurred in the mercury of a gauge attached to an exhausted tube as soon as the current passed. In the second and third paragraphs of Note C, in the Appendix, is the account of his experiments.

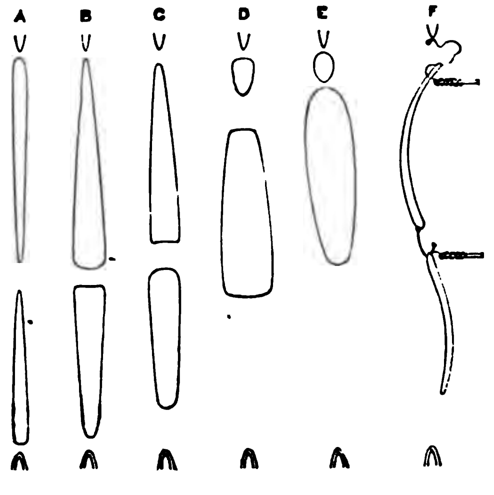
As the pressure was diminished the arc widened out until at last the entire surface of the negative disc was covered with a luminous discharge. From the first it will be seen that the central spindle did not extend quite up to the negative, as shown in fig. 19, at atmospheric pressure; that in fig. 18 the central bright part detached itself so as to form an entity; in figs. 17, 16, and 15 distinct strata were formed; then that, as is shown in figs. 14 and 15, the central bright portion diminished, the dark discharge near the negative increasing; while at the lower pressures, represented in figs. 12 and 11, there existed no central spindle, and that with the exception of the luminosity on the terminals the rest of the discharge was dark. It should be mentioned that part of the details have, in some instances, been supplied from sketches made when the photographs were obtained, for there was sometimes a movement in the discharge which produced a confused picture. No observations of the expansion of the gas were attempted.

The appearance of the discharge in carbonic acid is shown twice the original size in Plate 10, figs. 20, 21, 22, and 23; in all but the last will be observed the peculiar bell-shape form which is characteristic of the arc in carbonic acid (Part I., page 96, 3 of fig. 23).

Fig.	M.	cells.	
20	983,158	10,960	from a drawing.
21	504,211	6300	" "
22	126,052	2400	" "
23	39,474	1200	" "

In fig. 20 is seen a central spindle surrounded by a bell-shape fainter discharge; in fig. 21 the central spindle is composed of well-defined strata; in fig. 22 there is a bright luminosity in contact with the positive disc, then a dark interval with an illumination of the negative surface, the whole surrounded by the bell-shape glow; in fig. 23 the central bright portion and the bell-shape glow have disappeared, and

Fig. 82.



the surfaces of the positive and negative discs alone are illuminated, the glow, especially on the negative, having become much wider.

The following cases of the discharge between two points in air, illustrated by the outline sketches A, B, C, D, E, F, fig. 82, one-third of the full size, will explain what is meant by the arc breaking up into distinct entities; the outlines represent merely the bright portions of the arc: these are always surrounded with a less bright portion of highly heated gas, as shown in Plate 8, figs. 1 to 33 inclusive.

DISTANCE of terminals, two brass points, 6 inches=152 m.m. in air, 11,000 cells.

Fig.	Pressure.		Current.
	m.m.	M.	W.
A	36	47,369	0·02456
B	14	18,421	0·02456
C	14	18,421	0·02209
D	7	9211	0·02634
E	7	9211	not observed
F	56	73,684	0·01575

The following experiments in air were made with the points still 6 inches distant:—

Pressure 63 m.m., 82,895 M. Two wires, 0·001 inch diameter, had been previously fixed in the direction the arc would take, one at 15 m.m. the other at 80 m.m. from the positive terminal (F, fig. 82); both instantaneously fused and served as new intermediate terminals; the heat appeared to be greatest at the wire most distant 80 m.m. The mean increase of temperature in the jar was at the rate of 0°·485 per second, and the cooling at the rate of 0°·14, so that the real increase would have been 0·485+0·14=0°·625 supposing no cooling had occurred.

Pressure 8 m.m., 10,526 M, 11,000 cells, current 0·01771 W. On making connexion the mercury in the gauge was depressed 2 m.m., so that the volume, or rather the pressure, increased in the ratio 1·25 to 1. The appearance of the arc is represented in Plate 8, fig. 33; the pale portion surrounding the brighter part was quite invisible in subdued daylight. By keeping the current on for about two minutes the air in the bell-jar rose from 17° C. to 95° C.

Pressure 67 m.m., 88,158 M, 11,000 cells, current not measurable with the galvanometer, depression on connecting 22·2 m.m., increase of volume (pressure) in the ratio of 1·331 to 1; the arc is shown in Plate 8, fig. 32. Even at a pressure of 100 m.m., 139,474 M, there was a glow visible on both the terminals.

Pressure 8 m.m., 10,526 M, 11,000 cells, current 0·02634 W, depression on connecting 2·5 m.m., increased pressure as 1·312 to 1. In this experiment wires were placed across the arc; surrounding that one 15 m.m. distant from the positive was a cylindrical brighter glow resembling the glow around a negative terminal; the arc was crossed with close paper-like strata, as in Plate 9, fig. 9.

Pressure 6 m.m., 7895 M, 11,000 cells, current 0·03138 W, depression 1·5 m.m., increase of pressure 1·250 to 1. The arc was also in this case crossed with close paper-like strata.

Pressure 7 m.m., 9211 M, 11,000 cells, current 0·03138 W; appearance represented in Plate 9, fig. 9, from a photograph taken in 3 minutes.

Pressure 60 m.m., 78,947 M, current not measured; represented in Plate 9, fig. 10, from a photograph taken in 1 minute.

The arc in air between a point positive and a disc 6 inches distant is shown in Plate 8, fig. 31; pressure 56 m.m., 73,684 M, 11,000 cells, current 0·00993 W. The central spindle is curved and divided near the centre.

Increase of pressure on making connexion with the battery of 11,000 cells, and producing a discharge between points 6 inches distant.

Pressure.	Depression.	Ratio of increased to normal pressure.	Current.
M.	m.m.		W.
10,526	2·0	1·25	0·01771
88,158	22·2	1·33	not measured
10,526	2·5	1·31	. .
7895	1·5	1·25	0·03138

The arc in air between two points at various distances and pressures with a constant number of cells. Temp. 12°·7 C.

PLATE 8.

11,000 cells, distance 0·54 inch, pressure atmospheric, current 0·02456 W; the total resistance of battery and arc was found to be 461,500 ohms, that of the arc 27,550, by substituting wire resistance; whence the potential between the terminals was 657 cells. The appearance of the arc is shown in fig. 20; it exhibits clearly the tendency to break up into luminous entities; the photograph of which this is a copy is nearly full size and was obtained in 20 seconds. All the other photographs are on a reduced scale. As the batteries were undergoing the annual overhauling, the number of cells, some being removed from time to time, was somewhat less in the following experiments, namely, 10,940.

Pressure atmospheric 748·6 m.m., 985,000 M, distance 0·58 inch, current not observed, no depression of the mercury in the gauge was noticed; indeed, it will be seen that at the higher pressures the depression is generally less than at the lower, up to a certain point. The arc is shown in fig. 1 from a photograph obtained in 10 seconds.

Distance 0·58 × 2 = 1·16 inch, pressure 294·9 m.m., 388,026 M, current 0·02881 W, depression 16 m.m., total pressure 294·9 + 16 = 310·9; ratio of increased to normal pressure as 1·054 to 1. The arc is represented in fig. 2 from a photograph in

15 seconds. It will be seen that the central spindle has become bifurcated about midway.

Distance × 3=1·74 inch, pressure 191·3 m.m., 251,711 **M**, current 0·04060 **W**, depression 17 m.m., total pressure 208·3 m.m.; ratio of increased pressure 1·089. The arc is represented in fig. 3 from a photograph in 15 seconds. The bifurcation is apparent in this also.

Distance × 4=2·32 inches, pressure 142·6 m.m., 187,631 **M**, current 0·04474 **W**, depression 19 m.m., total pressure 161·6 m.m.; ratio of increased pressure 1·133. The arc is represented in fig. 4, in which the central spindle is broken up into several luminosities.

Distance × 5=2·9 inches, pressure 112·6 m.m., 148,157 **M**, current 0·03459 **W**, depression 19 m.m., total pressure 131·6; ratio of increased pressure 1·169. The arc is represented in fig. 5 from a photograph in 15 seconds; in this the central spindle is split up into bright entities connected by less bright portions.

Distance × 6=3·48 inches, pressure 99·4 m.m., 130,789 **M**, current 0·03071 **W**, depression 21 m.m., total pressure 120·4 m.m.; ratio of increased pressure 1·211. The arc is represented in fig. 6 from a photograph in 15 seconds. The luminous entities still seen, but are less marked.

Distance × 7=4·06 inches, pressure 85·9 m.m., 113,026 **M**, current 0·03259 **W**, depression 22 m.m., total pressure 107·9 m.m.; ratio of increased pressure 1·256. Fig. 7 from a photograph in 15 seconds. The central spindle is divided into two luminosities, with a tendency to form a third near the negative.

Distance × 8=4·64 inches, pressure 71·6 m.m., 94,210 **M**, current 0·02693 **W**, depression 22 m.m., total pressure 93·6 m.m.; ratio of increased pressure 1·307. Fig. 8 from a photograph in 15 seconds. The central spindle nearly of the same character as fig. 7.

Distance × 9=5·22 inches, pressure 65·5 m.m., 86,184 **M**, current 0·02693 **W**, depression 22 m.m., total pressure 87·5 m.m.; ratio of increased pressure 1·336. Fig. 9 from a photograph in 15 seconds. The bright entities show a tendency to break up into less bright portions.

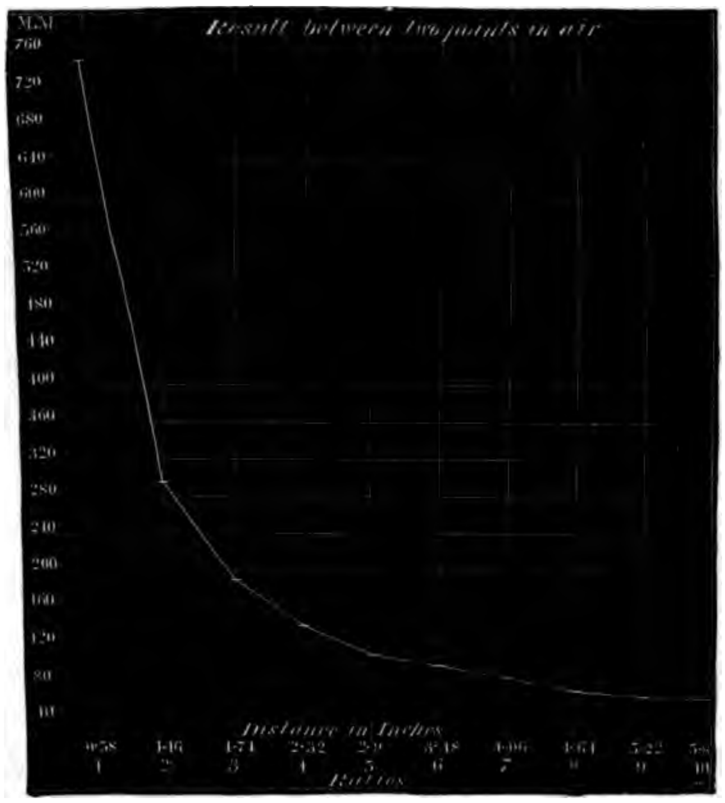
Distance × 10=5·8 inches, pressure 64·4 m.m., 84,737 **M**, current 0·03071 **W**, depression 20 m.m., total pressure 84·4 m.m.; ratio of increased pressure 1·310. Fig. 10 from a photograph in 15 seconds. The arc resembles that seen in fig. 9.

Distance 5·8 inches, pressure 4 m.m., 5263 **M**, current 0·02544, a single nearly globular luminosity about 1 inch in diameter near the positive. The negative terminal completely surrounded with a brilliant purple glow.

Distance 5·8 inches, pressure less than 4 m.m., but could not be exactly observed, current 0·02544. Near the positive three distinct strata had formed.

The curve of the discharge between two points in air, with a constant number of cells, the distance increasing as 1, 2 . . . 10, and the pressures diminishing, is shown in fig. 83.

Fig. 83.



It will be seen by the following table that while the distances increase as 1, 2 . . . 10, the pressures have to decrease in a greater ratio, namely, as 1, 2 . . . 10 multiplied by a factor greater than unity in order to permit a discharge to take place.

Col. I. Ratio of distances.	Col. II. Ratio of diminution of pressure.	Col. III. $\frac{\text{Col. II.}}{\text{Col. I.}}$
1	1	
2	2·54	1·27
3	3·91	1·30
4	5·25	1·31
5	6·65	1·35
6	7·51	1·25
7	8·71	1·24
8	10·45	1·30
9	11·43	1·27
10	11·61	1·16
		1·27 mean

The expansion which arises when a discharge takes place at various distances and pressures :—

Pressure.	Depression.	Ratio of increased to normal pressure.	Current.	
M.	m.m.		W.	ratios.
985,000	None			
388,026	16	1·054	0·02881	1·07
251,711	17	1·089	0·04060	1·51
187,631	19	1·133	0·04474	1·66
148,157	19	1·169	0·03459	1·28
130,789	21	1·211	0·03071	1·14
113,026	22	1·256	0·03258	1·21
94,210	22	1·307	0·02693	1·00
86,184	22	1·336	0·02693	1·00
84,737	20	1·310	0·03071	1·14

The arc in air between two spherical surfaces, 3 *inches radius, and* 1·5 *inch diameter at various distances and pressures, with* 10,960 *cells. Temp.* 15°·2 C.

PLATE 10.

(All the representations are from drawings.)

Distance 0·16 inch (the current would not pass at 0·165 inch), pressure 761 m.m., 1,001,317 M, current 0·02371 W, no depression apparent on making connexion with the battery, fig. 1.

Distance × 2=0·32 inch, pressure 272·1 m.m., 358,027 M, current 0·02456 W, no depression, fig. 2 ; the central spindle is broken up into nebulosities.

Distance × 3=0·48 inch, pressure 181·3 m.m., 238,553 M, current 0·02209 W, depression 0·25, total pressure 181·55 m.m., ratio of increased pressure 1·0002, fig. 3, one detached luminosity near negative.

Distance × 4=0·64 inch, pressure 149·8 m.m., 197,111 M, current 0·02131 W, depression 1 m.m., total pressure 150·8 m.m., ratio of increased pressure 1·007, fig. 4 ; the discharge no longer occurred at the shortest distance, and moreover took a curved formation, and was of unequal brightness in the central portion.

Distance × 5=0·80 inch, pressure 140 m.m., 184,211 M, current 0·02209 W, depression 3·5 m.m., total pressure 143·5 m.m., ratio of increased pressure 1·025, fig. 5 ; the discharge was curved, there was a small dark space near the negative on which there was a glow.

Distance × 6=0·96 inch, pressure 117·9 m.m., 155,131 M, current 0·02249 W, depression 2·5 m.m., total pressure 120·4 m.m., ratio of increased pressure 1·021 ; fig. 6. The discharge was pointed towards the negative.

Distance × 7=1·12 inch, pressure 118·4 m.m., 155,789 M, current 0·02209 W, depression 2 m.m., total pressure 120·4 m.m., ratio of increased pressure 1·016; fig. 7, similar to the preceding.

Distance × 8=1·28 inch, pressure 118·4 m.m., 155,789 M, current 0·02170 W, depression 3 m.m., total pressure 121·4 m.m., ratio of increased pressure 1·025; fig. 8, the luminosity on the negative increasing.

Distance × 9=1·44 inch, pressure 118·4 m.m., 155,789 M, current 0·02170 W, depression 3 m.m., total pressure 121·4 m.m., ratio of increased pressure 1·025, fig. 9. The central bright portion was divided into two luminosities, that nearest the negative being the less bright.

Distance × 10=1·60 inch, pressure 118·4 m.m., 155,789 M, current 0·02056 W, depression 3 m.m., total pressure 121·4 m.m., ratio of increased pressure 1·025, fig. 10. The central portion was divided into two luminosities.

Distance × 11=1·76 inch, pressure 111·3 m.m., 146,448 M (the current would not pass at 118·4 m.m.), current 0·01948 W, depression 4 m.m., total pressure 115·3 m.m., ratio of increased pressure 1·036, fig. 11. The central portion was more decidedly divided into two portions, and assuming an upper-lip like form.

Distance × 20=3·2 inches, pressure 74·6 m.m. 98,158 M, current 0·01840 W., depression 8 m.m, total pressure 82·6 m.m., ratio of increased pressure 1·107, fig. 12. The discharge had formed near the peripheries of the terminals. The luminosity on the negative had increased.

Distance × 40=6·4 inches, pressure 46·6 m.m., 63,947 M, current 0·01390 W, depression 9 m.m., total pressure 55·6 m.m., ratio of increased pressure 1·193, fig. 13. The central bright part reached at first nearly to the negative, then after a short interval retreated first to about 2 inches and then to 3 inches from it.

Distance 6·4 inches, pressure 26·9 m.m., 35,395 M, current 0·01840 W, depression 10 m.m., total pressure 36·9 m.m., ratio of increased pressure 1·372, fig. 14. The central portion reached to the negative and did not divide into entities.

Distance 6·4 inches, pressure 13·7 m.m., 18,026 M, current 0·02056 W, depression 3 m.m., total pressure 16·7 m.m., ratio of increased pressure 1·219, fig. 15. The central portion reached at first to within 2 inches, and then retreated to 3 inches from the negative.

Distance 6·4 inches, pressure 7·1 m.m., 9342 M, current 0·02131 W. The jar-potential was found to be 2069 cells. The discharge had increased in width 1½ inches in diameter, and was 3 inches long. A purple glow on the negative terminal, fig. 17.

Distance 6·4 inches, pressure 5·5 m.m., 7237 M, current not taken, depression not appreciable. There was a barely visible discharge between the terminals, the positive was illuminated all over the surface opposed to the negative, which terminal was completely covered on the top and under surface with a purple glow about a quarter of an inch deep, fig. 16.

Distance 6·4 inches, pressure 4·1 m.m., 5395 M, current 0·02456 W, the jar-potential was 1448 cells, its resistance 70,000 ohms. 2400 rod cells passed, giving a current of 0·05353 W, and produced thin strata near the positive, the negative being completely covered with a purple glow, fig. 18.

Distance 6·4 inches, pressure 2·6 m.m., 1316 M, 1200 cells, the zinc rods of which had been recently scraped to remove the oxychloride of zinc, gave a current of 0·17230 W, and produced a discharge one and a half inches wide, nearly reaching the negative, which was covered with a purple light, surrounding the top and bottom surfaces to a depth of three-eighths of an inch.

Distance.	Pressure.	Depression.	Ratio of increased to normal pressure.	Current.	
inches.	M.	m.m.		W.	ratios.
0·16	1,001,317	None	..	0·02371	1·71
0·32	358,027	None	..	0·02456	1·77
0·48	238,553	0·25	1·0002	0·02209	1·59
0·64	197,111	1·00	1·007	0·02131	1·54
0·80	184,211	3·5	1·025	0·02209	1·59
0·96	155,131	2·5	1·021	0·02249	1·62
1·12	155,789	2·0	1·016	0·02209	1·59
1·28	155,789	3·0	1·025	0·02170	1·56
1·44	155,789	3·0	1·025	0·02170	1·56
1·60	155,789	3·0	1·025	0·02056	1·48
1·76	146,448	4·0	1·036	0·01948	1·40
3·20	98,158	8·0	1·107	0·01840	1·32
6·40	63,947	9·0	1·193	0·01390	1·00
6·40	35,395	10·0	1·372	0·01840	1·32
6·40	18,026	3·0	1·219	0·02056	1·48
6·40	7237	None	..	Not taken	

The arc in hydrogen between two spherical surfaces, 3 *inches radius, and* 1·5 *inch diameter, at various distances and pressures, chiefly with* 10,960 *cells. Temp.* 13°·3 C.

Distance 0·225 inch (the current would not pass at 0·23 inch), pressure 762·8 m.m., 1,003,685 M, current 0·02456 W, no depression apparent on making connexion with the battery. The central spindle did not quite reach the negative terminal. A, fig. 84 ; C and F lines visible with the spectroscope.

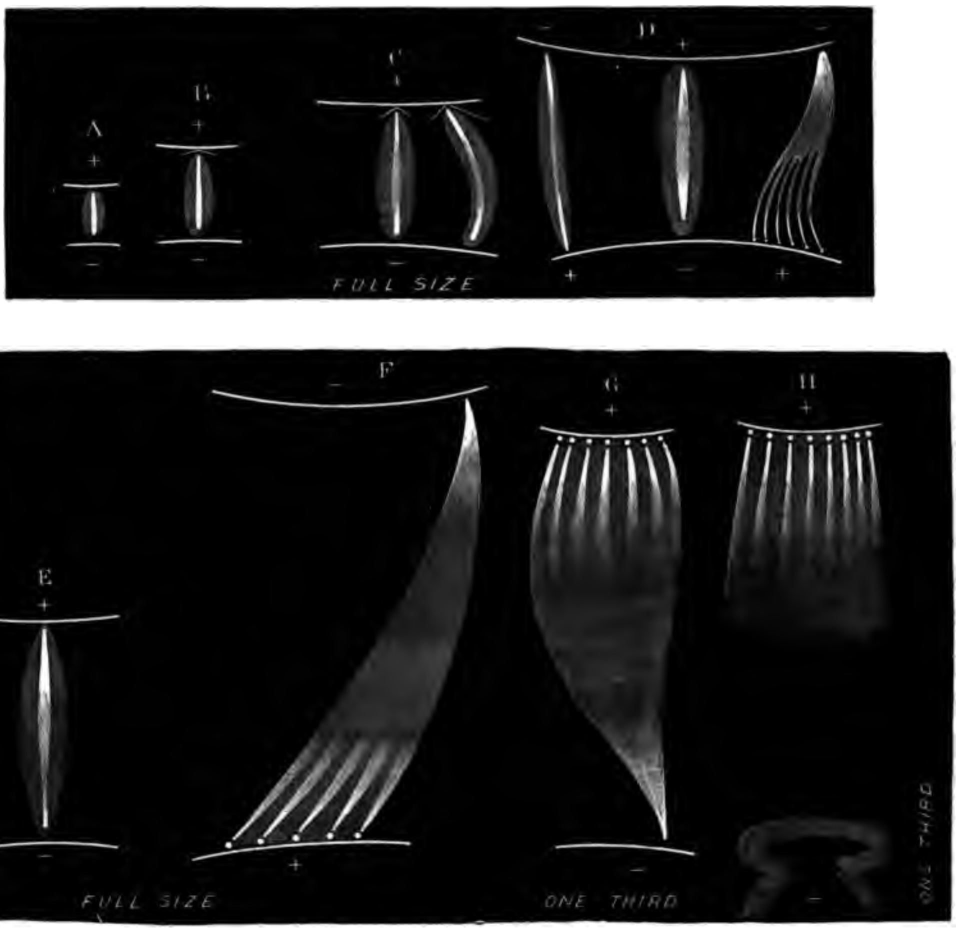
Distance × 2=0·45 inch, pressure 339 m.m., 446,053 M, current 0·02371 W, depression 0·1 m.m., total pressure 339·1 m.m., ratio of increased pressure 1·0002. The central spindle did not quite reach the negative terminal. The arc appeared to be crossed by very close lines; at the positive, winged appendages were formed B, fig. 84.

Distance × 3=0·675 inch, pressure 256 m.m., 336,842 M, current 0·02456 W, depression 0·5 m.m., total pressure 256·5 m.m., ratio of increased pressure 1·002. The arc at first formed in the centre as on the left of C, fig. 84, then moved towards the periphery, as on the right of the same figure. Winged appendages were formed as represented.

Distance × 4=0·90 inch, pressure 255 m.m., 335,526 M, current 0·02544 W, depression 0·5 m.m., total pressure 255·5 m.m., ratio of increased pressure 1·002. The appearance as shown in D, fig. 84, the arc first formed in the centre, then ran all over the surfaces of the terminals, lastly becoming steady, as seen in edge view on the left, and side view on the right.

Distance × 5=1·125 inch, pressure 200·8 m.m., 264,211 **M**, current 0·02634 **W**, depression not observed. The arc at first formed in the centre, as seen in E, fig. 84, and then ran all over the surface, as in D.

Fig. 84.



Distance × 10=2·250 inches, pressure 164·1 m.m., 215,921 **M**, current 0·02056 **W**. The discharge as in F, fig. 84; on the positive there formed a number of beads of light.

Distance × 25=5·62 inches, pressure 61·8 m.m., 81,316 **M**, current 0·01272 **W**. The discharge in form like a fig; a number of beads of light forming on the positive, and the arc ending in a rounded point (the stem of the fig) on the negative G, fig. 84, reduced to one-third.

Distance 5·62 inches, pressure 2·3 m.m., 3026 **M**, 2400 cells, current 0·08923 **W**.

Beads still formed on the positive, the light became so faint an inch or two from the positive as to be barely visible, H, fig. 84, reduced to one-third.

Distance.	Pressure.	Depression.	Ratio of increased to normal pressure.	Current.	
inches.	M.	m.m.		W.	ratios.
0·225	1,003,685	..	..	0·02456	1·93
0·450	446,053	0·1	1·0002	0·02371	1·86
0·675	336,842	0·5	1·002	0·02456	1·93
0·900	335,526	0·5	1·002	0·02544	2·00
1·125	264,211	..	..	0·02634	2·07
2·250	215,921	..	..	0·02056	1·62
5·520	81,316	..	..	0·01272	1·00
5·520	3026	..	(with 2400 rod cells)	0·08933	7·02

The arc in hydrogen between two points. Temp. 16°·2 C., 10,940 *cells.*

Distance 0·75 inch, pressure 745 m.m., 980,263 M, current 0·01575 W, the appearance is represented in Plate 8, figs. 21 and 22, the first copied from a photograph obtained in 5 seconds, the second in 15 seconds. The central spindle breaks into a brush-like form towards the negative, there is then a dark interval between it and the glow on the negative.

Distance 0·9 inch, pressure 745 m.m., 980,263 M, the discharge passed intermittently, so that the current could not be read off on the galvanometer. The appearance is represented full size in Plate 8, fig. 26, from a drawing.

Distance 0·9 × 2, 1·8 inch, pressure 385·6 m.m., 507,368 M, current 0·01575 W, depression 14 m.m., total pressure 399·6 m.m., ratio of increased pressure 1·04. Plate 8, fig. 23, from a photograph in 13 seconds. The distance between the brush-like termination of the central spindle and the glow on the negative has relatively increased. Figs. 27 and 28 are other representations copied from drawings.

Distance × 3, 2·7 inches, pressure 317·8 m.m., 418,158 M, current 0·00580 W, depression 13 m.m., total pressure 330·8 m.m., ratio of increased pressure 1·04. Plate 8, fig. 24, from a photograph in 15 seconds. The central spindle relatively still shorter. At times only the terminals were illuminated, but sometimes strata formed on the positive terminal. Fig. 29 is another representation copied from a drawing.

Distance × 4, 3·6 inches, pressure 170·5 m.m., 224,342 M, 6300 cells, current not measured, the appearance is represented in Plate 8, fig. 25; the central spindle has decreased relatively still more.

Distance × 7, 6·3 inches, pressure 85·8 m.m., 112,895 M, 10,940 cells, current 0·01393 W, only a glow on both terminals, the intermediate discharge being dark.

Distance 6·3 inches, pressure 10 m.m., 13,158 M, 10,940 cells, current 0·02456 W, depression 14 m.m., total pressure 24, ratio of increased pressure 2·4, only a glow on terminals, the rest of the discharge dark.

Distance 6·3 inches, pressure 3·3 m.m., 4342 M, 2400 cells, current 0·03896 W, strata at the positive extending half way towards the negative, then there is an intervening dark discharge, and the negative point and the whole of its holder is surrounded with a bright violet halo.

Distance 6·3 inches, pressure 3 m.m., 3947 M, 1200 cells, the bottom point positive current 0·03896 W, a splendid stratification though somewhat unsteady; it is represented in Plate 8, fig. 35, partly copied from a photograph, partly from drawings. It was thought at first that well-defined strata would not be formed in a jar of such large diameter, but this experiment shows that this conjecture was unfounded. The negative glow completely fills the neck of the jar.

Distance 6·3 inches, pressure 2·4 m.m., 3158 M, 1200 cells, current 0·02728 W, a very steady stratification when the bottom point was positive; this curious stratification completely overlapped the whole surface of the bottom point and the brass holder, as shown in Plate 10, fig. 26, the glow around the negative completely filling the upper portion of the jar.

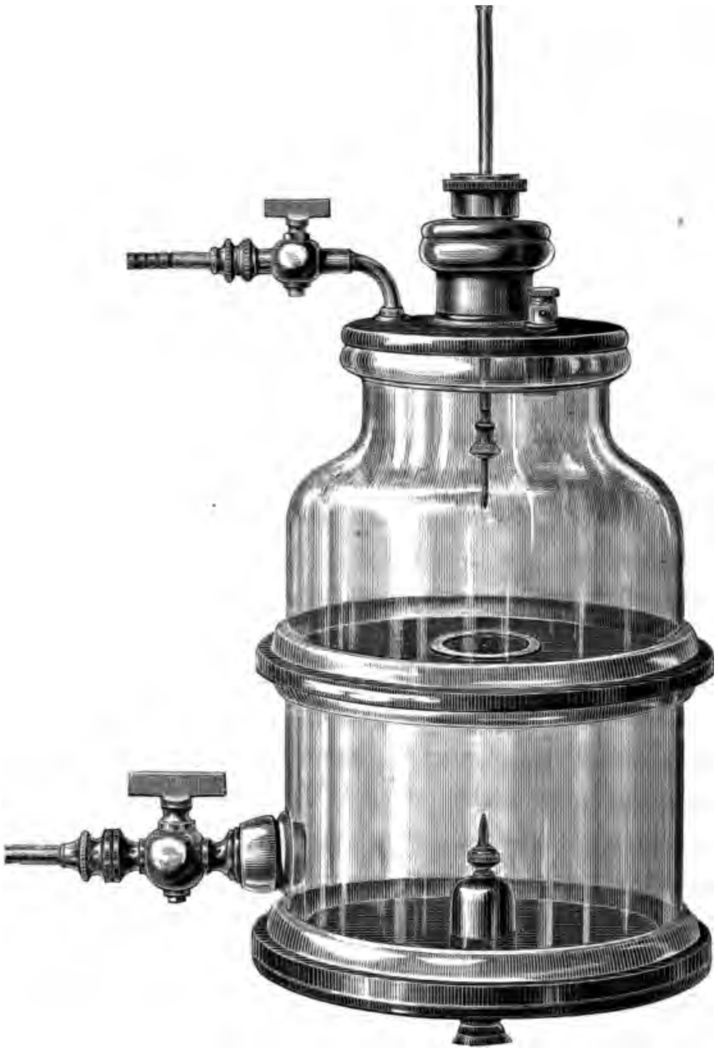
Air having leaked into the receiver it was removed from the plate and a tube inserted, as shown in fig. 79, p. 84, in order to ascertain whether the contraction of the space surrounding the discharge would have any effect on the production of strata. It will be remembered that a number of holes had been drilled in opposite sides of the tube, which is 8 inches long and 1·8 inch in diameter. These holes were drilled with the object of straining very fine platinum wires across at different heights for ascertaining the temperature of the arc at these positions, but in the experiments about to be described there were no wires.

The bell jar was refilled with hydrogen and exhausted; distance of points 6·3 inches, pressure 5·7 m.m., 7500 M, 2700 cells, current 0·13340 W; notwithstanding this large current no depression of the mercury in the gauge occurred on making connexion. The discharge is mainly cylindrical, with a spherical termination half an inch from the negative, and completely fills the tube; then there is a small dark space, and the negative is completely surrounded with a violet halo.

Pressure 2 m.m., 2632 M, 2400 cells; when the top point was positive there was a production of ordinary strata, as is shown in Plate 8, fig. 35. But when the bottom was positive, a very remarkable phenomenon was observed, namely, the protrusion of strata through the small holes, $\frac{1}{8}$th inch in diameter, in the walls of the inner tube, this being accompanied by an overpouring of negative discharge above the top of it, Plate 8, fig. 34. It seemed as if the positive discharge sought a complete neutralization with negative electricity beyond the confines of the tube, the area of which was too small to permit of complete relief. The close confinement of the discharge at the bottom end of the tube which rests on the glass plate of the pump, may account for the non-oozing out of strata through the holes when the top point was positive, because in the latter case the negative could not overflow the end of the tube and seek positive neutralization.

Some gas let in, pressure 4 m.m., 5263 M, 2400 cells, current 0·15470 W, a well-defined stratification occurred when the bottom point was positive, but no oozing out through the holes in the tube, Plate 10, fig. 25.

Fig. 85.



In order to ascertain whether there was any difference in the increased pressure on the positive or negative side of the discharge on connecting the terminals with the battery, we had a divided bell-jar constructed as shown in fig. 85, both ends of the two halves being accurately ground, and a glass disc for dividing the chamber into parts was provided; in this there is a hole half an inch in diameter surrounded by a raised rim, in order that plates of mica with holes of different diameters might be cemented with Canada balsam centrally to the diaphragm. The capacity of the upper half was found to be 1530 cub. centims., that of the lower half 1755, total 3285, or somewhat less than the jar, fig. 79 (3787).

Instead of putting separate gauges in connexion with the two chambers, as was originally contemplated, the lower chamber only was connected to the gauge, as in fig. 81, p. 86, and the lower terminal was made + or − alternately.

Air. Points distant 5 *inches.*

In a preliminary trial with the central hole 0·5 inch diameter, and a pressure which was not recorded, but at which 8840 cells just passed, the depression of the gauge when the lower terminal was negative or positive, was—

Lower terminal.

	−	+
	m.m.	m.m.
	27	19
	21	19
	23	20
Mean	23⅔	19⅓

Central hole 0·125 inch diameter, temperature 15° C., pressure 60·9 m.m., 80,132 **M**, 11,000 cells, current 0·03657 **W**, the jar-potential was found to be 3550 cells, depression—

Lower terminal.

	−	+
	m.m.	m.m.
	28	24
	26	23
Mean	27	23·5
Ratio of increased to normal pressure	1·44	1·39

The appearance of the discharge is shown in Plate 10, fig. 24, which is copied from a drawing.

Pressure 65·2 m.m., 85,789 **M**, temperature 13°·9 C., 11,000 cells, current 0·04686 **W**, jar-potential 3188 cells, depression—

Lower terminal.

	−	+
	m.m.	m.m.
	23	27
	22	26
Mean	22·5	26·5
Ratio of increased pressure	1·34	1·41

Central hole 0·02 inch, pressure 83·5 m.m., 109,869 M, 11,000 cells, current 0·04686 W, depression—

LOWER terminal.

	−	+
	m.m.	m.m.
	11	17
Ratio of increased pressure .	1·13	1·20

Central hole 0·02 inch, pressure 78 m.m., 102,632 M, 11,000 cells, current 0·05121, jar-potential 3477 cells, depression—

LOWER terminal.

	−	+
	m.m.	m.m.
	17	26
	18	27
Mean	17·5	26·5
Ratio of increased pressure .	1·22	1·34

Central hole 0·02 inch, pressure 68·5 m.m., 90,132 M, 11,000 cells, current when the lower terminal was positive 0·04901 W, when it was negative 0·04686 W ; in order to avoid any errors from reversing with the key, the contacts of which might possibly be more perfect in one or other direction, the connecting wires were reversed and connexion made by moving the key always in one direction ; the jar-potential was 3188 cells, depression—

LOWER terminal.

	−	+
	m.m.	m.m.
	26	28
	26	27
Mean	26	27·5
Ratio of increased pressure .	1·38	1·40

The hole in the centre of the diaphragm was covered with a disc of thin brass cemented on the glass diaphragm. The top point distant 2·5 inches from the brass disc, the bottom point distant from the bottom of the disc 2·55 inches, this makes a total distance between the points of 5·05 inches. The top and bottom cocks were connected together by tubing. Temperature 14° C., pressure 69·2 m.m., 91,053 M, 11,000 cells, current, with 40,000 ohms additional resistance, making a total of 279,000 ohms, 0·04060 W ; the connexion between the top and bottom chambers was left open, depression—

LOWER terminal.

	−	+
	m.m.	m.m.
	11	8
Ratio of increased pressure .	1·16	1·11

The communication between the top and bottom chambers was now closed, pressure 60·2 m.m., 79,210 **M**, 11,000 cells, current bottom negative 0·04474 **W**, bottom positive 0·03459 **W**, jar-potential, both chambers, 4538 cells, depression—

LOWER terminal.

	−	+
	m.m.	m.m.
	12	10
	11	10
Mean	11·5	10
Ratio of increased pressure .	1·19	1·16

The discharge took place when the top was negative in a curved arch from the upper rim of the disc to the point, and from the lower point it was axial to the centre of the underside of the disc; the underside of the disc formed a new negative, while its upper side formed a new positive terminal, Plate 8, fig. 30.

Summary.—Ratio of increased to normal pressure.

LOWER chamber.

−	+
1·44	1·39
1·34	1·41
1·13	1·20
1·22	1·34
1·38	1·40
1·16	1·11
1·19	1·16
1·266	1·287

It would appear therefore that the dilatation of the gas is the same both in the positive and negative chambers.

In order to prosecute our experiments in a vessel of still greater capacity, we had constructed a larger jar with a neck at each end, or, more properly speaking perhaps, a tube as represented in fig. 86, supported horizontally on ebonite crutches. It is 37 inches, 94 centims., long and $5\frac{13}{16}$ inches, 14·8 centims., in diameter, and its cubical content 14,435 cubic centims., or 3·8 times that of the bell-jar, fig. 79, p. 84. The ends are ground and are closed by caps with necks, C′, C′, also ground flat; the holes in the necks are likewise ground so as to receive two tubes T′, T′; in these tubes are sealed wires on which terminals of any form may be screwed; this has to be done after they have been inserted through the caps and before the caps have been placed in position; by this arrangement terminals nearly as large as the neck of the tube may be used. The tubes which hold the terminals are furnished with glass cocks. The joints are made air-tight by means of grease, and the caps are kept in their places by ebonite outer rings held by bolts which screw into half-rings at the back of the neck-shaped openings of the tube. This tube is numbered 199; the distance between

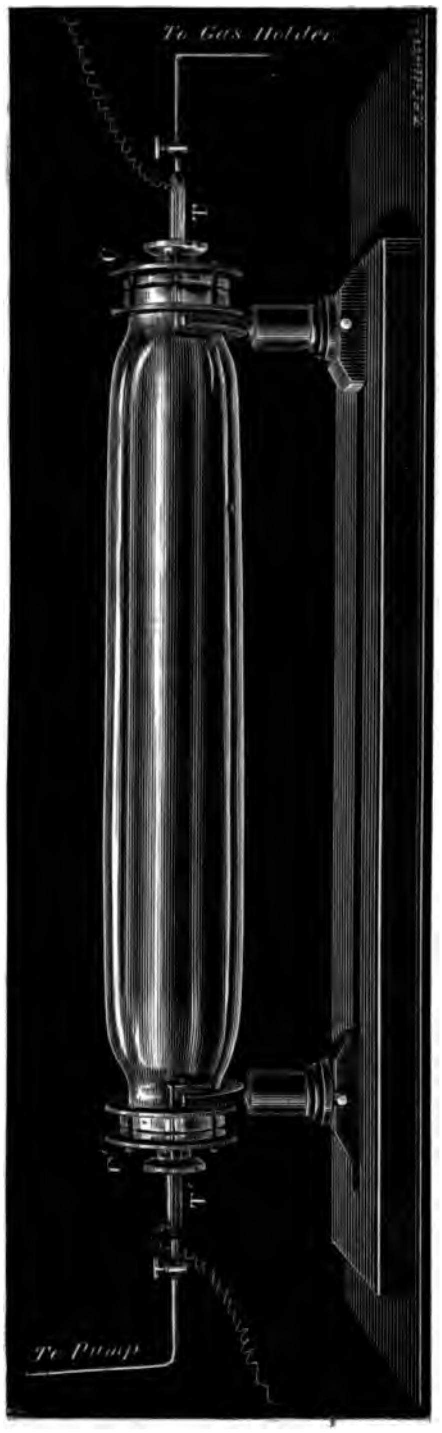


Fig. 86.

the terminals, a ring, and a straight wire is 33·5 inches, 85 centims. Another tube, No. 200, 34 inches, 86·4 centims., long and 1¾ inch, 4·4 centims., diameter, 30 inches, 76·2 centims., between the terminals, a ring, and a straight wire, is also connected to the pumps in order to permit of the study of the effect of different bores on the phenomena of the discharge when both tubes are at the same pressure.

The experiments with Tube 199 will necessarily occupy a considerable period, partly on account of the long time it takes to exhaust it after each set, partly on account of the variety of experiments it is intended to make with it. We now propose to describe only a few of the first results we have hitherto obtained.

Tube 199, *Air.*

Pressure 3 m.m., 3947 M, 6300 cells. Two luminosities were formed, as shown in Plate 9, fig. 5, the ring negative being surrounded with a nebulosity which completely filled the end of the tube. The tube glowed brilliantly with a blue fluorescent light, which proved to have great actinic power. The figure is copied from a dry-plate photograph obtained in 5 seconds; it records a very curious phenomenon, namely, that the outer boundary of the luminosity appears *darker* than the tube. It is to be remarked that while the discharge was reddish (nitrogen), the fluorescence of the tube was blue; the effect appears to be due to the absorption of a portion of the fluorescent light emanating from the back of the tube in passing through the red luminosity. The effect was quite unexpected, and it was thought at first that it might have arisen from some peculiarity in the development of the dry plate; it was not therefore until the result had been confirmed by other photographs that we ventured on the explanation given.

Tube 199, *Hydrogen.*

Pressure 40 m.m., 56,632 M, 11,000 cells, current 0·00087 W. A mere speck of light both on the positive and negative terminals; in tube No. 200 there was not the slightest glow when the battery was connected with it. The whole of tube 199 glowed with a blue fluorescent light, notwithstanding that there was only a mere speck of illumination on the terminals, and no appearance of light between them.

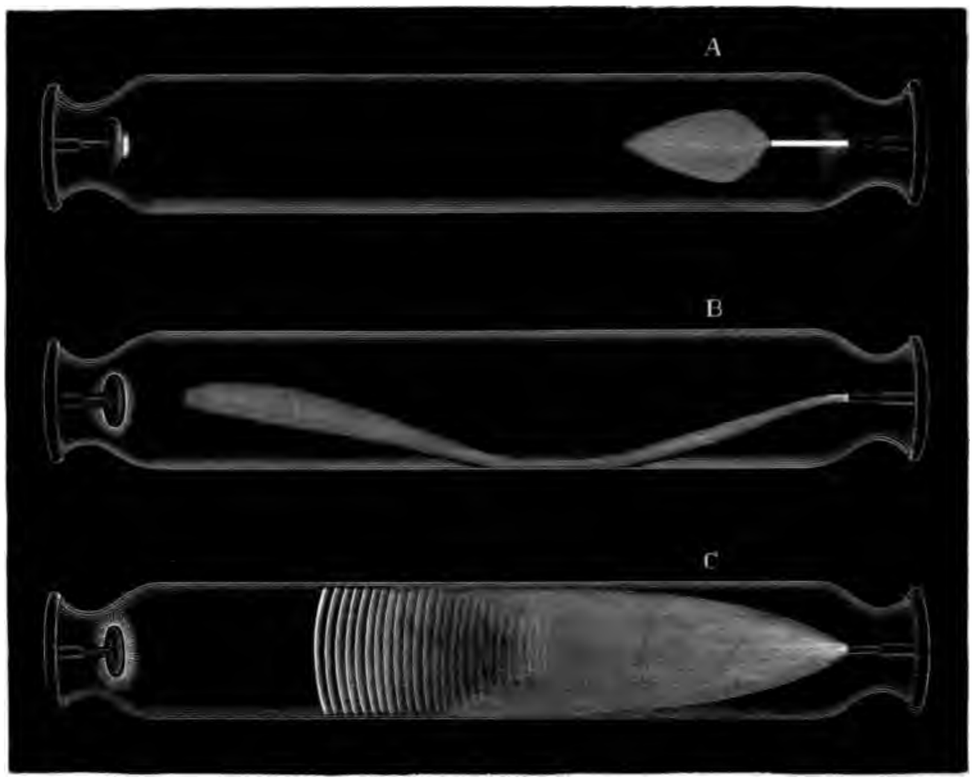
Pressure 28 m.m., 36,842 M, 11,000 cells, current 0·00527 W. A spear-head shaped luminosity about half an inch long on the positive straight terminal, a mere speck of light on the negative ring. No discharge occurred in tube 200 at the same pressure.

Pressure 26 m.m., 34,211 M, 11,000 cells, current 0·00879 W. A fan-shape luminosity about one inch long at the positive, a mere speck of light about one-eighth of an inch in length on the negative ring. The tube glowed with a blue fluorescent light throughout its length. 6300 cells would not pass.

Pressure 22 m.m., 28,948 M, 11,000 cells, current 0·01412 W. The glow on negative extended to three-eighths of an inch, a spear-head luminosity on the positive wire, to which it was attached by a very bright wire-like stem not greater in diameter than the terminal, A, fig. 87.

Pressure 15 m.m., 19,737 **M**, 11,000 cells, current 0·03071 **W**. A spindle-shaped luminosity at the positive about 1½ inch long, and the negative ring completely surrounded with a glow which had increased considerably since A. After a short time the spindle on the positive lengthened out and nearly reached the negative, hugging the underside of the tube, as in B, fig. 87. It was not sensitive to the approach of the finger, although close to the glass; 6300 cells produced the same phenomena.

Fig. 87.



Pressure 13 m.m., 17,105 **M**, 11,000 cells, current 0·01772 **W**. At the positive, a spear with a staff not greatly exceeding the diameter of the wire, the spear-head reaching to the middle of the tube. The glow on negative less brilliant than the preceding phase in consequence of a less current passing.

Pressure 9 m.m., 11,842 **M**, 6,300 cells, current 0·02693 **W**. A well-formed arrow-like luminosity at the positive, the negative ring three parts illuminated. After a while the luminosity shot forward towards the negative and lost its arrow shape, and then retreated to the positive, assuming the form of a long pear.

Pressure 8 m.m., 10,526 **M**, 6300 cells, current 0·01772 **W**. A long discharge reaching to the middle of the tube, in the centre of which was a very bright wire-like luminosity like that in A, fig. 87, which in the latter case was attached to the positive.

Pressure 5 m.m., 6579 M, 6300 cells, current 0·02693 W.

Pressure 4·5 m.m., 5921 M, 6300 cells, current 0·03657 W.

Pressure 4 m.m., 5263 M, 6300 cells, current 0·03459 W. The discharge in the latter case was partially stratified.

Pressure 3 m.m., 3947 M, 6300 cells, current 0·04474 W. Similar phenomena to those at a pressure of 4 m.m., except that the negative glow was more extended and terminated in a fringe-like contour composed of filaments of light.

Pressure 3 m.m., 3947 M, 6300 cells, current 0·04901 W. Two stratified luminosities as shown in Plate 9, fig. 6, copied from a dry-plate photograph obtained in 25 seconds, and a drawing made at the time; the negative was completely and very brilliantly illuminated. After a short time the two luminosities coalesced to form one at the positive terminal, and between it and the negative there was a long dark discharge. The outer boundary of the discharge from the positive is less bright than the fluorescent glow of the tube, as was the case with a discharge in air. Plate 10, fig. 19, shows the phenomena of the discharge in tube 200 at the same pressure, with a current of 0·03071 W; this exemplifies well the influence of the diameter of the tube on the phenomena of stratification. In fact, in tube 199 we have not yet succeeded in obtaining a steady stratification, while in tubes of a less diameter perfectly steady strata are always obtained, although their form and number depend partly on the dimensions of the tube, Part II., fig. 62a, p. 209.

Pressure 2 m.m., 2632 M, 6300 cells, current 0·04686 W. Strata for three-fourths of the length of the tube, commencing at the positive in a point and enlarging to about half the diameter of the tube as it approached the negative.

Pressure 1·5 m.m., 1974 M, 6300 cells, current 0·05801 W. The strata completely filling the tube towards the negative in form like C, fig. 87. The negative still more brilliantly illuminated, the glow terminating as a fringe. The resistance of the tube was found to be 63,000 ohms.

Pressure 0·5 m.m., 657 M, 4800 cells, current 0·03657 W. The discharge was as in Plate 9, fig. 7, copied from a dry-plate photograph obtained in 25 seconds. The glow around the negative completely filled the tube as if the discharge experienced difficulty in emanating from that terminal, and that the gas was driven with force against the inner wall of the tube. We have already alluded in Part II. to the projection of matter from the negative terminal in a direction normal to its surface, and not necessarily towards the positive; GASSIOT[*] has recorded similar phenomena, and CROOKES' remarkable researches have shown in the most convincing manner that this occurs, and that the matter projected may be brought to a focus by curved surfaces.

The great extent of the dark discharge which sometimes obtains is very remarkable and requires special study. We have stated in Part II., p. 157, &c., that the strata sometimes thread themselves completely on the negative terminal, and that after a time they retreat towards the positive. In the course of our experiments we have frequently observed the stratification extending quite up to the negative, and after a certain time only filling about half of the tube, as, for example, in tube 128, containing hydrogen, the discharge in which, with 2400 cells, the current being 0·01639 W, is shown in Plate 9, fig. 4, copied from a photograph obtained in 5 seconds.

---

[*] Part II., Appendix, Note C.

It will have been noticed that the extent of illumination of the negative in tube 199 gradually increased as the exhaustion and the current became greater; it would seem that the negative discharge requires a greater extent of outlet than the positive, especially as the current increases; this is well exemplified by tube 51 containing hydrogen; it is 7 inches long and 2 inches in diameter; it contains a small ring terminal half an inch in diameter, the other being a spiral supported on three glass rods, and formed of a wire 19 inches long, and making four turns. When connected with a battery of 1200 cells, the spiral being negative, it became illuminated to a small extent near the positive, then the luminosity extended backwards as the current was increased, but not in the same ratio.

Through a resistance of	Illumination of negative.		Current.	
ohms.	inches.	ratios.	W.	ratios.
500,000	2·00	1·00	0 00238	1·00
400,000	2·50	1·25	0·00263	1·10
300,000	4·50	2·25	0·00277	1·16
200,000	8·00	4·00	0·00347	1·34
100,000	9·00	4·50	0·00555	2·33
50,000	13·50	6·25	0·01023	4 29
40,000	15·75	7·87	0·01158	4·86
20,000	19·00	9·50	0·01575	6·62
0	19·00 more brilliantly	9·50	0·03138	13·18

The resistance of the tube was found by substitution to be 10,500 ohms.

Plate 10, fig. 27, represents the appearance with 500,000 ohms resistance in circuit; it is copied from a photograph obtained in 10 seconds; fig. 28, the appearance without resistance, from a photograph in 5 seconds.

The experiments described in Part III. lead to the following conclusions:—

1. *For all gases there is a pressure which offers the least resistance to the passage of an electric discharge. After the minimum has been reached, the resistance to a discharge rapidly increases as the pressure of the medium decreases.* With hydrogen the minimum is 0·64 m.m., 842 M; at 0·002 m.m., 3 M, it is as great as at 35 m.m., 46,000 M.
2. *There is neither condensation nor dilatation of a gaseous medium in contiguity with charged terminals.*

3. *When the discharge takes place there is a sudden dilatation of the medium in addition to and distinct from that caused by heat. This dilatation ceases instantaneously when the discharge ceases.*
4. *The potential necessary to produce a discharge between parallel flat surfaces at a constant distance and various pressures, or at a constant pressure and various distances, may be represented by hyperbolic curves. The resistance to the discharge between parallel flat surfaces being as the number of molecules intervening between them.*
5. *This law does not hold with regard to points. In Part I. it has been shown that the potential necessary to produce a discharge at the atmospheric pressure and various distances is as the square root of the distances; while with a constant potential and various distances, the pressure has to be diminished in a greater ratio than that of the increase of distance in order to permit a discharge to take place.*
6. *The electric arc and the stratified discharge in vacuum tubes are modifications of the same phenomenon.*

We propose later on to follow up this communication with an account of some researches on the conditions of the dark discharge and of special phenomena of the negative discharge. We have also made preparations for constructing the terminals of the tubes of very small platinum tube, in order to permit of gas being admitted through them during the discharge.

We have again pleasure in thanking Professor STOKES for his much-valued advice during the course of our investigations. To our assistant, Mr. FRAM, we are indebted for his able co-operation, and we have to thank Mr. H. REYNOLDS for his aid and skill in taking photographs.

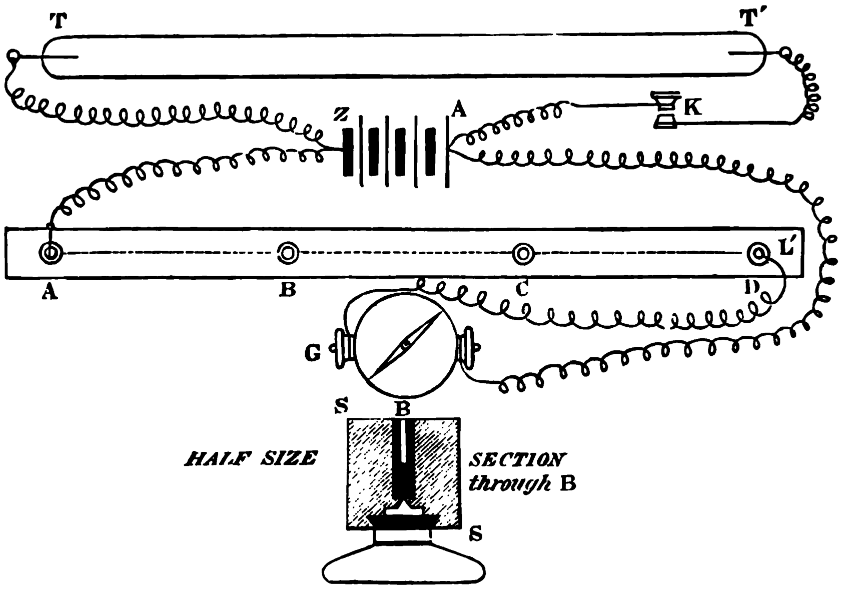
APPENDIX.

NOTE A, RELATING TO PAGE 69.

In order to avoid the injurious running down of the life of the battery in making observations of the tube-potential, two new methods of observation were adopted subsequently to that described in pp. 65–68; one by means of the galvanometer, the other by means of the electrometer. The first method consists in observing, in the first place, the deflection, D, of a THOMSON galvanometer through an adjustable resistance, so high in relation to the internal resistance of the battery that the latter might be neglected, then in observing the deflection, d, when the tube is connected with the terminals; the second deflection d, multiplied by the number of cells employed, B, and divided by the first deflection, D, gives the tube-potential V, $V=\frac{dB}{D}$.

The resistance made use of is a line drawn with a hard (H) blacklead pencil along a **V**-shaped groove in a quadrangular prism of ebonite, one inch square in section, as shown in S S (fig. 88), half size. In the prism, on the under side, a recess is made not quite reaching the ends of the prism, and at the bottom of this recess the **V** groove is formed. A dovetail groove is also made, in which an ebonite cover slides, to completely cover the recess so as to keep out dust. Into the cover two ebonite feet are screwed. Four brass rods, pierced partly through with holes to receive the terminals, are inserted through the upper surface so as to reach the apex of the V, and at the end of each rod, at the level of the apex, a small flat is formed which is rubbed over with blacklead to ensure contact with the pencil line. The length of the pencil line from A to B is 9·1 inches, B to C 9·05 inches, and C to D 9·05 inches.

Fig. 88.



The resistance of A to D was found to be 149,670 megohms.
" " A " C " " 50,789 "
" " A " B " " 8098 "

The deflection produced by different potentials, per 100 cells, was as follows:—

Number of cells used.	Sections.		
	A to D.	A to C.	A to B.
	divisions.	divisions.	divisions.
10,980	1·226	3·613	22·66
9780	1·339	3·569	19·84
8580	0·9790	3·368	18·53
7380	0·7995	3·035	16·12
6180	0·5825	2·655	14·40
4700	0·2979	2·106	11·68
3240	0·1235	1·666	9·23
2160	0·0000	1·250	7·36
1080	0·0000	0·000	5·64

The deflection is read for the battery without the tube and again when the tube is connected. If the tube-potential approaches that of the whole number of cells it may at once be calculated, but if the tube-potential is much less than that of the battery a shorter length of the line-resistance is taken and a new reading obtained with the tube connected; this new deflection is reproduced approximately by lessening the number of cells and reading the deflection again without the tube; the tube-potential is then calculated from the new data. The pencil-line-resistance has been found to change from time to time, but this is not of the least moment, because, in every observation, the tube-potential is balanced, as it were, at the time it is made.

In fig. 88 T T′, is the tube; Z A, the battery; L L′, the line-resistance; A C, B C, C D, sections of this resistance; G, the galvanometer; K, a key for connecting on the tube when required. The movable wire is shown to be inserted into the hole D so as to include the whole resistance A D, but it may be inserted into either C or B.

NOTE B, RELATING TO PAGE 69.

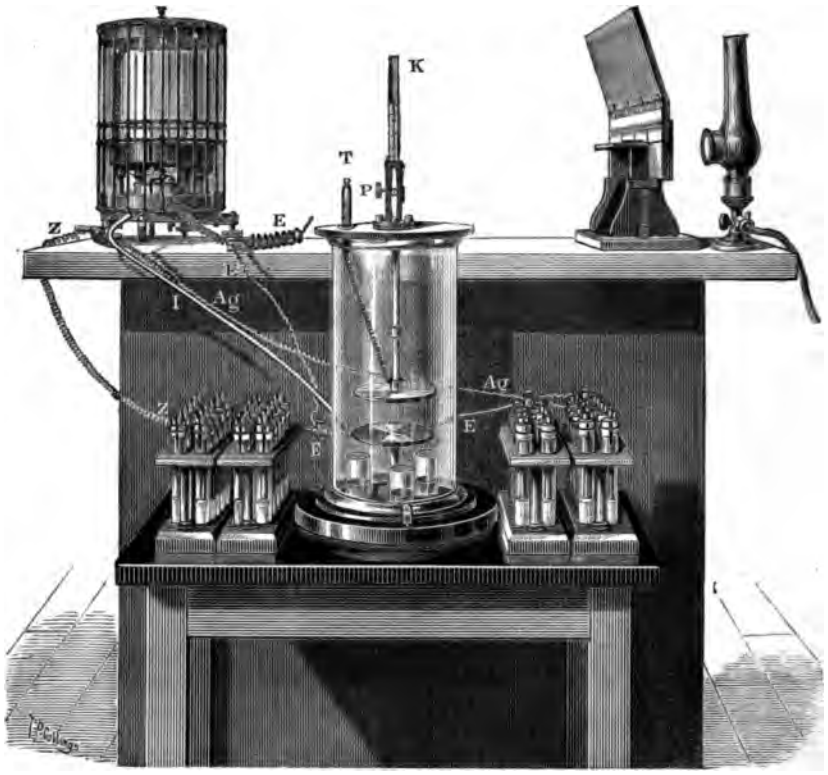
The second method of obtaining the tube-potential, suggested to us by Professor MASCART, is a modification of that described (Part II., p. 165); the change consisting in detaching the induction apparatus from the electrometer, as shown in fig. 89; for it was found in the former arrangement that the influence of the induction-plate on the pair of quadrants opposed to that over which it was placed was detrimental, especially when much raised. Moreover, it was found to be advantageous to charge the *needle* and not one of the quadrants with the induced charge; and, on the other hand, to connect each pair of quadrants respectively with the + or — terminal of two chloride of silver batteries, each of 20 or more cells. The other terminal of each battery being connected to the earth; each pair of quadrants becomes constantly charged to the same potential, one with positive the other with negative electricity, and their influence on the needle is perfectly symmetrical.

The figure shows clearly the arrangement. The lower plate is in connexion with the needle through the insulated wire I; the Z terminal of one battery, and the A*g* terminal of the other are connected to their respective quadrants. When an observation has to be made, one terminal of the battery of high potential is brought into connexion with T, which is in metallic communication with the upper or inducing plate of the induction apparatus, the other terminal of the battery being to earth; the swing of the needle is observed and the apparatus is afterwards discharged by connecting the needle and the inducing plate to earth, the tube is then attached to the terminals and the swing observed when T is again touched. The ratio of the latter to the former gives the ratio of the tube-potential to the number of cells used. If the two deflections are very different, it is advantageous to balance the swing with the tube approximately by a less number of cells, and to calculate the tube-potential with the fresh data.

In order to prevent worrying oscillations of the needle a slight but important change is made in the attachment of the damper to the needle. Soldered rigidly to the needle is a ⊥-shaped wire, to the horizontal ends of which is hung a Y-shaped wire to form a hinge; to the vertical projection of the Y are attached three cross-wires which dip into the acid, forming in this way a damper as well as a conductor. As the damper is rigidly connected to the needle in the plane of the swing it moves steadily, and its extreme swing may be taken for the reading.

The inducing and induced plate are both 4 inches in diameter, and may be made to touch or be separated a distance of 7 inches.

Fig. 89.



The following readings were obtained with the plates 4 inches apart; the quadrants charged to a potential of 20 cells:—

Cells.	Deflection in scale divisions.		
	Total.	Per 1000 cells.	
1200	22	18·3	17·4 average
2400	40	16·7	
3600	64	17·9	
4800	79	16·5	
6300	108	17·1	
7760	139	17·8	
8840	185	20·9	
9920	234	23·6	
11,000	288	26·2	
	with the quadrants charged to a potential of 30 cells		
11,000	338	30·7	

By increasing the number of cells in connexion with the quadrants, say to 100, the deflection is considerably augmented; it may also be increased to any requisite extent by bringing the plates of the induction apparatus closer together.

NOTE C, RELATING TO PAGE 88.

DE LA RIVE, Genève, Mem. Soc. Phys. XVII., 1863, pp. 73–74.—"Le phénomène dans les fluides élastiques raréfiés consisterait dans des contractions et dilatations alternatives du milieu gaseux, produites par la série des décharges toujours plus ou moins discontinues dont le jet électrique est formé. En effet, que ce soit par l'appareil RUHMKORFF ou bien par une machine électrique ordinaire, ou une machine hydro-électrique d'ARMSTRONG, et même par une pile voltaique à haute tension, qu'on produise les stratifications, on n'a jamais une décharge continue, mais bien une série de décharges qui peuvent se succéder assez rapidement pour que la discontinuité ne soit pas accusée même par un galvanomètre; mais cette discontinuité n'existe pas moins, comme M. GASSIOT l'a montré en opérant avec une pile de GROVE à haute tension qui, avec les mêmes électrodes et dans le même milieu, peut donner naissance, d'abord à des stratifications, puis plus tard à un arc voltaique, quand le courant est devenu continu.

"Au reste l'action mécanique de la série des décharges sur le fluide élastique raréfié peut être constatée directement par les oscillations très prononcées de la colonne de mercure du manomètre mis en communication avec la fluide élastique qui accompagnent la propagation de l'électricité dans ce fluide. Ces oscillations s'élèvent jusqu'à deux ou trois dixièmes de millimètre dans l'hydrogène sous la pression de 16 millimètres; elles commencent à être sensibles dès que le jet passe, c'est-à-dire à 36 millimètres de pression; elles atteignent leur maximum, qui est de trois dixièmes de millimètre, entre 20 et 12 millimètres de pression; elles diminuent rapidement à partir de 12 jusqu'à 5 millimètres de pression, sous laquelle elles n'ont pas lieu. Avec l'azote et avec l'air atmosphérique, et en se servant du même tube de 16 centimètres de longueur et de 5 de diamètre, les oscillations commencent à se montrer en même temps que le jet passe, sous la pression de 20 millimètres environ; elles atteignent leur maximum de 4 à 5 dixièmes de millimètre entre 12 et 8 millimètres de pression; puis elles vont en diminuant jusqu'à 2 ou 3 millimètres pression, sous laquelle elles ne sont plus sensible.

"Avec le tube d'un mètre de longueur, et même avec celui de 50 centimètres, je n'ai pu observer aucune apparence d'oscillation accompagnant la transmission du jet électrique, quel que fût le gaz renfermé dans ces tubes et quelle que fût la pression à laquelle il fut soumis. Par contre, j'ai obtenu des très prononcées, de 1 à 2 dixièmes de millimètre dans un bocal de 20 centimètres de hauteur (7·8 inches),

sur 16 (3·2 inches) de diamètre rempli d'hydrogène raréfié, et dans lequel le jet électrique passait d'une boule centrale à un anneau concentrique à cette boule, de 12 centimètres (4·68 inches) de diamètre. Ce dernier résultat montre que l'absence d'oscillation dans les longs tubes tient moins au volume de la couche gazeuse, qui est plus faible qu'elle ne l'est dans le bocal de la dernière expérience, qu'à l'influence des parois des tubes qui gênent le mouvement du gaz. C'est aussi une preuve que les oscillations proviennent bien d'une action mécanique, et non d'une élévation de température. Quant à l'intensité des oscillations, elles dépendent évidemment de la résistance plus ou moins grande que le milieu gazeux oppose à la transmission du jet électrique, puisque les oscillations sont plus considérables avec l'azote qu'avec l'hydrogène, et qu'elles diminuent en même temps que la pression à partir d'une certaine pression, qui est celle où la décharge peut s'opérer d'une manière complète et à laquelle l'intensité des oscillations atteint son maximum.

"La stratification de la lumière électrique serait donc un phénomène analogue à la production des ondes sonores, c'est-à-dire un phénomène mécanique provenant d'une succession d'impulsions isochrones excercées sur la colonne gazeuse raréfiée, par la série des décharges électriques se succédant très rapidement les unes aux autres. Nous trouvons une nouvelle preuve en faveur d'envisager le phénomène dans la perturbation qu'apporte aux stratifications un déplacement de la matière gazeuse, et par conséquent dans la disposition du fluide élastique qui permet leur apparition. Il suffit, pour produire cette perturbation d'introduire dans le tube où l'on a un fluide élastique raréfié, pendant que l'électricité s'y propage une quantité additionelle du même gaz qui s'y trouve déjà renfermé de manière à augmenter la pression de $\frac{1}{4}$ ou $\frac{1}{2}$ millimètre ou plus. Voici ce qui se passe avec l'hydrogène, les effets sont les mêmes avec les trois tubes de 15, de 50, et 100 centimètres de longueur." . . . . .

INDEX TO PART III.

PAGE.
Air—Phenomena of arc in 91–94
  Potential necessary to produce discharge between discs in 77
Arc, Electric—Phenomena of, in air between points, Plate 8, figs. 1–10 89
  „ in air between spherical surfaces, Plate 10, figs. 1–18 94–96
  „ in carbonic acid, Plate 10, figs. 20–23 89
  „ in hydrogen, Plate 8, figs. 11–19 88
  „ „ between spherical surfaces, fig. 84, A, B, C, D, E, F, G, H 97
  „ „ between points, Plate 8, figs. 21–29; 34–35; Plate 10, figs. 25–26 99–100
  Temperature of, supposing the sudden dilatation of a gas when a discharge occurs to be solely due to development of heat 84–87
Carbonic Acid—Phenomena of arc in, Plate 10, figs. 20–23.. 89
  Potential necessary to produce a discharge in 78
Condensation, none occurs in a gaseous medium in contiguity with charged terminals, fig. 74 .. 75–76
Curves representing the potential necessary to produce a discharge in gases 70–71; 74
De La Rive on the nature of the stratified discharge 113
Dilatation of gases when discharge takes place is mainly due to a projection of molecules and not to increase of temperature 87–88
  „ of gases (expansion) the same at both terminals, Plate 10, fig. 24 101
  „ „ none previous to the discharge in contiguity with charged terminals, fig. 74 75–76
  „ sudden, of a gaseous medium when a discharge takes place, fig. 81; Plate 9, figs. 8–10 .. .. 85–86; 91; 94; 96; 98–99
Discharge between points in air with a constant potential, and different distances and pressures 91–93
  „ dark, extent of, Plate 9, fig. 4 108
Electrometer, the quadrants charged to the same potential but of opposite signs; the needle with the potential to be measured (Mascart's arrangement).. 111
Electro-motive force, table of, in volts, necessary for various striking distances in centimetres in air between discs at atmospheric pressure 83
Expansion. *See* Dilatation.
Hydrogen, potential necessary to produce a discharge in 78
  „ phenomena of arc in 88; 96; 98–99
Hyperbolic curve represents the experimental results of the potential necessary to produce a discharge between discs at a constant distance and various pressures in air, hydrogen, and carbonic acid 77; 81
  „ „ also holds good for a constant pressure and various distances, fig. 78.. .. 82
Illumination of negative terminals, extent of, dependent on amount of current, Plate 10, figs. 27–28.. 107–108
Mascart, Professor 75; 111
Pencil, blacklead-line resistance, use of in determining a tube-potential during a discharge .. 110
Potential, difference of, between terminals during a discharge in a vacuum tube (tube-potential) 93
  „ necessary to produce a discharge in air 75–77
  „ „ „ „ carbonic acid.. 78
  „ „ „ „ hydrogen 78

PAGE.
Resistance, blacklead-pencil-line .. 110
Stokes, Professor .. 79; 109
Stratification, nature of, De la Rive .. 113–114
Tube of large dimensions, Plate 9, figs. 5, 6, 7; fig. 87; A, B, C .. 104; 106
„ peculiar discharge in a, Plate 9, fig. 3 .. 75
Tube-potential. *See* Potential.
  „ calculation from deflections of a galvanometer .. 65
  „ decreases with decrease of pressure at first, then increases .. 69
  „ for hydrogen, least at a pressure of 0·64 m.m., 845 **M** .. 69
  „ measurement of, by means of the electrometer .. 111
  „ „ „ an enormous pencil-line resistance .. 110

Plates 8, 9, 10, references to text.

Plate 8, figs. 1–10 .. 91
„ figs. 11–19 .. 88
„ fig. 20 .. 91
„ figs. 21–29 .. 98
„ fig. 30 .. 103
„ fig. 31 .. 91
„ figs. 32–33 .. 90
„ figs. 34–35 .. 99
Plate 9, figs. 1, 2, 3 .. 73, 75
„ fig. 4 .. 108
„ figs. 5–7 .. 104–106
„ figs. 8–10 .. 87; 90–91
Plate 10, figs. 1–18 .. 94–96
„ fig. 19 .. 107
„ figs. 20–23 .. 89
„ fig. 24 .. 101
„ fig. 25 .. 100
„ fig. 26 .. 99
„ figs. 27–28 .. 108

V. *On the Conduction of Heat in Ellipsoids of Revolution.*

By C. NIVEN, *M.A., Professor of Mathematics in Queen's College, Cork.*

Communicated by J. W. L. GLAISHER, *F.R.S.*

Received May 7—Read May 29, 1879.

THE object of the present paper is to investigate the expressions which present themselves in the solution of the problem of the conduction of heat in an ellipsoid of revolution. For although the question of the stationary temperature of ellipsoids in general has been completely solved by means of the functions introduced by GREEN and LAMÉ, the corresponding problem of conduction has not been so successfully dealt with. M. MATHIEU, indeed, in his 'Cours de Physique Mathematique,' has shown how to reduce the solution to ordinary differential equations, and for the special case of an ellipsoid of revolution has shown how to approximate to their solutions. His method, which is novel and remarkable, enables him to calculate a few terms of the expressions, but does not afford a view of their general constitution and properties. In the present paper the question is treated in a more direct and general manner.

Choosing with M. MATHIEU, as coordinates of a point, the azimuth ϕ of the meridional section through it and the parameters α and β of the ellipsoid and hyperboloid confocal to the surface which intersect in the point, it is first shown how to transform the general equation of conduction to these coordinates. This equation is then satisfied by a series of terms of the form $e^{-\lambda^2 t}\cos m\phi\, \vartheta_m^k(\beta)\Omega_m^k(\alpha)$, in which k is determined by an equation whose roots are infinite in number.

The function ϑ is expanded in what Mr. TODHUNTER, translating HEINE's term, calls associated functions of $\cos\beta$, and we shall also follow HEINE in denoting by $P_m^n(\mu)$ the expression $(\mu^2-1)^{\frac{m}{2}}(\mu^{n-m}-$ &c.). The language of harmonic analysis has been greatly benefited by Professor MAXWELL's introduction of the words type and degree into the specification of a tesseral harmonic, though we prefer to replace the term type by order. We shall therefore call the product $P_m^n(\mu)\cos m\phi$ the tesseral harmonic of the m^{th} order and n^{th} degree, and the factor $P_m^n(\mu)$ the associated function of the m^{th} order and n^{th} degree. The expansion of the function ϑ will then consist of a series of associated functions of the m^{th} order.

It is shown that the roots of the equation in k fall into two classes, and that the corresponding expressions for ϑ take different forms, for one of which the difference between the degree and order of the associated functions involved is an even number and for the other odd. The values of $\Omega(\alpha)$ divide themselves in like manner into two classes. These values are expanded, in the first instance, in terms of the minor axis of the confocal ellipsoid, and afterwards in terms of the major axis, the former series proceeding by functions which satisfy the differential equation

$$\frac{d^2u}{dx^2}+\frac{2}{x}\frac{du}{dx}+\left(1-\frac{n.\overline{n+1}}{x^2}\right)u=0.$$

Of the two solutions of this equation, which are both finite in form, one S_n is finite when $x=0$, while the other T_n becomes infinite.

The expression S_n plays for spheres the same part that BESSEL's function plays for circular cylinders, and as HEINE has employed the term cylinder-function for the latter, it would seem consistent with analogy to use the term spherical-function for S_n. When Ω is expressed in terms of the major axis of the confocal ellipsoid two expansions are given, one in terms of spherical functions and the other in terms of associated functions, and it is shown that each of these series possesses special advantages in relation to particular points which arise in the problem of the conduction of heat.

The properties of these functions are afterwards further considered.

The above expansions being found, I have next discussed the system of equations which determine k, following more or less closely the analysis which HEINE has given of a similar system, and it is proved that the values of k are all real and definite in position, and that for these values the expansions of ϑ and Ω converge rapidly when a sufficiently large number of terms is taken.

I then show how to express, by successive approximation, the roots of the equation in k in powers of a quantity ϵ which depends on the eccentricity, and have entered with some fulness of detail into the numerical calculation of a few of the smaller roots and of the corresponding coefficients of the functions ϑ and Ω, more especially in the case of the first of the two classes into which they fall. Besides these particular values, however, the general formulæ are given, which will furnish them to a certain degree of approximation for all values of m and for any value of k.

With regard to the special problem of the conduction of heat, the boundary condition is supposed to be either that the surface is kept at a constant temperature, or that the body is cooling by radiation. The former is mathematically the simpler, and we might imagine it realised in the case of a body kept in the midst of an infinite fluid after a sufficient time has elapsed for the surface to take the temperature of the fluid. With this assumption the different values of λ might be found from the equation $\Omega_m{}^k=0$; and the roots of $\Omega_0{}^0=0$ are investigated up to e^4. The general condition of radiation is next considered, and it is shown how it may be brought theoretically within the range of analysis. I have not, however, thought it necessary to do more in this direction than indicate how the successive approximations may be found.

Although the calculations were undertaken in the first instance for the case of an ellipsoid whose major axis is that of revolution, it is proved that the expressions and results may be easily transferred to the case of a planetary ellipsoid. And as the chief object of the paper is to examine the forms and properties of the functions which present themselves in the mathematical treatment of the subject, I have not thought it necessary to enter on the discussion of the movement of heat in a shell bounded by two confocal ellipsoids, or in an infinite solid from which an ellipsoidal cavity has been taken—questions, however, which might be easily treated by introducing the functions T_n as well as S_n. The analysis would seem also to be applicable to other physical problems relating to ellipsoids of revolution.

Perhaps I should explain that if I have entered somewhat minutely into points which are not new, I have done so for the purpose of rendering the argument clearer and more coherent.

2. When a solid body is heated in any manner and left to cool, the equations which have to be satisfied are, first, the general equation of conduction

$$\frac{d^2V}{dx^2}+\frac{d^2V}{dy^2}+\frac{d^2V}{dz^2}=\frac{1}{\mathfrak{k}}\frac{dV}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

wherein V is the temperature at any point, $\mathfrak{k}$ the "thermal diffusivity" of the body; 2^0 the boundary condition, which we shall suppose to be one or other of the forms

$$V=0 \text{ or } \frac{dV}{dN}+\mathfrak{h}V=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

wherein $\frac{dV}{dN}=l\frac{dV}{dx}+m\frac{dV}{dy}+n\frac{dV}{dz}$, l m n being the direction cosines of the normal N to the surface measured outwards, and $\mathfrak{h}$ is a constant.

The first of these conditions corresponds to the case where the surface of the body is maintained everywhere at uniform temperature, for we may suppose the zero of temperature so chosen as to coincide with the given temperature, whatever it may be. The second is the usual condition of radiation according to NEWTON's law of cooling.

We have also a third condition, that which gives the initial state of the body:—

$$V=V_0=f(x, y, z), \text{ when } t=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The general course of solution, whatever the solid may be, is to put

$$V=\Sigma A.e^{-\mathfrak{k}\lambda^2 t}.v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where v is a function of x y z, so chosen as to satisfy the equation

$$\frac{d^2v}{dx^2}+\frac{d^2v}{dy^2}+\frac{d^2v}{dz^2}=-\lambda^2 v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and the boundary condition

$$v=0 \text{ or } \frac{dv}{d\mathrm{N}}+\mathfrak{h}v=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

When the appropriate functions v have been found, in general triply infinite in number, to satisfy (5) and (6), the constants A may be determined from condition (3). Now it is obvious from the nature of the case that any solution of the equations which satisfies (1) and (2) and reproduces (3) when $t=0$, will be the solution sought, and the same conclusion can be readily demonstrated by analysis. With regard to equation (6), it serves two purposes: first in enabling us to select the appropriate form of v, and secondly in furnishing the values of λ, which determine the types of heat-movement which take place. POISSON has shown, in a very elegant way, that the values of λ are always real, and as his results are of importance as showing also how the constants A are to be found, I shall here reproduce them.

Let v and v' be two functions of $x\ y\ z$ satisfying the equations (5) and (6), and let ∇^2 stand for $\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}$, then

$$(\lambda'^2-\lambda^2)\int vv'dxdydz=\int(v'\nabla^2v-v\nabla^2v')dxdydz$$
$$=\int\left(v'\frac{dv}{d\mathrm{N}}-v\frac{dv'}{d\mathrm{N}}\right)ds,$$

ds being an element of the surface; hence, if either form of (6) be true,

$$\int vv'dxdydz=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

It follows from this equation that the equation in λ cannot have imaginary roots; for, v being always a function of λ, if $\lambda=p+q\sqrt{-1}$, there will be another root $\lambda'=p-q\sqrt{-1}$, and the corresponding values of v will be respectively $\mathrm{L}+\mathrm{M}\sqrt{-1}$ and $\mathrm{L}-\mathrm{M}\sqrt{-1}$. Equation (7) now becomes

$$\int(\mathrm{L}^2+\mathrm{M}^2)dxdydz=0,$$

which is clearly impossible. We may also employ (7) to find A, for

$$\mathrm{V}_0=\Sigma\mathrm{A}v$$
$$\mathrm{A}=\frac{\int\mathrm{V}_0v d\mathrm{E}}{\int v^2 d\mathrm{E}},\ d\mathrm{E}=dxdydz \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

in which the integration is extended throughout the whole of the solid.

3. For the case of a solid sphere whose radius $r=r_0$, these equations become, in polar coordinates,

$$\frac{d^2V}{dr^2}+\frac{2}{r}\frac{dV}{dr}+\frac{1}{r^2}\left\{(1-\mu^2)\frac{d^2V}{d\mu^2}-2\mu\frac{dV}{d\mu}+\frac{1}{1-\mu^2}\frac{d^2V}{d\phi^2}\right\}=\frac{1}{\mathfrak{k}}\frac{dV}{dt},$$

$$V=0 \text{ or } \frac{dV}{dr}+\mathfrak{h}V=0, \text{ when } r=r_0,$$

$$V_0=f(r, \mu, \phi),$$

$$dE=-dr.r^2d\phi d\mu, \text{ where } \mu=\cos\theta.$$

The general type of the solution is

$$V=e^{-\lambda^2\mathfrak{k}t}(A\cos m\phi+B\sin m\phi)P_m^nR_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

in which m may have all integral values from 0 up to ∞, $P_m^n\cos m\phi$ is the tesseral harmonic of the m^{th} order and n^{th} degree, and R_n satisfies the equation

$$\frac{d^2R_n}{dr^2}+\frac{2}{r}\frac{dR_n}{dr}+\left(\lambda^2-\frac{n(n+1)}{r^2}\right)R_n=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The two particular solutions of this equation are

$$S_n=r^n\left(\frac{1}{r}\frac{d}{dr}\right)^n\frac{\sin\lambda r}{r},\ T_n=r^n\left(\frac{1}{r}\frac{d}{dr}\right)^n\frac{\cos\lambda r}{r},$$

(as will be presently shown), of which the former only is appropriate to the case of a solid sphere, the other becoming infinite at the centre. We have therefore to choose $R_n=S_n$.

With regard to the integer n, it must be at least as great as m, but may have any value from m up to ∞. The form of the solution being now ascertained, the values of λ may be found from the condition that at the boundary

$$R_n=0 \text{ or } \frac{dR_n}{dr}+\mathfrak{h}R_n=0 \text{ when } r=r_0,$$

and the arbitrary constants A, B . . . may be found from the initial distribution.

4. We shall now determine the appropriate transformations of (1) and (2) for an ellipsoid of revolution, and shall confine ourselves in the first instance to the case of an ovoid ellipsoid, reserving that of a planetary ellipsoid for subsequent treatment.

The axis of z being that of revolution, put $x=\rho\cos\phi$, $y=\rho\sin\phi$; equations (1) and (2) become

$$\frac{d^2V}{dz^2}+\frac{d^2V}{d\rho^2}+\frac{1}{\rho}\frac{dV}{d\rho}+\frac{1}{\rho^2}\frac{d^2V}{d\phi^2}=\frac{1}{\mathfrak{k}}\frac{dV}{dt}$$

$$V=0 \text{ or } l\frac{dV}{d\rho}+n\frac{dV}{dz}+\mathfrak{h}V=0,$$

in which l n are here the direction cosines of the normal to a meridian section. Let us now replace ρ and z by

$$\rho=c\sinh\alpha\sin\beta,\ z=c\cosh\alpha\cos\beta\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\qquad(11)$$

where α and β are the thermometric parameters of the confocal system which includes the principal elliptic section of the bounding surface for which $\alpha=\alpha_0$, and whose axes are therefore $2c\sinh\alpha_0$, $2c\cosh\alpha_0$.

Since ρ and z are conjugate functions of α, β,

$$\frac{d^2V}{d\alpha^2}+\frac{d^2V}{d\beta^2}=\left[\left(\frac{d\rho}{d\alpha}\right)^2+\left(\frac{d\rho}{d\beta}\right)^2\right]\left(\frac{d^2V}{d\rho^2}+\frac{d^2V}{dz^2}\right)=c^2(\cosh^2\alpha-\cos^2\beta)\left(\frac{d^2V}{d\rho^2}+\frac{d^2V}{dz^2}\right).$$

Moreover

$$\frac{dV}{d\alpha}=c\cosh\alpha\sin\beta\frac{dV}{d\rho}+c\sinh\alpha\cos\beta\frac{dV}{dz},$$

$$\frac{dV}{d\beta}=c\sinh\alpha\cos\beta\frac{dV}{d\rho}-c\cosh\alpha\sin\beta\frac{dV}{dz},$$

whence

$$c(\cosh^2\alpha\sin^2\beta+\sinh^2\alpha\cos^2\beta)\frac{dV}{d\rho}=\cosh\alpha\sin\beta\frac{dV}{d\alpha}+\sinh\alpha\cos\beta\frac{dV}{d\beta},$$

and

$$\cosh^2\alpha\sin^2\beta+\sinh^2\alpha\cos^2\beta=\cosh^2\alpha-\cos^2\beta.$$

By the help of these formulæ we may transform the general equation of conduction into

$$\frac{d^2V}{d\alpha^2}+\frac{d^2V}{d\beta^2}+\coth\alpha\frac{dV}{d\alpha}+\cot\beta\frac{dV}{d\beta}+\left(\frac{1}{\sin^2\beta}+\frac{1}{\sinh^2\alpha}\right)\frac{d^2V}{d\phi^2}=\frac{c^2}{\mathfrak{k}}(\cosh^2\alpha-\cos^2\beta)\frac{dV}{dt}\ \ .\qquad(12)$$

In a similar manner, the equation to be satisfied at the boundary becomes

$$V=0 \text{ or else}\frac{dV}{d\alpha}+\mathfrak{h}c\sqrt{\cosh^2\alpha-\cos^2\beta}V=0,\text{ when }\alpha=\alpha_0.\ \ .\ \ .\ \ .\qquad(13)$$

We must also find the space element, dE,

$$dE=\rho d\phi.d\rho dz=\rho d\phi d\alpha d\beta\begin{vmatrix}\frac{d\rho}{d\alpha}, & \frac{d\rho}{d\beta}\\ \frac{dz}{d\alpha}, & \frac{dz}{d\beta}\end{vmatrix}$$

and finally,

$$dE=c^3\sinh\alpha\sin\beta(\cosh^2\alpha-\cos^2\beta)d\phi d\alpha d\beta \quad . \quad . \quad . \quad . \quad . \quad (14)$$

5. We proceed to find the solution of (12) which is appropriate. We may satisfy it by putting $V=\Sigma(\cos m\phi U_1+\sin m\phi U_2)$, where U_1 and U_2 both satisfy

$$\frac{d^2U}{d\alpha^2}+\frac{d^2U}{d\beta^2}+\coth\alpha\frac{dU}{d\alpha}+\cot\beta\frac{dU}{d\beta}-m^2\left(\frac{1}{\sinh^2\alpha}+\frac{1}{\sin^2\beta}\right)U=\frac{c^2}{\mathfrak{f}}(\cosh^2\alpha-\cos^2\beta)\frac{dU}{dt}.$$

And with regard to m, it must be observed that it cannot be other than a whole number, since the value of V must repeat itself in going round the surface of the ellipsoid in the ϕ-direction; that is to say,

$$m=0,\ 1,\ 2\ \ldots\infty \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

We may also put

$$\left.\begin{array}{l} U=e^{-\lambda^2\mathfrak{f}t}.v \\ v=\vartheta_m{}^k(\beta).\Omega_m{}^k(\alpha) \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where ϑ and Ω are functions of β alone and of α alone respectively, determined from the equations

$$\frac{d^2\vartheta}{d\beta^2}+\cot\beta\frac{d\vartheta}{d\beta}-\frac{m^2}{\sin^2\beta}\vartheta=\lambda^2c^2\cos^2\beta\vartheta-k\vartheta \quad . \quad . \quad . \quad . \quad . \quad . \quad (18,)$$

$$\frac{d^2\Omega}{d\alpha^2}+\coth\alpha\frac{d\Omega}{d\alpha}-\frac{m^2}{\sinh^2\alpha}\Omega=-\lambda^2c^2\cosh^2\alpha\Omega+k\Omega \quad . \quad . \quad . \quad . \quad . \quad (19,)$$

wherein k is a constant, as yet undetermined. In the sequel it will appear that k has an infinite number of values for a given value of m and a given value of λ, and one of the objects of the present investigation is to furnish the equation which determines it, and to approximate to its different values when the eccentricity of the ellipsoid is small. In this respect the present problem differs essentially from the corresponding one for a sphere in which k is independent of λ; it is then given by

$$k=n(n+1),\ \text{where}\ n=m,\ m+1,\ m+2,\ \ldots\infty \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

If we put $\cos\beta=\nu$, the equation in ϑ may be written

$$(1-\nu^2)\frac{d^2\vartheta}{d\nu^2}-2\nu\frac{d\vartheta}{d\nu}-\frac{m^2}{1-\nu^2}\vartheta=\lambda^2c^2\nu^2\vartheta-k\vartheta \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and, if we write $\xi=\lambda c\sinh\alpha$, the equation in Ω is

$$(\xi^2+\lambda^2c^2)\frac{d^2\Omega}{d\xi^2}+2\xi\frac{d\Omega}{d\xi}+\frac{\lambda^2c^2}{\xi}\frac{d\Omega}{d\xi}-\frac{m^2c^2\lambda^2}{\xi^2}\Omega=-(\lambda^2c^2+\xi^2)\Omega+k\Omega$$

or, more conveniently,

$$\xi^2\frac{d^2\Omega}{d\xi^2}+2\xi\frac{d\Omega}{d\xi}+\xi^2\Omega-k\Omega+\lambda^2c^2\left(\frac{d^2\Omega}{d\xi^2}+\frac{1}{\xi}\frac{d\Omega}{d\xi}+\Omega-\frac{m^2}{\xi^2}\Omega\right)=0 \quad . \quad . \quad . \quad (19)$$

It will be observed that these transformations are suggested by the consideration that, when $c=0$, the equations in ϑ and Ω should reproduce the corresponding equations in P_m^n and R_n for a sphere; $\frac{\xi}{\lambda}$ is the semi-axis minor of the confocal ellipse, and we shall use $\frac{\eta}{\lambda}$ to denote in like manner the semi-axis major.

6. I shall now show how equation (18) may be satisfied by a series of associated functions of the order m. If we compare the functions P_m^{n+1}, P_m^n, P_m^{n-1}, *in which the constant multipliers are so chosen as to make the coefficient of the highest power of* ν *in their rational factors unity,* it is known that

$$\nu P_m^n=P_m^{n+1}+\frac{n^2-m^2}{4n^2-1}P_m^{n-1},$$

and hence

$$\nu^2P_m^n=P_m^{n+2}+\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}P_m^n+\frac{(n^2-m^2)(\overline{n-1}^2-m^2)}{(4n^2-1)(4\overline{n-1}^2-1)}P_m^{n-2} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

It appears from this that the last term will vanish both when $n=m$ and when $n=1+m$, the theorem reducing then to

$$\nu^2P_m^m=P_m^{m+2}+\frac{1}{2m+3}P_m^m,$$

and to

$$\nu^2P_m^{m+1}=P_m^{m+3}+\frac{3}{2m+5}P_m^{m+1}.$$

It is clear, generally, that $\nu^{2i}P_m^n$ can be expanded in a series of associated functions of an even or odd degree, according as n is even or odd. Bearing in mind that

$$\left[(1-\nu^2)\frac{d^2}{d\nu^2}-2\nu\frac{d}{d\nu}-\frac{m^2}{1-\nu^2}+k\right]P_m^n=-(n(n+1)-k)P_m^n,$$

we can obviously satisfy (18) by the expression

Similarly, when $n-m$ is odd, the values of k are the roots of

$$b_\infty=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

and we have to show that these roots converge to fixed values.

But before entering on this discussion it will be convenient to take up the consideration of equation (19), and show that it can be satisfied by a definite series of known functions corresponding to the same values of k; and, preparatory to doing so, I shall digress briefly into the solutions of the equation

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+\left(\lambda^2-\frac{n(n+1)}{r^2}\right)R=0.$$

My reason for doing so is that, for the transformations which follow, a connected view of the properties of these solutions is necessary.

7. If we write $x=\lambda r$, the equation may be written

$$\frac{d^2R}{dx^2}+\frac{2}{x}\frac{dR}{dx}+\left(1-\frac{n.n+1}{x^2}\right)R=0.$$

We shall now show that the equation is satisfied by

$$R=x^n\left(\frac{1}{x}\frac{d}{dx}\right)^n\frac{\sin x}{x}=S_n, \text{ or by } R=T_n=x^n\left(\frac{1}{x}\frac{d}{dx}\right)^n\frac{\cos x}{x} \quad . \quad . \quad . \quad . \qquad (25)$$

If we write $R=x^n.u_n$, we find

$$\frac{d^2u_n}{dx^2}+2(n+1)\frac{1}{x}\cdot\frac{du_n}{dx}+u_n=0.$$

If we differentiate this equation, we may readily put the result into the form

$$\frac{d^2}{dx^2}\left(\frac{1}{x}\frac{du_n}{dx}\right)+2(n+2)\cdot\frac{1}{x}\cdot\frac{d}{dx}\left(\frac{1}{x}\frac{d}{dx}u_n\right)+\frac{1}{x}\frac{du_n}{dx}=0;$$

and, comparing this with the former equation, we find

$$u_{n+1}=\frac{1}{x}\frac{d}{dx}\cdot u_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (26)$$

We thus obtain, generally, $u_n=\left(\frac{1}{x}\frac{d}{dx}\right)^n u_0$.

And it is easily seen that u_0 is given by

$$\frac{d^2(u_0x)}{dx^2}+u_0x=0,$$

or

$$u_0=\mathrm{A}\,\frac{\sin x}{x}+\mathrm{B}\,\frac{\cos x}{x}.$$

The two values of R are therefore those given by (25).

We may readily find these values as expansions in powers of x, by putting t for x^2 and expanding $\frac{\sin\sqrt{t}}{\sqrt{t}}$ and $\frac{\cos\sqrt{t}}{\sqrt{t}}$.

$$\left.\begin{aligned}\mathrm{S}_n&=\frac{(-1)^nx^n}{1.3\ldots(2n+1)}\left\{1-\frac{1}{1.(2n+3)}\,\frac{x^2}{2}+\frac{1}{1.2.(2n+3)(2n+5)}\,\frac{x^4}{4}-\ldots\right\}\\ \mathrm{T}_n&=\frac{(-1)^n1.3\ldots(2n-1)}{x^{n+1}}\left\{1+\frac{1}{1.(2n-1)}\,\frac{x^2}{2}+\frac{1}{1.2.(2n-1)(2n-3)}\,\frac{x^4}{4}+\ldots\right\}\end{aligned}\right\}\;. \qquad (27)$$

Since the differential equation is unaltered by replacing n by $-(n+1)$, it follows that S_{-n-1} and T_{-n-1} are also solutions of it; and in fact it is clear, in comparing corresponding terms, that

$$\mathrm{S}_{-n-1}=(-1)^n\mathrm{T}_n$$
$$\mathrm{T}_{-n-1}=(-1)^n\mathrm{S}_n,$$

the constants, introduced by integration in S_{-n-1}, being so adjusted as to make the first $\overline{n+1}$ terms agree; no constants are to be introduced in determining T_{-n-1}.

The form $\mathrm{S}_n=2^nt^{\frac{n}{2}}\left(\frac{d}{dt}\right)^n\cdot\frac{\sin\sqrt{t}}{\sqrt{t}}$ also indicates the expansion as a **Bessel's** function, quoted by Lord **Rayleigh**,

$$\mathrm{S}_n=(-1)^n\sqrt{\frac{\pi}{2}}x^{-\frac{1}{2}}\mathrm{J}_{n+\frac{1}{2}}(x)\;;$$

so

$$\mathrm{T}_n=(-1)^n\sqrt{\frac{\pi}{2}}x^{-\frac{1}{2}}\mathrm{K}_{n+\frac{1}{2}}(x).$$

The finite expansions of S_n and T_n are

$$\left.\begin{aligned}\mathrm{S}_n=&\left(\frac{1}{x}-\frac{n'(n'-1')}{1.2}\cdot\frac{1}{2^2x^3}+\frac{n'(n'-1')(n'-2')(n'-3')}{1.2.3.4}\cdot\frac{1}{2^4x^5}-\ldots\right)\sin\left(x+\frac{n\pi}{2}\right)\\ &+\left(\frac{n'}{1}\cdot\frac{1}{2x^2}-\frac{n'(n'-1')(n'-2')}{1.2.3}\cdot\frac{1}{2^3x^4}+\ldots\right)\cos\left(x+\frac{n\pi}{2}\right)\end{aligned}\right\}. \qquad (28)$$

$$\begin{aligned}\mathrm{T}_n=&\left(\frac{1}{x}-\frac{n'(n'-1')}{1.2}\cdot\frac{1}{2^2x^3}+\ldots\right)\cos\left(x+\frac{n\pi}{2}\right)\\ &-\left(\frac{n'}{1}\cdot\frac{1}{2x^2}-\frac{n'(n'-1')(n'-2')}{1.2.3}\cdot\frac{1}{2^3x^4}+\ldots\right)\sin\left(x+\frac{n\pi}{2}\right)\end{aligned}$$

in which n' stands for $n(n+1)$, and $n'-r'=(n-r)(n+r+1)$.

Lord **Rayleigh** has given another form for S_n in terms of differential coefficients with regard to x only, but what we are principally concerned with here are the expressions (27) which show that S_n alone is finite at the centre, and with equation (26) which gives rise to the following *formulæ of reduction.*

8. Replacing in (26) u_n by S_n, we find successively

$$\frac{dS}{dx}=S_{n+1}+\frac{n}{x}S_n,$$

$$\frac{d^2S_n}{dx^2}=S_{n+2}+\frac{2n+1}{x}S_{n+1}+\frac{n(n-1)}{x^2}S_n.$$

If we substitute these expressions in the equation which S_n satisfies, we find

$$S_{n+2}+\frac{2n+3}{x}S_{n+1}+S_n=0.$$

Whence

$$\frac{2n+1}{x}S_n+S_{n+1}+S_{n-1}=0,$$

$$(2n+1)\frac{dS_n}{dx}=(n+1)S_{n+1}-nS_{n-1};$$

and finally,

$$\left.\begin{aligned}\frac{S_n}{x^2}&=\frac{S_{n+2}}{(2n+1)(2n+3)}+\frac{2S_n}{(2n-1)(2n+3)}+\frac{S_{n-2}}{(2n-1)(2n+1)}\\ \frac{1}{x}\frac{dS_n}{dx}&=-\frac{n+1}{(2n+1)(2n+3)}S_{n+2}+\frac{S_n}{(2n-1)(2n+3)}+\frac{nS_{n-2}}{(2n-1)(2n+1)}\end{aligned}\right\}\quad . \quad . \quad (29)$$

We have to substitute these values in

$$\frac{d^2S_n}{dx^2}+\frac{1}{x}\frac{dS_n}{dx}+S_n\left(1-\frac{m^2}{x^2}\right),$$

which may also be written

$$\frac{n(n+1)-m^2}{x^2}S_n-\frac{1}{x}\frac{dS_n}{dx}$$

$$=\frac{(n+1)^2-m^2}{(2n+1)(2n+3)}S_{n+2}+\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}S_n+\frac{n^2-m^2}{(2n-1)(2n+1)}S_{n-2}$$

If we now write

$$S_n=\frac{1.3.5\ldots(2n-1)}{((n-1)^2-m^2)(\overline{n-3}^2-m^2)\ldots(\overline{m-1}^2-m^2)}\Sigma_n\quad .\quad .\quad .\quad .\quad .\quad (30)$$

$$\frac{d^2\Sigma_n}{d\xi^2}+\frac{1}{\xi}\frac{d\Sigma_n}{d\xi}+\Sigma_n\left(1-\frac{m^2}{\xi^2}\right)=\Sigma_{n+2}+\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}\Sigma_n+\frac{(n^2-m^2)(\overline{n-1}^2-m^2)}{(4n^2-1)(\overline{4n-1}^2-1)}\Sigma_{n-2}\quad (31)$$

If we now turn to equation (19), and compare the result just obtained with (20), we see that we can satisfy the equation (19) by

$$\Omega=C_1(a_0\Sigma_m-a_1\Sigma_{m+2}+a_2\Sigma_{m+4}-\ldots). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (31)$$

Where a_0, a_1, a_2 . . . are connected by the same relations as before—namely, I. of (22), and the values of k are still, as before, the roots of $a_\infty=0$.

It will be observed that this result depends on the identity of the forms of the right-hand members of (31) and (20). But the transformation (30) depends essentially on the hypothesis that $n-m$ is even. For the values of k which depend on $n-m$ odd, and give rise to the second class of expressions for ϑ, this method fails completely; in other words, distributions of heat which are not symmetrical about the equator of the ellipsoid cannot be represented by Ω-functions of the type we have just found. I was led however to expect, from other expressions which will be given presently, that the true form in this case was to be discovered by putting

$$\Omega=\frac{\sqrt{\xi^2+\lambda^2c^2}}{\xi}\Omega' \quad . \quad . \quad . \quad . \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (32)$$

After substitution and reductions, we obtain

$$\xi^2\frac{d^2\Omega'}{d\xi^2}+2\xi\frac{d\Omega'}{d\xi}+\xi^2\Omega'-k\Omega'+\lambda^2c^2\left(\frac{d^2\Omega'}{d\xi^2}-\frac{1}{\xi}\frac{d\Omega'}{d\xi}+\Omega'+\frac{1-m^2}{\xi^2}\Omega'\right)=0.$$

Now the value of the expression

$$\frac{d^2S_n}{d\xi^2}-\frac{1}{\xi}\frac{dS_n}{d\xi}+S_n+\frac{1-m^2}{\xi^2}S_n=-\frac{3}{\xi}\frac{dS_n}{d\xi}+\frac{n(n+1)+1-m^2}{\xi^2}S_n,$$

by (29)

$$=\frac{\overline{(n+2)}^2-m^2}{(2n+1)(2n+3)}S_{n+2}+\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}S_n+\frac{(n-1)^2-m^2}{(2n-1)(2n+1)}S_{n-2}$$

If we put

$$S_n=\frac{1.3.5\ldots(2n-1)}{(n^2-m^2)(\overline{n-2}^2-m^2)\ldots(\overline{m+1}^2-m^2)}Q_n$$

where $n-m$ is an odd number, we find

$$\left(\frac{d^2}{d\xi^2}-\frac{1}{\xi}\frac{d}{d\xi}+1+\frac{1-m^2}{\xi^2}\right)Q_n=Q_{n+2}+\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}Q_n+\frac{(n^2-m^2)(\overline{n-1}^2-m^2)}{(4n^2-1)(4\overline{n-1}^2-1)}Q_{n-2}.$$

It appears therefore that Ω' can be expanded in a series of the form

$$\Omega'=b_0Q_{m+1}-b_1Q_{m+3}+b_2Q_{m+5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33)$$

9. But we may also express Ω as a function of η, the major semi-axis of the confocal ellipsoid. And, if we turn to the original equation in Ω (19) and put therein $\eta=\lambda c\cosh\alpha$, we find

$$(\eta^2-\lambda^2c^2)\frac{d^2\Omega}{d\eta^2}+2\eta\frac{d\Omega}{d\eta}-\frac{m^2c^2\lambda^2}{\eta^2-\lambda^2c^2}\Omega=-\eta^2\Omega+k\Omega\ .\ \ .\ \ .\ \ .\ \ .\qquad(19')$$

We shall first replace Ω by W, where $\Omega=(\eta^2-\lambda^2c^2)^{\frac{m}{2}}\mathrm{W}$, the equation becoming after substitution and reduction,

$$(\eta^2-\lambda^2c^2)\frac{d^2\mathrm{W}}{d\eta^2}+2(m+1)\eta\frac{d\mathrm{W}}{d\eta}+m(m+1)\mathrm{W}=-\eta^2\mathrm{W}+k\mathrm{W}.$$

If we now replace W by w where $\mathrm{W}=\eta^{-m}w$, we find after reduction

$$\eta^2\frac{d^2w}{d\eta^2}+2\eta\frac{dw}{d\eta}+\eta^2w-kw=\lambda^2c^2\left(\frac{d^2w}{d\eta^2}-\frac{2m}{\eta}\frac{dw}{d\eta}+\frac{m(m+1)}{\eta^2}w\right).$$

and we can satisfy this equation by a series of S-functions.

Before doing so, we calculate the value of the expression

$$\frac{d^2\mathrm{S}_n}{d\eta^2}-\frac{2m}{\eta}\frac{d\mathrm{S}_n}{d\eta}+\frac{m(m+1)}{\eta^2}\mathrm{S}_n=-\mathrm{S}_n-\frac{2.(m+1)}{\eta}\frac{d\mathrm{S}_n}{d\eta}+\frac{n(n+1)+m(m+1)}{\eta^2}\mathrm{S}_n$$

$$=\frac{(n+m+1)(n+m+2)}{(2n+1)(2n+3)}\mathrm{S}_{n+2}-\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}\mathrm{S}_n+\frac{(n-m)(n-m-1)}{(2n-1)(2n+1)}\mathrm{S}_{n-2},\text{ by (29).}$$

Putting

$$\mathrm{S}_n=\frac{1.3.5\ldots(2n-1)}{1.2.3\ldots(n+m)}\mathrm{H}_n\ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\qquad(34)$$

whether $n-m$ be even or odd, we obtain

$$\left(\frac{d^2}{d\eta^2}-\frac{2m}{\eta}\frac{d}{d\eta}+\frac{m(m+1)}{\eta^2}\right)\mathrm{H}_n=\mathrm{H}_{n+2}-\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}\mathrm{H}_n+\frac{(n^2-m^2)(\overline{n-1}^2-m^2)}{(4n^2-1)(4.\overline{n-1}^2-1)}\mathrm{H}_{n-2}$$

The equation in w may, therefore, be satisfied by either of the forms

$$\left.\begin{array}{l} w=a_0\mathrm{H}_m+a_1\mathrm{H}_{m+2}+a_2\mathrm{H}_{m+4}+\ldots \\ w=b_0\mathrm{H}_{m+1}+b_1\mathrm{H}_{m+3}+b_2\mathrm{H}_{m+5}+\ldots \\ \text{and}\qquad \Omega=\dfrac{(\eta^2-\lambda^2c^2)^{\frac{m}{2}}}{\eta^m}w \end{array}\right\}\ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\qquad(35)$$

It was the consideration of these formulæ, including as they do both the cases of $n-m$ even and odd, which suggested the transformation (32).

10. We shall now sum up the results of the last two articles and shall replace, in doing so, the symbols Σ, Q, H by S. Neglecting unnecessary constants, we obtain

(1) $n-m$ even, distribution symmetrical on opposite sides of the equator,

$$\left.\begin{aligned}\vartheta&=a_0\mathrm{P}_m{}^m-a_1\mathrm{P}_m{}^{m+2}+a_2\mathrm{P}_m{}^{m+4}-a_3\mathrm{P}_m{}^{m+6}+\dots\\ \Omega(\xi)&=a_0\mathrm{S}_m-\frac{1}{2m+3}a_1\mathrm{S}_{m+2}+\frac{1.3}{(2m+5)(2m+7)}a_2\mathrm{S}_{m+4}\\ &\qquad-\frac{1.3.5}{(2m+7)(2m+9)(2m+11)}a_3\mathrm{S}_{m+6}+\dots\\ \Omega(\eta)&=\frac{(\eta^2-\lambda^2c^2)^{\frac{m}{2}}}{\eta^m}\left\{a_0\mathrm{S}_m+\frac{2(m+1)}{2m+3}a_1\mathrm{S}_{m+2}+\frac{2^2(m+1)(m+2)}{(2m+5)(2m+7)}a_2\mathrm{S}_{m+4}\right.\\ &\qquad\left.+\frac{2^3(m+1)(m+2)(m+3)}{(2m+7)(2m+9)(2m+11)}a_3\mathrm{S}_{m+6}+\dots\right\}\end{aligned}\right\}. \quad (36)$$

(2) $n-m$ odd, distribution equal and opposite on opposite sides of the equator,

$$\left.\begin{aligned}\vartheta&=b_0\mathrm{P}_m{}^{m+1}-b_1\mathrm{P}_m{}^{m+3}+b_2\mathrm{P}_m{}^{m+5}-b_3\mathrm{P}_m{}^{m+7}+\dots\\ \Omega(\xi)&=\frac{(\xi^2+\lambda^2c^2)^{\frac{1}{2}}}{\xi}\left\{b_0\mathrm{S}_{m+1}-\frac{3}{2m+5}b_1\mathrm{S}_{m+3}+\frac{3.5}{(2m+7)(2m+9)}b_2\mathrm{S}_{m+5}\right.\\ &\qquad\left.-\frac{3.5.7}{(2m+9)(2m+11)(2m+13)}b_3\mathrm{S}_{m+7}+\dots\right\}\\ \Omega(\eta)&=\frac{(\eta^2-\lambda^2c^2)^{\frac{m}{2}}}{\eta^m}\left\{b_0\mathrm{S}_{m+1}+\frac{2(m+1)}{2m+5}b_1\mathrm{S}_{m+3}+\frac{2^2(m+1)(m+2)}{(2m+7)(2m+9)}b_2\mathrm{S}_{m+5}\right.\\ &\qquad\left.+\frac{2^3(m+1)(m+2)(m+3)}{(2m+9)(2m+11)(2m+13)}b_3\mathrm{S}_{m+7}+\dots\right\}\end{aligned}\right\}. \quad (37)$$

It will be observed that all these expressions for Ω, notwithstanding the factors in the denominators, are necessarily finite at the centre.

The first equation of last article (19′) suggests yet another form for Ω which we shall find of service in calculating the coefficients.

Putting $\zeta=\cosh\alpha=\frac{\eta}{\lambda c}$, that equation may be written

$$(1-\zeta^2)\frac{d^2\Omega}{d\zeta^2}-2\zeta\frac{d\Omega}{d\zeta}-\frac{m^2}{1-\zeta^2}\Omega=\lambda^2c^2\zeta^2\Omega-k\Omega;$$

and a comparison of this equation with the corresponding one (18) in ϑ shows that we may satisfy it by or by

$$\left.\begin{aligned}\Omega&=a_0\mathrm{P}_m{}^m(\zeta)-a_1\mathrm{P}_m{}^{m+2}(\zeta)+\ldots+(-1)^r a_r\mathrm{P}_m{}^{m+2r}(\zeta)+\ldots\\ \Omega&=b_0\mathrm{P}_m{}^{m+1}-b_1\mathrm{P}_m{}^{m+3}+\ldots\end{aligned}\right\}\quad\ldots\quad(38)$$

according as $n-m$ is even or odd.

The function $\mathrm{P}_m{}^n(\zeta)$ may be written $(\zeta^2-1)^{\frac{m}{2}}\mathfrak{P}_m{}^n(\zeta)$ and we know that

$$\mathfrak{P}_m{}^n(1)=\frac{(n+m)!}{2^m m!\,1.3.5\ldots(2n-1)}$$

$$=2^r\frac{(m+1)(m+2)\ldots(m+r)}{(2m+2r+1)(2m+2r+3)\ldots(2m+4r-1)},\ n=m+2r$$

and

$$=2^r\frac{(m+1)(m+2)\ldots(m+r)}{(2m+2r+3)(2m+2r+5)\ldots(2m+4r+1)},\ n=m+2r+1.$$

This relation enables us to find the constant factor by which $\Omega(\zeta)$ differs from $\Omega(\xi)$. For when ζ is infinitely near to unity,

$$\Omega(\zeta)=\frac{(\eta^2-\lambda^2c^2)^{\frac{m}{2}}}{\lambda^m c^m}(a_0\mathrm{P}_m{}^m(1)-a_1\mathrm{P}_m{}^{m+2}(1)+\ldots),\ n-m\text{ even}$$

$$=\frac{(\eta^2-\lambda^2c^2)^{\frac{m}{2}}}{\lambda^m c^m}(b_0\mathrm{P}_m{}^{m+1}(1)-b_1\mathrm{P}_m{}^{m+3}(1)+\ldots),\ n-m\text{ odd,}$$

while $\Omega(\xi)$ then reduces to

$$-\frac{a_0\xi^m(-1)^m}{1.3\ldots(2m+1)},\ n-m\text{ even}$$

$$\frac{\lambda c b_0\xi^m(-1)^{m+1}}{1.3\ldots(2m+3)},\ n-m\text{ odd,}$$

terms above ξ^{m+1} being neglected in both sets of formulæ.

Hence if

$$\left.\begin{aligned}\Omega(\xi)&=\mathrm{K}.\Omega(\zeta),\ n-m\text{ even}\\ &=\mathrm{K}'\Omega(\zeta),\ n-m\text{ odd,}\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(39)$$

$$\left.\begin{aligned}(-1)^m\lambda^m c^m a_0\mathrm{K}&=1.3\ldots(2m+1)\Big[a_0-\frac{2(m+1)}{2m+3}a_1+\frac{2^2(m+1)(m+2)}{(2m+5)(2m+7)}a_2\\ &\qquad-\frac{2^3(m+1)(m+2)(m+3)}{(2m+7)(2m+9)(2m+11)}a_3+\ldots\Big]\\ (-1)^{m+1}\lambda^{m+1}c^{m+1}b_0\mathrm{K}'&=1.3\ldots(2m+3)\Big[b_0-\frac{2(m+1)}{2m+5}b_1+\frac{2^2(m+1)(m+2)}{(2m+7)(2m+9)}b_2\\ &\qquad-\frac{2^3(m+1)(m+2)(m+3)}{(2m+9)(2m+11)(2m+13)}b_3+\ldots\Big]\end{aligned}\right\}\quad(40)$$

$\frac{1}{\epsilon}(\kappa_{r-1}-\kappa_{r-2})>1+\frac{1}{16}$. When this is true it follows, *à fortiori*, that $\frac{1}{\epsilon}(\kappa_{r-1}-\kappa_{r-3})$, $\frac{1}{\epsilon}(\kappa_{r-1}-\kappa_{r-4}) \ldots \frac{1}{\epsilon}(\kappa_r-\kappa_{r-1})$, $\frac{1}{\epsilon}(\kappa_{r+1}-\kappa_r) \ldots$ are all greater than $\frac{17}{16}$.

Let us also suppose, for the sake of clearness, that r is even; then, if we substitute $k=\kappa_{r-1}$ in the equations, a_0, a_1, a_2, $\ldots$ a_{r-2} are alternately positive and negative and each is less than $\frac{1}{16}$ of the one after it; we have, in fact,

$$p_1a_1=-\frac{1}{\epsilon}(\kappa_{r-1}-\kappa_0)a_0$$

$$p_2a_2=-\frac{1}{\epsilon}(\kappa_{r-1}-\kappa_1)a_1-a_0, \text{ \&c.};$$

and since when $k=\kappa_{r-1}$, $p_ra_r=-a_{r-2}$, a_r will also be negative. But, in the same manner, when we substitute $k=\kappa_r$ or any greater quantity, the series a_0 $a_1 \ldots a_r$ are alternately positive and negative, and remain so as k changes from κ_r to $+\infty$. We infer, therefore, that all the roots of $a_r=0$ lie below κ_r and that one root lies between κ_{r-1} and κ_r.

(*c*) Moreover, only one root of $a_r=0$ lies between κ_{r-1} and κ_r. For when $k=\kappa_{r-1}$ the series $a_0 \ldots a_{r-1}$ have $\overline{r-1}$ changes of sign and retain these ever afterwards as k increases; therefore the subsequent changes of k can introduce only one more change of sign into the series a_0, $a_1 \ldots a_r$.

(*d*) When $k=\rho$, one of the roots of $a_r=0$, the expressions $\frac{1}{\epsilon}(\kappa_{r+1}-\rho)$, $\frac{1}{\epsilon}(\kappa_{r+2}-\rho) \ldots$ are all positive and greater than $1+\frac{1}{16}$; and, for this value of k, $p_{r+2}a_{r+2}=\frac{1}{\epsilon}(\kappa_{r+1}-\rho)a_{r+1}$, $p_{r+3}a_{r+3}=\frac{1}{\epsilon}(\kappa_{r+2}-\rho)a_{r+2}-a_{r+1} \ldots$; thus a_{r+1}, $a_{r+2} \ldots$ have all the same signs and each is less than $\frac{1}{16}$ of the one after it, and these signs are opposite to that of a_{r-1}. But as k increases from one root ρ to the next ρ' of $a_r=0$, a_{r-1} must have undergone one change of sign; hence a_{r+1}, $a_{r+2} \ldots$ must have each undergone one change of sign. In other words, each of the equations $a_{r+1}=0$, $a_{r+2}=0 \ldots$ has one root between each pair of $a_r=0$.

(*e*) It is clear, therefore, that each of the functions a_2, $a_3 \ldots$ vanishes once for values of k lying in the intervals between $-\infty$, κ_0, $\kappa_1 \ldots$ and but once. Let us therefore conceive these lengths cut off on the axis of k, and construct the curves $a_{r+1}=f_1(k)$, $a_r=f_2(k)$, $a_{r-1}=f_3(k) \ldots$ When $a_r=0$, $a_{r+1}=-16a_{r-1}$ ultimately when r becomes very great; and when $a_{r+1}=0$, a_r and a_{r-1} have like signs and $a_r=\frac{\epsilon}{\kappa_r-k}a_{r-1}$. a_r therefore becomes indefinitely small compared to a_{r-1}, and reference to equations (22) shows that then a_{r-1} is indefinitely small compared to a_{r-2}. $\ldots$ The points, therefore, in which the curve $a_r=f_2(k)$ cuts the axis converge to fixed points as r becomes infinitely

The effect of having only *non-adjacent* terms is that no ϕ occurs in a higher degree than the first, as would otherwise be the case.

The equation which determines k is therefore

$$\phi_0\phi_1 \ldots \phi_\infty(1-\epsilon^2{}_\infty S_1+\epsilon^4{}_\infty S_2-\ldots)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

If we neglect ϵ altogether, the several factors reproduce simply the values of k given by $k=n(n+1)$, $r=0, 1 \ldots \infty$.

Before proceeding to approximate more closely to these roots, I shall state two properties of these sums which will present themselves in the reductions, the proof of which is so simple that it is scarcely necessary to dwell on it—

$${}_rS_1={}_{r-1}S_1+\frac{p_r}{\phi_{r-1}\phi_r},$$

$${}_rS_2=\frac{p_r}{\phi_{r-1}\phi_r}{}_{r-1}S_1+{}_{r-1}S_2-\frac{p_{r-1}p_r}{\phi_{r-2}\phi^2{}_{r-1}\phi_r}.$$

These properties will be of great service in proceeding to a second approximation, and in calculating the coefficients.

Let us concentrate our attention on the $(r+1)^{th}$ root, for which an approximate value is given by $\phi_r=0$, viz.,

$$k=n(n+1)+\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}\epsilon \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

The equation may be written in full

$$1-\epsilon^2\left(\frac{p_1}{\phi_0\phi_1}+\frac{p_2}{\phi_1\phi_2}+\ldots+\frac{p_r}{\phi_{r-1}\phi_r}+\frac{p_{r+1}}{\phi_r\phi_{r+1}}+\ldots\right)$$
$$+\epsilon^4\left[\frac{p_1p_3}{\phi_0\phi_1\phi_2\phi_3}+\frac{p_1p_4}{\phi_0\phi_1\phi_3\phi_4}+\ldots+\left(\frac{p_r}{\phi_{r-1}\phi_r}+\frac{p_{r+1}}{\phi_r\phi_{r+1}}\right)\left(\frac{p_1}{\phi_0\phi_1}+\ldots+\frac{p_{r-1}}{\phi_{r-2}\phi_{r-1}}+\ldots\right)\right]\ldots=0$$

it being observed that no two consecutive p's can occur in the products.

Multiplying the equation by ϕ_r, it becomes

$$\phi_r-\epsilon^2\left[\frac{p_r}{\phi_{r-1}}+\frac{p_{r+1}}{\phi_{r+1}}+\phi_r\left(\frac{p_1}{\phi_0\phi_1}+\ldots\right)\right]+\epsilon^4(\ldots)+\ldots=0.$$

We see from this that up to ϵ^3 the value of ϕ_r will be

$$\phi_r=\epsilon^2\left(\frac{p_r}{\phi'_{r-1}}+\frac{p_{r+1}}{\phi'_{r+1}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Wherein ϕ'_{r-1} and ϕ'_{r+1} represent what ϕ_{r-1} and ϕ_{r+1} become when we substitute for k its first approximate value (44).

To obtain a third approximation, we must substitute this value of k in the above equation (43).

The third series of terms may be arranged

$$\epsilon^4\frac{p_r}{\phi_{r-1}}\left(\frac{p_1}{\phi_0\phi_1}+\frac{p_2}{\phi_1\phi_2}+\ldots+\frac{p_{r-2}}{\phi_{r-3}\phi_{r-2}}+\frac{p_{r+2}}{\phi_{r+1}\phi_{r+2}}+\ldots\right)$$
$$+\epsilon^4\frac{p_{r+1}}{\phi_{r+1}}\left(\frac{p_1}{\phi_0\phi_1}+\frac{p_2}{\phi_1\phi_2}+\ldots+\frac{p_{r-1}}{\phi_{r-2}\phi_{r-1}}+\frac{p_{r+3}}{\phi_{r+2}\phi_{r+3}}+\ldots\right),$$

while the coefficient of ϕ_r in the second series is

$$\epsilon^2\left(\frac{p_1}{\phi_0\phi_1}+\frac{p_2}{\phi_1\phi_2}+\ldots+\frac{p_{r-1}}{\phi_{r-2}\phi_{r-1}}+\frac{p_{r+2}}{\phi_{r+1}\phi_{r+2}}+\ldots\right),$$

On putting, therefore, the value of ϕ_r given by (45) in these terms, we see that we shall have

$$\phi_r=\epsilon^2\left(\frac{p_r}{\phi''_{r-1}}+\frac{p_{r+1}}{\phi''_{r+1}}\right)+\epsilon^4\left(\frac{p_{r-1}p_r}{\phi'_{r-2}\phi'^2_{r-1}}+\frac{p_{r+1}p_{r+2}}{\phi'^2_{r+1}\phi'_{r+2}}\right)+\ .\ .\ .\ .\ .\ . \quad (46)$$

where, in ϕ''_{r-1}, ϕ''_{r+1} we have to substitute the value of k given by the second approximation. This gives k as far as ϵ^5.

I proceed to find its value up to ϵ^4.

Having $n=m+2r$, $n'=m+2r'$, we shall put

$$D_{r,r}=n'(n'+1)-n(n+1)$$
$$\delta_{r,r}=\frac{2n'^2+2n'-2m^2-1}{(2n'-1)(2n'+3)}-\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}\ ;$$

and, for a first approximation, obtain

$$k=n(n+1)+\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}\ \epsilon\ ;$$

for a second approximation,

$$k=n(n+1)+\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}\ \epsilon-\left(\frac{p_r}{D_{r-1,r}}+\frac{p_{r+1}}{D_{r+1,r}}\right)\epsilon^2+\left(\frac{p_r\delta_{r-1,r}}{D^2_{r-1,r}}+\frac{p_{r+1}\delta_{r+1,r}}{D^2_{r+1,r}}\right)\epsilon^3+\ldots\ . \quad (47)$$

and, as a third,

$$\left.\begin{aligned}k=n(n+1)&+\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}\ \epsilon-\left(\frac{p_r}{D_{r+1,r}}+\frac{p_{r+1}}{D_{r+1,r}}\right)\epsilon^2+\left(\frac{p_r\delta_{r-1,r}}{D^2_{r-1,r}}+\frac{p_{r+1}\delta_{r+1,r}}{D^2_{r+1,r}}\right)\epsilon^3\\&+\epsilon^4\left\{\left(\frac{p_r}{D_{r-1,r}}+\frac{p_{r+1}}{D_{r+1,r}}\right)\left(\frac{p_r}{D^2_{r-1,r}}+\frac{p_{r+1}}{D^2_{r+1,r}}\right)-\frac{p_r\delta^2_{r-1,r}}{D^3_{r-1,r}}-\frac{p_{r+1}\delta^2_{r+1,r}}{D^3_{r+1,r}}\right.\\&\left.\qquad-\frac{p_{r-1}p_r}{D_{r-2,r}D^2_{r-1,r}}-\frac{p_{r+1}p_{r+2}}{D^2_{r+1,r}D_{r+2,r}}\right\}+\ldots\end{aligned}\right\}\ .\ . \quad (48)$$

This formula becomes simplified for the first of the series of roots corresponding to a given value of m, that, namely, for which $r=0$; in the above expression $p_r=0$, and the value of k is

$$k_0=m(m+1)+\frac{1}{2m+3}\,\epsilon-\frac{p_1}{D_{1,0}}\,\epsilon^2+\frac{p_1\delta_{1,0}}{D_{1,0}^2}\,\epsilon^3+\left(\frac{p_1^2}{D_{1,0}^3}-\frac{p_1\delta_{1,0}^2}{D_{1,0}^3}-\frac{p_1p_2}{D_{1,0}^2D_{2,0}}\right)\epsilon^4+\dots$$

To reduce these expressions we observe that

$$\begin{gathered}D_{r,r}=(n'-n)(n'+n+1),\\ D_{r+1,r}=2(2n+3),\ D_{r+2,r}=4(2n+5),\\ D_{r-1,r}=-2(2n-1),\ D_{r-2,r}=-4(2n-3).\end{gathered}$$

Furthermore, since $\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}=\frac{1}{2}\left(1-\frac{4m^2-1}{(2n-1)(2n+3)}\right)$,

$$\delta_{r,r}=\frac{2(4m^2-1)(n'-n)(n'+n+1)}{(2n-1)(2n+3)(2n'-1)(2n'+3)},$$

$$\delta_{r+1,r}=\frac{4(2m+1)(2m-1)}{(2n-1)(2n+3)(2n+7)},\quad \delta_{r+2,r}=\frac{8(2m+1)(2m-1)(2n+5)}{(2n-1)(2n+3)(2n+7)(2n+11)}$$

$$\delta_{r-1,r}=-\frac{4(2m+1)(2m-1)}{(2n-5)(2n-1)(2n+3)},\quad \delta_{r-2,r}=-\frac{8(2m+1)(2m-1)(2n-3)}{(2n-9)(2n-5)(2n-1)(2n+3)}$$

Wherein, as before, $n=m+2r$.

We may also write p in factors as follows :—

$$p_r=\frac{(n-m)(n-m-1)(n+m)(n+m-1)}{(2n-3)(2n-1)^2(2n+1)},$$

$$p_{r+1}=\frac{(n+2-m)(n+1-m)(n+m+2)(n+m+1)}{(2n+1)(2n+3)^2(2n+5)}.$$

Putting in these we find, for the first root of the series,

$$k_0=m(m+1)+\frac{1}{2m+3}\,\epsilon-\frac{2(m+1)}{(2m+3)^3(2m+5)}\,\epsilon^2+\frac{4(m+1)(2m+1)}{(2m+3)^5(2m+5)(2m+7)}\,\epsilon^3+\dots\quad (49)$$

and, in general,

$$k=n(n+1)+\frac{2n^2+2n-2m^2-1}{(2n-1)(2n+3)}\,\epsilon-k_2\epsilon^2+k_3\epsilon^3+\dots\quad (50)$$

where

$$k_2=\tfrac{1}{2}\left\{\frac{(n-m+2)(n-m+1)(n+m+2)(n+m+1)}{(2n+1)(2n+3)^3(2n+5)}-\frac{(n-m)(n-m-1)(n+m)(n+m-1)}{(2n-3)(2n-1)^3(2n+1)}\right\},$$

$$k_3=(4m^2-1)\left\{\frac{n-m+2)(n-m+1)(n+m+2)(n+m+1)}{(2n-1)(2n+1)(2n+3)^5(2n+5)(2n+7)}\right.$$
$$\left.-\frac{(n-m)(n-m-1)(n+m)(n+m-1)}{(2n-5)(2n-3)(2n-1)^5(2n+1)(2n+3)}\right\}.$$

13. The roots of the equation in k being thus approximately found, I proceed to show how to calculate the coefficients. When we consider that the series for $\vartheta_m{}^k$ reduces to its first term when $r=0$, and to the term $a_r\,\mathrm{P}_m{}^n$ when $k=n(n+1)$ and we suppose the ellipsoid to become a sphere, it is clear that, for the first root, a_0 must be the leading coefficient, and that for the $(r+1)^{th}$ root the coefficient a_r must be the leading one of the series. Taking the general case first, we observe that, since none of the expressions for $\frac{a_1}{a_0}, \frac{a_2}{a_0} \dots \frac{a_{r-1}}{a_0}, \frac{a_r}{a_0}$ contains ϕ_r, it follows that a_0 contains no terms lower than ϵ^r, a_1 none lower than $\epsilon^{r-1} \dots$ and a_{r-1} none lower than ϵ, it being understood that a_r is finite and of the degree ϵ^0. It is also true, as we shall see, although it is not so evident at first sight, that a_{r+1} is at least of the degree ϵ, a_{r+2} of degree ϵ^2, and so on. We shall work out the first three coefficients on either side as far as ϵ^3; though in the subsequent reductions the coefficients of ϵ^3 in a_{r+1} and a_{r-1} are too complicated to be worked out fully in general.

We have seen that

$$p_1p_2\dots p_r\epsilon^r\frac{a_r}{a_0}=\phi_0\phi_1\dots\phi_{r-1}(1-{}_{r-1}\mathrm{S}_1\epsilon^2+{}_{r-1}\mathrm{S}_2.\epsilon^4+\dots),$$

$$p_1p_2p_3\dots p_{r+1}\epsilon^{r+1}\frac{a_{r+1}}{a_0}=\phi_0\dots\phi_r(1-{}_r\mathrm{S}_1.\epsilon^2+{}_r\mathrm{S}_2.\epsilon^4\dots).$$

But we have also proved that

$${}_r\mathrm{S}_1={}_{r-1}\mathrm{S}_1+\frac{p_r}{\phi_{r-1}\phi_r},$$

$${}_r\mathrm{S}_2={}_{r-1}\mathrm{S}_2+\frac{p_r}{\phi_{r-1}\phi_r}{}_{r-1}\mathrm{S}_1-\frac{p_{r-1}p_r}{\phi_{r-2}\phi^2{}_{r-1}\phi_r};$$

and that, up to ϵ^5,

$$\phi_r=\epsilon^2\left(\frac{p_r}{\phi''{}_{r-1}}+\frac{p_{r+1}}{\phi''{}_{r+1}}\right)+\epsilon^4\left(\frac{p_{r-1}p_r}{\phi'{}_{r-2}\phi'^2{}_{r-1}}+\frac{p_{r+1}p_{r+2}}{\phi'^2{}_{r+1}\phi'{}_{r+2}}\right);$$

hence, putting in this value in the expression for a_{r+1},

$$p_1\dots p_{r+1}\epsilon^{r+1}\frac{a_{r+1}}{a_0}=\phi_0\dots\phi_{r-1}\left\{\epsilon^2\frac{p_{r+1}}{\phi''{}_{r+1}}+\epsilon^4\frac{p_{r+1}}{\phi'{}_{r+1}}\left(\frac{p_{r+2}}{\phi'{}_{r+1}\phi'{}_{r+2}}-{}_{r-1}\mathrm{S}_1\right)+\dots\right\};$$

and hence, dividing by the expression for $\frac{a_r}{a_0}$, we find

$$\frac{a_{r+1}}{a_r}=\epsilon\left(\frac{1}{\phi''_{r+1}}+\frac{\epsilon^2 p_{r+2}}{\phi'^2_{r+1}\phi'_{r+2}}\right),\text{ up to }\epsilon^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

In a similar manner

$$p_1 \ldots p_{r+2}.\epsilon^{r+2}\frac{a_{r+2}}{a_0}=\phi_0 \ldots \phi_{r-1}.\phi_{r+1}\{\phi_r-\epsilon^2{}_{r+1}\mathrm{S}_1.\phi_r+\epsilon^4{}_{r+1}\mathrm{S}_2\phi_r- \ldots\} \quad . \quad (52)$$

But

$${}_{r+1}\mathrm{S}_1={}_{r-1}\mathrm{S}_1+\left(\frac{p_r}{\phi_{r-1}}+\frac{p_{r+1}}{\phi_{r+1}}\right)\cdot\frac{1}{\phi_r},$$

and

$${}_{r+1}\mathrm{S}_2={}_{r-1}\mathrm{S}_2+\frac{1}{\phi_r}\left(\frac{p_r}{\phi_{r-1}}+\frac{p_{r+1}}{\phi_{r+1}}\right){}_{r-1}\mathrm{S}_1-\frac{p_{r-1}p_r}{\phi_{r-2}\phi^2_{r-1}\phi_r}.$$

Putting these values into the series on the right-hand side of (52) it becomes, up to ϵ^5,

$$\frac{\epsilon^4 p_{r+1}p_{r+2}}{\phi'^2_{r+1}\phi'_{r+2}},$$

and finally

and therefore

$$\left.\begin{aligned}\frac{a_{r+2}}{a_r}&=\epsilon^2\cdot\frac{1}{\phi'_{r+1}\phi'_{r+2}}\\ \frac{a_{r+3}}{a_r}&=\epsilon^3\cdot\frac{1}{\phi'_{r+1}.\phi'_{r+2}.\phi'_{r+3}}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

The series for $\frac{a_{r-1}}{a_r}$, $\frac{a_{r-2}}{a_r}$, $\frac{a_{r-3}}{a_r}$ can be easily found; for

$$\frac{a_{r-1}}{a_r}=\frac{\epsilon p_r}{\phi_{r-1}}\cdot\frac{1-{}_{r-2}\mathrm{S}_1\epsilon^2+\ldots}{1-{}_{r-1}\mathrm{S}_1\epsilon^2+\ldots}=\frac{\epsilon p_r}{\phi''_{r-1}}\cdot[1+({}_{r-1}\mathrm{S}_1-{}_{r-2}\mathrm{S}_1)\epsilon^2+\ldots],$$

and therefore,

$$\frac{a_{r-1}}{a_r}=p_r\epsilon\left(\frac{1}{\phi''_{r-1}}+\epsilon^2\frac{p_{r-1}}{\phi'^2_{r-1}\phi'_{r-2}}\right),$$

$$\frac{a_{r-2}}{a_r}=p_r p_{r-1}\epsilon^2.\frac{1}{\phi'_{r-1}\phi'_{r-2}}.$$

Expanding these expressions in powers of ϵ, we obtain

$$\left.\begin{aligned}
a_{r+1}\div a_r&=\frac{\epsilon}{\mathrm{D}_{r+1,r}}-\frac{\epsilon^2.\delta_{r+1,r}}{\mathrm{D}^2{}_{r+1,r}}+\frac{\epsilon^3}{\mathrm{D}^2{}_{r+1,r}}\left(-\frac{p_r}{\mathrm{D}_{r-1,r}}-\frac{p_{r+1}}{\mathrm{D}_{r+1,r}}+\frac{p_{r+2}}{\mathrm{D}_{r+2,r}}+\frac{\delta^2{}_{r+1,r}}{\mathrm{D}_{r+1,r}}\right)+\ldots\\
a_{r-1}\div a_r&=\frac{p_r\epsilon}{\mathrm{D}_{r-1,r}}-\frac{p_r\epsilon^2\delta_{r-1,r}}{\mathrm{D}^2{}_{r-1,r}}+\frac{p_r\epsilon^3}{\mathrm{D}^2{}_{r-1,r}}\left(-\frac{p_r}{\mathrm{D}_{r-1,r}}-\frac{p_{r+1}}{\mathrm{D}_{r+1,r}}+\frac{p_{r-1}}{\mathrm{D}_{r-2,r}}+\frac{\delta^2{}_{r-1,r}}{\mathrm{D}_{r-1,r}}\right)+\ldots\\
a_{r+2}\div a_r&=\frac{\epsilon^2}{\mathrm{D}_{r+1,r}\,\mathrm{D}_{r+2,r}}\left(1-\left(\frac{\delta_{r+1,r}}{\mathrm{D}_{r+1,r}}+\frac{\delta_{r+2,r}}{\mathrm{D}_{r+2,r}}\right)\epsilon\right)+\ldots\\
a_{r-2}\div a_r&=\frac{p_rp_{r-1}\epsilon^2}{\mathrm{D}_{r-1,r}\,\mathrm{D}_{r-2,r}}\left(1-\left(\frac{\delta_{r-1,r}}{\mathrm{D}_{r-1,r}}+\frac{\delta_{r-2,r}}{\mathrm{D}_{r-2,r}}\right)\epsilon\right)+\ldots\\
a_{r+3}\div a_r&=\frac{\epsilon^3}{\mathrm{D}_{r+1,r}\,\mathrm{D}_{r+2,r}\,\mathrm{D}_{r+3,r}}+\ldots\\
a_{r-3}\div a_r&=\frac{p_rp_{r-1}p_{r-2}}{\mathrm{D}_{r-1,r}\,\mathrm{D}_{r-2,r}\,\mathrm{D}_{r-3,r}}.\epsilon^3+\ldots
\end{aligned}\right\}\quad.\quad(54)$$

The determination of the ratios of the remaining coefficients is thus reduced to algebraical manipulation : the last terms, however, of a_{r-1} and a_{r+1} are very complicated, and I have not succeeded in giving them a simple form, but they may, of course, be found in any case where assigned numbers are given for m and n.

For the others we obtain, on substitution,

$$a_{r+1}\div a_r=\frac{\epsilon}{2(2n+3)}-\frac{(2m+1)(2m-1)}{(2n-1)(2n+3)^3(2n+7)}\epsilon^2+\mathrm{B}_3\epsilon^3+\ldots$$

$$a_{r+2}\div a_r=\frac{\epsilon^2}{8(2n+3)(2n+5)}-\tfrac{1}{2}.\frac{4m^2-1}{(2n-1)(2n+3)^3(2n+5)(2n+11)}.\epsilon^3+\ldots$$

$$a_{r+3}\div a_r=\frac{\epsilon^3}{48(2n+3)(2n+5)(2n+7)}+\ldots$$

$$a_{r-1}\div a_r=-\frac{(n^2-m^2)(\overline{n-1}^2-m^2)}{(2n+1)(2n-1)^3(2n-3)}\left[\frac{\epsilon}{2}-\frac{4m^2-1}{(2n-5)(2n-1)^2(2n+3)}\epsilon^2+\mathrm{C}_3\epsilon^3+\ldots\right]$$

$$a_{r-2}\div a_r=\frac{(n^2-m^2)(\overline{n-1}^2-m^2)(\overline{n-2}^2-m^2)(\overline{n-3}^2-m^2)}{8(2n-7)(2n-5)^2(2n-3)^3(2n-1)^3(2n+1)}\left[\epsilon^2-4.\frac{4m^2-1}{(2n-9)(2n-1)^2(2n+3)}\epsilon^3+\ldots\right]$$

$$a_{r-3}\div a_r=-\frac{(n^2-m^2)(\overline{n-1}^2-m^2)(\overline{n-2}^2-m^2)(\overline{n-3}^2-m^2)(\overline{n-4}^2-m^2)(\overline{n-5}^2-m^2)}{48(2n-11)(2n-9)^2(2n-7)^2(2n-5)^3(2n-3)^3(2n-1)^3(2n+1)}.\epsilon^3+\ldots$$

14. We proceed now to the numerical calculation of several of the roots and coefficients.

I. Let $m=0$.

(1) Let $r=0$, $n=0$; a_0 is here the leading coefficient and we may put it $=1$.

$$k=\frac{1}{3}\epsilon-\frac{2}{135}\epsilon^2+\frac{4}{3^6.5.7}\epsilon^3+\frac{182}{3^7.5^3.7^3}\epsilon^4+\ldots$$

$$a_1=\frac{\epsilon}{6}-\frac{\epsilon^2}{189}+\frac{91}{2.3^4.5^2.7^2}\epsilon^3+\ldots$$

$$a_2=\frac{\epsilon^2}{120}-\frac{1}{2.3^3.5.11}.\epsilon^3+\ldots$$

$$a_3=\frac{1}{4.3^2.5.7}\epsilon^3+\ldots$$

(2) Let $r=1$, $n=2$, a_1 the leading term put equal to unity.

$$k=6+\frac{11}{21}\epsilon+\frac{94}{3^3.7^3}\epsilon^2-\frac{21388}{3^2.7^5.11}\epsilon^3+\ldots$$

$$a_2=\frac{\epsilon}{14}+\frac{1}{3.7^3.11}\epsilon^2+\ldots$$

$$a_3=\frac{\epsilon^2}{504}+\frac{1}{2.3.7^3.9.15}\epsilon^3+\ldots$$

$$a_0=-\frac{2}{135}\epsilon+\frac{4}{3^5.5.7}\epsilon^2+\ldots$$

(3) Let $r=2$, $n=4$, leading term a_2 put equal to unity,

$$k=20+\frac{39}{77}\epsilon+\frac{77674}{5.7^3.11^3.13}\epsilon^2-\frac{2805228}{7^5.11^5.13.15}.\epsilon^3+\ldots$$

$$a_3=\frac{1}{22}\epsilon+\frac{1}{7.11^3.13}\epsilon^2+\ldots$$

$$a_4=\frac{\epsilon^2}{1144}+\frac{1}{2.7.11^3.13.19}\epsilon^3+\ldots$$

$$a_1=-\frac{8}{1715}\epsilon-\frac{16}{3.5.7^5.11}\epsilon^2+\ldots$$

$$a_0=\frac{8}{3^2.5^3.7^3}.\epsilon^2-\frac{32}{3^2.5^3.7^5.11}\epsilon^3+\ldots$$

II. Let $m=1$.

(1) For $r=0$, $n=1$, leading term a_0 put equal to unity.

$$k=2+\frac{1}{5}\epsilon-\frac{4}{5^3.7}\epsilon^2+\frac{24}{5^5.7.9}.\epsilon^3+\ldots$$

$$a_1=\frac{\epsilon}{10}-\frac{1}{375}\epsilon^2+\frac{31}{5^3.7^2.10.11}\epsilon^3+\ldots$$

$$a_2=\frac{\epsilon^2}{70}-\frac{3}{2.5^3.7.13}\epsilon^3+\ldots$$

(2) $r=1$, $n=3$, leading term a_1 put equal to unity.

$$k=12+\frac{7}{15}\epsilon+\frac{1064}{3^4.5^3.7.11}\epsilon^2-\frac{808976}{3^7.5^5.7.11.13}\epsilon^3+\ldots$$

$$a_2=\frac{\epsilon}{18}-\frac{1}{5.3^5.13}\epsilon^2+\ldots$$

$$a_3=\frac{\epsilon^2}{792}-\frac{3}{9^3.10.11.15}\epsilon^2+\ldots$$

$$a_0=-\frac{4}{7.5^3}\epsilon+\frac{8}{3.5^6.7}\epsilon^2+\ldots$$

(3) Let $r=2$, $n=5$

$$k=30+\frac{19}{39}\epsilon+\frac{12188}{3^4.7.11.13^3}\epsilon^2+\ldots$$

III. Let $m=2$.

(1) If $r=0$, $n=2$, leading term a_0 put equal to unity.

$$k=6+\frac{1}{7}\epsilon-\frac{2}{3.7^3}\epsilon^2+\frac{20}{3.7^5.11}\epsilon^3+\ldots$$

$$a_1=\frac{\epsilon}{14}-\frac{5}{7^3.11}\epsilon^2+\ldots$$

$$a_2=\frac{\epsilon^2}{504}-\frac{5}{7^3.15.18}\epsilon^3+\ldots$$

(2) When $r=1$, $n=4$

$$k=20+\frac{31}{77}\epsilon-\frac{1270}{7^3.11^3.13}\epsilon^2+\ldots$$

(3) When $r=2$, $n=6$

$$k=42+\frac{75}{165}\epsilon+\frac{1534}{11^3.13.15.17}\epsilon^2+\ldots$$

15. The functions with which we have just been dealing belong to the first of the two classes of ϑ. There is, however, a second class in which ϑ is of the form

$$\vartheta=b_0\mathrm{P}_m^{m+1}-b_1\mathrm{P}_m^{m+3}+\ldots$$

The investigation of these functions will proceed on the same lines as those we have already treated, and the general formulæ for the roots and coefficients will still hold true, with the modification that we must here put $n=m+2s+1$, where $s=0, 1, 2\ldots$

For a given value of m, the first root of each series is

$$k=(m+1)(m+2)+\frac{3}{2m+5}\epsilon-\frac{6(m+1)}{(2m+5)^3(2m+7)}\epsilon^2+\frac{12(m+1)(2m-1)}{(2m+5)^5(2m+7)(2m+9)}\epsilon^3+\ldots \quad (55)$$

As the mode of formation of the coefficients has been already sufficiently illustrated, I shall confine myself to writing down a few of the smaller roots of this class.

When $m=0,\ s=0,\ k=\ 2+\frac{3}{5}\epsilon-\frac{6}{875}\epsilon^2-\frac{4}{3.5^6.7}\epsilon^3+\dots$

„ $m=0,\ s=1,\ k=12+\frac{23}{45}\epsilon+\frac{23114}{3^6.5^3.7.11}\epsilon^2+\dots$

„ $m=0,\ s=2,\ k=60+\frac{59}{117}\epsilon+\frac{696718}{3^6.5.7.11.13^3}\epsilon^2+\dots$

„ $m=1,\ s=0,\ k=\ 6+\frac{3}{7}\epsilon-\frac{4}{1029}\epsilon^2+\frac{8}{3.7^5.11}\epsilon^3+\dots$

„ $m=1,\ s=1,\ k=20+\frac{37}{77}\epsilon+\frac{21192}{3.7^3.11^3.13}\epsilon^2+\dots$

„ $m=1,\ s=2,\ k=42+\frac{27}{55}\epsilon+\frac{73892}{3.5^3.11^3.13.17}\epsilon^2+\dots$

„ $m=2,\ s=0,\ k=12+\frac{1}{3}\epsilon-\frac{2}{891}\epsilon^2+\frac{4}{3^7.11.13}\epsilon^3+\dots$

„ $m=2,\ s=1,\ k=30+\frac{17}{39}\epsilon+\frac{506}{3^4.11.13^3}\epsilon^2+\dots$

The larger m and n become the more nearly will the first few terms of the series represent the value of the root.

16. We shall now show how to find the types of heat-movement which take place when the surface of the ellipsoid is maintained at a constant temperature zero. The general equation of conduction is satisfied by any expression of the form

$$V=(A\cos m\phi+\beta\sin m\phi)e^{-\lambda^2\kappa t}\vartheta_m{}^n(\beta)\Omega_m{}^n(\alpha);$$

and, in order that the temperature may be constantly zero at the surface, we must have

$$\Omega_m{}^n(\alpha_0,\ \lambda,\ m)=0\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ . \qquad (56)$$

or, more fully,

$$a_0S_m-a_1S_{m+2}+\dots=0,$$

in which the leading term is $(-1)^r a_r S_{m+2r}$, where $m+2r=n$.

For given values of m and n this equation has an infinite number of roots; the total number of values of λ is therefore, apparently, triply infinite. Now, in the corresponding problem for the sphere, the equation is $S_n(\lambda r_0)=0$, the number being doubly infinite; and there is no theoretical difficulty in using this solution to approximate to the roots of $\Omega_m^n=0$, where the eccentricity of the ellipsoid is small. The precise manner in which each of the roots of the first problem resolves itself into several in the second is interesting. We observe that S_{m+2r} may be the leading term of Ω, either as the first term of $\Omega^0{}_{m+2r}$, the second of $\Omega^1{}_{m+2r-2}$, the third of $\Omega^2{}_{m+2r-4}$, . . . We infer therefore that each root of $S_0=0$ or of $S_1=0$ corresponds to one for the ellipsoid, each root of $S_2=0$ or $S_3=0$ to two, each of $S_4=0$ or $S_5=0$ to three; and, in general, that each root of

$$S_{2n}=0, \text{ or of } S_{2n+1}=0,$$

corresponds to $(n+1)$ roots of the equation in λ for the ellipsoid.

If, therefore, the total number of values of λ in the case of the sphere be nN, the total number for the ellipsoid is $\frac{n(n+1)}{2}$ N.

All the roots of $S_n=0$ may be found without difficulty by the general processes given by Professor STOKES (Camb. Phil. Trans., vol. ix.) and by Lord RAYLEIGH (Proc. Math. Soc., vol. v., p. 119); and, therefore, those of Ω_m^n can be expanded in powers of c^2. We shall confine ourselves to determining those of $\Omega_0{}^0=0$.

Employing the expression for $\Omega(\eta)$ and putting $a_0=1$, we have

$$S_0+\tfrac{2}{3}a_1S_2+\tfrac{8}{35}a_2S_4+\ldots=0\,.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\qquad(57)$$

If we neglect ϵ this equation reduces to $S_0(\lambda\mathrm{a})=0$, whose roots are given by $\lambda\mathrm{a}=i\pi$.

Let the full value of λa be $i\pi+l_1e^2+l_2e^4+\ldots$, e being the eccentricity of the ellipsoid $=\frac{c}{\mathrm{a}}$. The elements of the subsequent calculation are briefly as follows :—

$$\epsilon=\lambda^2c^2=i^2\pi^2e^2+2i\pi l_1e^4+\ldots$$

$$S_0(\lambda\mathrm{a})=\frac{l_1e^2}{i\pi}\cos i\pi+\left(\frac{l_2}{i\pi}-\frac{l_1{}^2}{i^2\pi^2}\right)\cos i\pi.e^4+\ldots$$

$$S_2(\lambda\mathrm{a})=-\frac{3}{i^2\pi^2}\cos i\pi-\left(\frac{1}{i\pi}-\frac{9}{i^3\pi^3}\right)l_1e^2\cos i\pi+\ldots$$

$$S_4(\lambda\mathrm{a})=\left(\frac{10}{i^2\pi^2}-\frac{105}{i^4\pi^4}\right)\cos i\pi+\ldots$$

$$\frac{2}{3}a_1=\frac{1}{9}i^2\pi^2e^2+\left(\frac{2}{9}i\pi l_1-\frac{2}{567}i^4\pi^4\right)e^4+\ldots$$

$$\frac{8}{35}a_2=\frac{i^4\pi^4}{525}e^4+\ldots$$

Substituting in the above equation, and putting equal to zero the coefficients of e^2 and e^4, we obtain finally

$$\lambda\mathrm{a}=i\pi+\frac{1}{3}i\pi e^2+\frac{i\pi}{405}(i^2\pi^2+27)e^4+\ldots\quad.\quad.\quad.\quad.\quad.\quad.\quad.\qquad(58)$$

17. When the ellipsoid cools by radiation, the equation to be fulfilled at the boundary is

$$\frac{dV}{d\alpha}+\mathfrak{h}c\sqrt{\cosh^2\alpha-\cos^2\beta}\,V=0,\ \text{when}\ \alpha=\alpha_0.$$

or

$$\frac{dV}{d\xi}+\frac{\mathfrak{h}}{\lambda}\sqrt{1-e^2\cos^2\beta}\,V=0\ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\qquad(59)$$

observing that $\frac{1}{\cosh\alpha_0}=\frac{c}{a}=e.$

In its present shape, the process of satisfying this condition is complicated. If, however, we neglect e^2, the condition then becomes

$$\frac{dV}{d\xi}+\frac{\mathfrak{h}}{\lambda}V=0.$$

The appropriate form of solution in this case is

$$V=(A\cos m\phi+B\sin m\phi)e^{-\lambda^2\kappa t}\vartheta_m{}^n\Omega_m{}^n,$$

λ being given by

$$\frac{d\Omega_m{}^n}{d\xi}+\frac{\mathfrak{h}}{\lambda}\Omega_m{}^n=0,\ \text{when}\ \xi=b.\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\qquad(60)$$

If, as before, we neglect e^2, this equation becomes simply

$$\frac{dS_n}{d\xi}+\frac{\mathfrak{h}}{\lambda}S_n=0,$$

the same equation as found for the case of a sphere. It is, therefore, only when we do not neglect $e^2\nu^2$ in the expression $\sqrt{1-e^2\nu^2}$, that we obtain results belonging specially to the ellipsoid. We must accordingly indicate how the problem in its more general form is to be dealt with. To do so we require various general properties of the ϑ-functions, to the discussion of which we now proceed. It should be understood that these properties are of a purely mathematical character, and have nothing to do with the special series of values which the physical conditions of the problem may ascribe to λ. We have seen that for given values of λ and m, the different values of k are perfectly definite, and it may facilitate the apprehension of these values to bear in mind the approximations which have been given for them in powers of ϵ.

The following theorems are well known:—

$$\int_{-1}^{+1}P_m{}^nP_m{}^{n'}d\nu=0,\ \int_{-1}^{+1}(P_m{}^n)^2d\nu=\frac{2}{2n+1}\cdot\frac{(n-m)!\,(n+m)!}{\{1.3\ldots(2n-1)\}^2}=j_m{}^n,\ \text{say}\ .\ \ .\ \ .\qquad(61)$$

and if we denote $\int_{-1}^{+1}\vartheta_m^n\vartheta_m^{n'}d\nu$ by (n, n', m, λ), or shortly by (n, n'), we obtain the following:—

$$\left.\begin{aligned}(n, n')&=a_0a'_0j_m^m+a_1a'_1j_m^{m+2}+a_2a'_2j_m^{m+4}+\ldots\\(n, n)&=(a_0)^2j_m^m+(a_1)^2j_m^{m+2}+\ldots\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (62)$$

It may be easily shown that

$$(n, n')=0,\ n \text{ not}=n' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

for the differential equations satisfied by ϑ_m^n, $\vartheta_m^{n'}$ are

$$\left.\begin{aligned}(1-\nu^2)\frac{d^2\vartheta^n}{d\nu^2}-2\nu\frac{d\vartheta^n}{d\nu}-\frac{m^2}{1-\nu^2}\vartheta^n&=\lambda^2c^2\nu^2\vartheta^n-k_n\vartheta^n\\(1-\nu^2)\frac{d^2\vartheta^{n'}}{d\nu^2}-2\nu\frac{d\vartheta^{n'}}{d\nu}-\frac{m^2}{1-\nu^2}\vartheta^{n'}&=\lambda^2c^2\nu^2\vartheta^{n'}-k_{n'}\vartheta^{n'}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (64)$$

and therefore

$$(k_n-k_{n'})\int_{-1}^{+1}\vartheta^n\vartheta^{n'}d\nu=\left[(1-\nu^2)\left(\vartheta^{n'}\frac{d\vartheta^n}{d\nu}-\vartheta^n\frac{d\vartheta^{n'}}{d\nu}\right)\right]_{-1}^{+1}=0.$$

From this result we infer that any function may be expanded in a series of ϑ-functions; and, first of all, we may invert the series which expresses the ϑ-functions of the a-group. This is manifest algebraically; for if we solve the equations for $\vartheta^0\ldots\vartheta^r$ in terms of $P_m^m\ldots P_m^{m+2r}$, we obtain for any one of the P's the following:—

$$\left.\begin{aligned}&P_m^n=f_0\vartheta^0+f_1\vartheta^1+f_2\vartheta^2+\ldots,\\\text{where}\qquad&f_s(s, s)=\int_{-1}^{+1}P_m^n\vartheta^s d\nu=(-1)^ra_rj_m^r.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

Now we may assume that any function of ν (at least any series of powers of ν commencing with ν^m) may be expressed in one or other of the three forms ΣAP_m^{m+2r}, ΣBP_m^{m+2r+1}, $\Sigma(AP_m^{m+2r}+BP_m^{m+2r+1})$, and therefore in ϑ-functions of the a-group, b-group, or a combination of these. In particular $\nu^2\vartheta$, $\nu^4\vartheta$, $\frac{d^2\vartheta}{d\nu^2}\ldots$ will give rise to ϑ-functions of the same type as ϑ, and $\nu\vartheta$, $\frac{d\vartheta}{d\nu}\ldots$ to functions of the opposite type. In general, if

$$F(\nu)=\Sigma C_i\vartheta^i,\ (i, i)C_i=\int_{-1}^{+1}F(\nu)\vartheta^i d\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

The expansion of $\nu^2\vartheta$ may be obtained without difficulty; for if we combine the first of equations (64) with

$$(1-\nu^2)\frac{d^2\mathrm{P}_m^n}{d\nu^2}-2\nu\frac{d\mathrm{P}_m^n}{d\nu}-\frac{m^2}{1-\nu^2}\mathrm{P}_m^n=-n(n+1)\mathrm{P}_m^n,$$

we obtain

$$\lambda^2c^2\int_{-1}^{+1}\vartheta_m^p\mathrm{P}_n^n\nu^2d\nu=(k_p-n(n+1))\int_{-1}^{+1}\vartheta^p\mathrm{P}^nd\nu$$
$$=(k_p-n(n+1))(-1)^ra_r^pj_m^r,\ n=m+2r,$$

in which p denotes any even integer.

But

$$\vartheta_m^{p'}=a_0^{p'}\mathrm{P}_m^m-a_1^{p'}\mathrm{P}_m^{m+2}+\ldots,$$

hence, remembering equation (63),

$$\lambda^2c^2\int_{-1}^{+1}\nu^2\vartheta^p\vartheta^{p'}d\nu=-[m(m+1)a_0a'_0j_m^0+a_1a'_1(m+2)(m+3)j_m^{m+2}+\ldots]\ . \quad (67)$$
$$=-\{p,p'\},\ \text{say}.$$

The expansion of $\nu^2\vartheta_m^p$ therefore becomes

$$\nu^2\vartheta_m^p=-\frac{1}{\epsilon}\left[\frac{\{p,0\}}{(0,0)}\vartheta_m^0+\frac{\{p,1\}}{(1,1)}\vartheta_m^1+\ldots\right]\ .\ .\ .\ .\ .\ .\ . \quad (68)$$

18. These investigations and developments place us in a position to deal with the boundary condition due to radiation; we may always put

$$\frac{\mathfrak{h}}{\lambda}\sqrt{1-e^2\nu^2}\vartheta^n=g_0^n\vartheta^0+g_1^n\vartheta^1+g_2^n\vartheta^2+\ldots\ .\ .\ .\ .\ .\ .\ . \quad (69)$$

and all the g's may be expanded in ascending powers of e^2, beginning with e^2, except g_n^n, which commences with ϵ^0. The general equation of conduction is to be satisfied in this case by the series

$$\mathrm{V}=(\mathrm{A}\cos m\phi+\mathrm{B}\sin m\phi)e^{-\lambda^2\kappa t}(\mathrm{C}_0\vartheta^0\Omega_0+\mathrm{C}_1\vartheta^1\Omega_1+\mathrm{C}_2\vartheta^2\Omega_2+\ldots),\ .\ . \quad (70)$$

in which the same λ occurs throughout.

At the boundary, where $\xi=\lambda\mathrm{b}$,

$$\mathrm{C}_0\left(\frac{d\Omega_0}{d\xi}+g_0^0\Omega_0\right)+\mathrm{C}_1g_0^1\Omega_1+\mathrm{C}_2g_0^2\Omega_2+\ldots=0,$$
$$\mathrm{C}_0g_1^0\Omega_0+\mathrm{C}_1\left(\frac{d\Omega_1}{d\xi}+g_1^1\Omega_1\right)+\mathrm{C}_2g_1^2\Omega_2+\ldots=0,$$
$$\mathrm{C}_0g_2^0\Omega_0+\mathrm{C}_1g_2^1\Omega_1+\mathrm{C}_2\left(\frac{d\Omega_2}{d\xi}+g_2^2\Omega_2\right)+\ldots=0,\ \&\text{c}.$$

These are the equations which determine the different values of λ and the corresponding ratios $C_0:C_1:C_2:\ldots.$ When we neglect powers of e^2 beyond e^4, the equation determining λ is

$$1=\sum_0^\infty\sum_0^\infty\frac{g_r^s g_s^r\Omega_r\Omega_s}{\left(\frac{d\Omega_r}{d\xi}+g_r^r\Omega_r\right)\left(\frac{d\Omega_s}{d\xi}+g_s^s\Omega_s\right)},\ r\text{ not}=s;\quad\ldots\ldots\quad(71)$$

If the first power of e^2 alone is retained, this equation breaks up into

$$\frac{d\Omega_r}{d\xi}+g_r^r\Omega_r=0,\ \frac{d\Omega_s}{d\xi}+g_s^s\Omega_s=0,\ \&c.\quad\ldots\ldots\ldots\quad(72)$$

The solutions of these equations may be easily found from the corresponding results for the sphere, and the quantities $g_r^r\ldots$ have been already found.

19. We shall not pursue this inquiry further, but shall now show how the arbitrary constants introduced into the solution may be determined. As already explained in Art. 2 of this paper, when the solution is represented by $V=\Sigma Ae^{-\lambda^2 kt}v$, A is to be found from

$$A\int v^2dE=\int vV_0dE,$$

and what we have to find is, therefore,

$$\int_0^{2\pi}\int_{-1}^{+1}\int_0^{\alpha_0}v^2\sinh\alpha\cos\beta\,(\cosh^2\alpha-\cos^2\beta)d\phi d\beta d\alpha\ldots\quad\ldots\quad(73)$$

If the surface is kept at a constant temperature zero, or if we adopt the simple law of radiation $\frac{dV}{d\xi}+\frac{\mathfrak{h}}{\lambda}V=0$, we may take

$$v=\cos m\phi\vartheta_m^n\Omega_m^n$$

as the type of solution: the above integral may then be resolved into the components $\int_{-1}^{+1}\vartheta\,.\,\vartheta d\nu$, $\int_{-1}^{+1}\nu^2\vartheta\,.\,\vartheta d\nu$, $\int_0^{\alpha_0}\sinh\alpha\Omega\,.\,\Omega d\alpha$, $\int_0^{\alpha_0}\sinh\alpha\cosh^2\alpha\Omega\,.\,\Omega d\alpha$; and of these the two former have been already found.

For the accurate solution of the problem of radiation we must take for v an expression of the form

$$\cos m\phi(c_0\vartheta^0\Omega^0+c_1\vartheta^1\Omega^1+c_2\vartheta^2\Omega^2+\ldots),$$

in which $c_0c_1\ldots$ are known constants, one of which is to be put equal to unity. When we substitute this expression for v we perceive that we have still another integral to evaluate, namely,

$$\int_0^{\alpha_0}\sinh\alpha\Omega\,.\,\Omega' d\alpha.$$

To effect the integrations it will be most convenient to suppose Ω expanded in terms of ζ in the form

$$\Omega = \mathrm{K}_n(a_0\mathrm{P}_m{}^m - a_1\mathrm{P}_m{}^{m+2} + \ldots) \text{ (Art. 10).}$$

But, as is well known,

$$\int_0^{\frac{1}{e}} \mathrm{P}_m{}^n.\mathrm{P}_m{}^{n'} d\zeta = \frac{(n-m)!\,(n'+m)}{(2n)!\,(2n')!}\int_0^{\frac{1}{e}}\left(\frac{d}{d\zeta}\right)^{n+m}(\zeta^2-1)^n\cdot\left(\frac{d}{d\zeta}\right)^{n'-m}(\zeta^2-1)^{n'}.d\zeta \quad . \quad . \quad (74)$$

and may be readily evaluated; the result we shall denote by $j(n, n')$.

We thus find

$$\left.\begin{array}{r}\dfrac{1}{\mathrm{K}_n.\mathrm{K}_{n'}}\displaystyle\int_0^{\frac{1}{e}}\Omega^n\Omega^{n'}d\zeta = \Sigma\Sigma(-)^{r+s}(a_r a'_s + a'_r a_s)j(m+2r,\ m+2s) + \Sigma a_r.a'_r j(m+2r,\ m+2r), \\ r \text{ not} = s\end{array}\right\}(75)$$

$\int\Omega^n.\Omega^n d\zeta$ may be found by putting $n=n'$, $a'_r = a_r \ldots$

For the remaining integral, combine the equations

$$(1-\zeta^2)\frac{d^2\Omega}{d\zeta^2} - 2\zeta\frac{d\Omega}{d\zeta} - \frac{m^2}{1-\zeta^2}\Omega = \lambda^2c^2\zeta^2\Omega - k\Omega$$

$$(1-\zeta^2)\frac{d^2\mathrm{P}_m{}^n}{d\zeta^2} - 2\zeta\frac{d\mathrm{P}_m{}^n}{d\zeta} - \frac{m^2}{1-\zeta^2}\mathrm{P}_m{}^n = \lambda^2c^2\zeta^2\mathrm{P}_m{}^n - n(n+1)\mathrm{P}_m{}^n;$$

whence

$$\lambda^2c^2\int_0^{\frac{1}{e}}\zeta^2\Omega\mathrm{P}_m{}^n d\zeta = (k - n.\overline{n+1})\int_0^{\frac{1}{e}}\Omega.\mathrm{P}_m{}^n d\zeta + \left[(1-\zeta^2)\left(\mathrm{P}_m{}^n\frac{d\Omega}{d\zeta} - \Omega\frac{d\mathrm{P}_m{}^n}{d\zeta}\right)\right]_0^{\frac{1}{e}} \quad . \quad . \quad (76)$$

But $\Omega(\zeta) = a_0\mathrm{P}_m{}^m - a_1\mathrm{P}_m{}^{m+2} + \ldots.$

Taking these together, we arrive at the value of $\int_0^{\frac{1}{e}}\zeta^2\Omega.\Omega d\zeta$.

20. The preceding investigations have related to the laws which govern the movement of heat in an ovoid ellipsoid. For a planetary ellipsoid, we must take

$$\rho = c\cosh\alpha\sin\beta,\ z = c\sinh\alpha\cos\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

The equation which V satisfies is

$$\frac{d^2\mathrm{V}}{d\alpha^2} + \frac{d^2\mathrm{V}}{d\beta^2} + \tanh\alpha\frac{d\mathrm{V}}{d\alpha} + \cot\beta\frac{d\mathrm{V}}{d\beta} + \left(\frac{1}{\sin^2\beta} - \frac{1}{\cosh^2\alpha}\right)\frac{d^2\mathrm{V}}{d\phi^2} = \frac{c^2}{\mathfrak{k}}(\cos^2\beta + \sinh^2\alpha)\frac{d\mathrm{V}}{dt},$$

to satisfy which we must put

$$\mathrm{V} = \cos m\phi e^{-\lambda^2\mathfrak{k}t}\vartheta_m{}^n(\beta)\Omega_m{}^n(\alpha) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

where ϑ and Ω are determined by the equations

$$\frac{d^2\vartheta}{d\beta^2}+\cot\beta\frac{d\vartheta}{d\beta}-\frac{m^2\vartheta}{\sin^2\beta}=-\lambda^2c^2\cos^2\beta\vartheta-k\vartheta\ .\ \ .\ \ .\ \ \ \ \ .\ \ .\quad(79)$$

$$\frac{d^2\Omega}{d\alpha^2}+\tanh\alpha\frac{d\Omega}{d\alpha}+\frac{m^2\Omega}{\cosh^2\alpha}=-\lambda^2c^2\sinh^2\alpha\Omega+k\Omega.$$

The latter of these equations, on putting $\xi=\lambda c\cosh\alpha$, becomes converted into

$$(\xi^2+\lambda^2c^2)\frac{d^2\Omega}{d\xi^2}+2\xi\frac{d\Omega}{d\xi}+\frac{\lambda^2c^2}{\xi}\ \frac{d\Omega}{d\xi}+\frac{m^2\lambda^2c^2\Omega}{\xi^2}=-(\xi^2-\lambda^2c^2)\Omega+k\Omega\ .\ \ .\ \ .\quad(80)$$

These two equations (79) and (80) are of precisely the same form as (18,) and (19) which we have been discussing, and differ from them only in having $-\lambda^2c^2$ instead of $+\lambda^2c^2$. The subsequent formulæ will, therefore, be the same almost in every instance, and the investigation need not therefore be repeated.

VI. *On some recent Improvements made in the Mountings of the Telescopes at Birr Castle.*

By the EARL OF ROSSE, *F.R.S.*

Received May 14,—Read June 19, 1879.

[PLATES 11-13.]

FOR some time in carrying on the work of the Observatory at Birr Castle the mountings of the instruments had been found yearly more and more deficient in the requirements of modern astronomy. The six-foot reflector still remained unequalled in aperture, and even the three-foot was in two or three cases only surpassed or even approached in size, but the mechanical appliances for working them were by no means equal in convenience to those fitted to most modern instruments, so that much time was unprofitably spent, and in some departments of the science we found it impossible to make progress. It was therefore decided, in the first place, to apply a clock-movement to the six-foot, the motion of which in Right Ascension had up to that time been given by the hand of an attendant.

The mounting of the six-foot reflector had proved perfectly successful as to steadiness and ease of working, and with the means available for its execution at the time, perhaps, it was the only one which could have been well carried out, but in its limited range of motion we latterly found a considerable inconvenience, which increased as the re-observation of HERSCHEL'S catalogue proceeded, and when the spectroscope came to be applied to astronomical investigation the drawbacks to it were found still greater.

Six-foot reflector.

In the year 1869 the clock-movement was applied to the six-foot instrument. This was a matter of some difficulty, as the resistance to be overcome was not only on an average greater, but also, owing to the counterpoising being approximate only, more variable[*] in amount than in any other example; still the clock has been so far successful as to have been of immense advantage in working with the filar micrometer.[†]

---

[*] In some positions of the telescope the unbalanced pressure with or against the motion in Right Ascension reaches the large amount of 400 lbs., while the weight to be moved—tube, speculum, &c.—probably exceeds 10 tons.

[†] The suspending chain admits of a very considerable departure of the telescope from a given declination, considerably more than the length of the slit of a spectroscope, owing to the variable amount of "sag" of the chain, so it was useless to seek for so accurate a motion in Right Ascension as would keep a star's image within the breadth of the slit.

The clock has a train of five arbors, the first, carrying the barrel, being worked by a weight of about 30 cwt. (acting through a single pulley) descending half a foot per minute. The fifth carries a ball-governor like that usually adopted but with a higher velocity, and springs are employed to assist gravity in acting in antagonism to centrifugal force. A pulley (A) fixed upon the third axle transmits the motion through the medium of an endless rope of galvanised wire to a pulley (D) (Plate 11, figs. 1, 2, and 3), fixed at the eastern end of a wrought-iron pipe (G H). A weighted lever with a pulley (C) at its extremity serves to keep the wire-rope at the requisite degree of tension. The pipe (G H) is supported near its centre of gravity from a beam (E E) by a standard (K) with a set of rollers in a live ring, and is prevented from moving endwise towards G by an annular bearing, and towards H by a centre bearing. The beam at its eastern end rests, through the intervention of a strong piece of T-iron, provided with rollers, against a circle of about 40 feet radius (which remains fixed to the eastern wall as heretofore). The beam runs through a socket provided with rollers (R R). The socket is attached to the tube (a portion of which, O O, P P, is shown) by a pin (M), and follows the angular motion of the beam as the tube of the telescope moves westwards. The pipe is fitted with a nut at G, into which a square-threaded screw (I G) of four threads to one inch works. The telescope can be advanced or drawn back in Right Ascension by the observer by turning the screw by means of the pulley I.* A clamping screw in the socket (J), through which the end of the screw passes, prevents the screw from turning round with the nut when the clock is going. The socket J is bolted to the socket R M, L R, and so connected with the telescope. The beam (E F) is faced with strong hoop iron. The greater part of the pressure of the beam, pipe, and screw in a direction tangential to the circle of motion in polar distance is carried by a lever and counterpoise attached at N, but not represented. The east end of the screw is turned down for a length of two or three inches, and is provided with a washer with rounded edge which keeps that end of the screw from lying upon the inside of the tube. Plate 11, fig. 1, shows the general arrangement of the screw in elevation, looking northwards, with the telescope directed to the zenith; fig. 2 is a cross section of the beam, &c., at G L; fig. 3 is a general view of the whole on a smaller scale, looking westwards.

It will be observed that the rate of the clock must be made to vary in proportion to the cosine of declination. No automatic contrivance has been applied for effecting this; but, as for obvious reasons we as far as possible restrict our observations on each night within a limited range of declination, the alteration of the rate by hand has not been found to cause serious inconvenience.

---

* This motion is rarely used, as the eye-piece has a motion of 15′ or 20′ range in an east and west direction, by means of which an object can be followed. The small speculum also having a clear aperture of nine inches, little or nothing of the pencil of rays is cut off within this range.

The three-foot reflector.

The three-foot reflector had up to the year 1874 been carried by an Altazimuth mounting constructed principally of wood, which, though possessing this advantage over that of the six-foot—that the instrument could be directed to any part of the visible heavens—was from its nature less convenient for general work, and did not possess the steadiness of that of the six-foot, or of a mounting composed wholly of metal and masonry. The woodwork was in need of extensive repair, and it was decided to remount the instrument as an equatorial of such strength and solidity as to be as far as possible free from vibration even when exposed to a moderate force of breeze.

The mounting now most in favour both for reflectors and refractors, which has been recently much improved in details by Mr. GRUBB and others, and modified by the late Mr. GRUBB in the case of the great Melbourne telescope by placing the upper bearing of the polar axis above instead of below its intersection with the declination axis, with the object of securing greater steadiness and ease of motion in so large a reflector, is that known as FRAUNHOFER'S.

In it the tube of the telescope is situated at one extremity of the declination axis, and it is counterpoised round the polar axis by a weight at the other extremity of the declination axis. In following an object with a telescope mounted on this plan from near the eastern towards the western horizon it is, in general, necessary that the tube shall be "reversed" from one side to the other of the polar axis at or near to the meridian (usually the best position for observation). This is certainly a cause of considerable inconvenience in some classes of work.

In the mounting about to be described "reversal" is also necessary, but only at the less advantageous positions for observation, the east and west points. The tube is balanced round the polar axis, and the only part which requires a counterpoise is the fork which carries the tube.

The mounting adopted by Mr. LASSELL for his two-foot and four-foot reflectors furnished the original idea, but with the kind assistance of Mr. BINDON B. STONEY, C.E., from whom many valuable suggestions were received, as also through the careful attention given to the designing of the details of the various parts by Mr. W. G. STRYPE, C.E., who was the first to suggest the form of fork ultimately adopted, and superintended the execution of the whole, it is hoped that the mounting may be found well adapted for the work required.*

In Mr. LASSELL'S mounting the polar axis consists of a cone of boiler-plate, the apex of which is directed towards the south pole and rests in a step or bearing. The upper end is covered in with a plate, upon which are erected two standards which support the two extremities of the declination axis. The base of the cone turns upon antifriction rollers.

---

* Mr. WILLIAM SPENCE, of Cork Street, Dublin, was contractor for the work, masonry only excepted.

In modifying this mounting we endeavoured to make the polar axis as much a continuous girder as the conditions to be fulfilled admitted of. The upper bearing was placed externally to the structure and was reduced to a small diameter. Unlimited freedom of motion round the polar axis was thus sacrificed; but it is believed that a more rigid and steady mounting has been obtained which, without the introduction of antifriction rollers, turns with at least equal ease in Right Ascension. The mounting (Plates 11, 12, and 13) is carried by a massive pier of stone laid in cement-mortar, on the southern inclined face of which rests a bed plate (A A, B B) of cast-iron, ribbed on its under side, imbedded in cement and fixed firmly down by six bolts, four of them near the lower and two near the upper edge of the plate, and all extending deep into the masonry. Upon this rests, with power of adjustment in both planes, the pedestal casting (C), which is bored to receive the polar axis, a wrought-iron bar with slightly conical bearing surfaces near its ends fitted into it. A cotter and wedge received by a slot through its centre press it down firmly home, and the same when reversed in position can be used to start it up from its place if at any time required. The pedestal casting is provided with a lip which catches the top edge of the bed-plate, keeps the former from sliding down, and takes off all side pressure from the adjusting screw, D. The centre of gravity of the whole overhangs the upper edge of the base-plate a little.

The fork is tubular and made of boiler-plate $\frac{1}{4}$-inch thick, firmly rivetted to angle iron, of $2\frac{1}{4} \times 2\frac{1}{4} \times \frac{5}{16}$-inch scantling, along each angle in the usual manner. The web of the lower part is carried up and connected to the base of the fork at F F to give greater rigidity. The fork, as will be seen from Plate 11, fig. 4, turns on conical bearings of small diameter. The bearings are of hard brass. Their external surfaces are cylindrical, and they are fitted into castings rivetted to the plating of the fork. The bearings can be turned round,* and thus, the common axis of the conical cavities being inclined at an angle of about 13′ from that of their external surfaces (the cavity in the lower one being eccentric by $\frac{5}{16}$ of an inch), an adjustment for perpendicularity of the declination to the polar axis can be made, and no provision for adjustment of the bearings of the declination axis is needed. The counterpoise of the fork is marked W in the drawings.

The tube turns on cylindrical trunnions which have conical ends fitted into castings rivetted to the sides of the tube. They are retained securely in their places by feathers, nuts, and lock-nuts. The pressure of the tube parallel to the declination axis is communicated through the shoulder of the lowermost trunnion to the corresponding branch of the fork, and also equally to the other branch through the washer, nut, and lock-nut on the uppermost trunnion. It will be observed that the sides of the tube are expanded to 50 inches at the trunnions to give greater steadiness. The diameter of the speculum is about 36 inches, but the mouth of the tube has a clear aperture of $39\frac{1}{2}$ inches, so that no light may be cut off within a field of 40′ diameter. It will be observed that, unlike Mr. LASSELL'S and that of the great Melbourne telescope, the tube is square. It also differs from Mr. LASSELL'S in this—that the brass ring (G, Plates 12 and 13) at its upper end is the only part which can be turned round the axis of collimation, but this motion meets all requirements of convenience in observing. The ring is retained in a nearly central position by three blocks of brass, provided with shoulders, against which its inner edge rests. The outer edge of the ring is chamfered off, and each block is provided with a screw (S) and a piece of brass, by simply tightening or slackening any one of which the ring is made free to move or firmly clamped.

---

* Both bearings are provided with graduations, so that they may be turned round by an exactly equal amount, and that thus the axis of each may continue parallel to the axis of the bar, the whole describing a conical surface round the common axis of the external surfaces of the bearings. The component of the motion in the plane of the declination axis is that which effects the adjustment, that perpendicular to that axis having no effect beyond a slight disturbance of equilibrium round the polar axis.

The tube is made of angle iron, the longitudinal bars $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{4}$-inch scantling, and the lattice bars $1\frac{1}{8} \times 1\frac{1}{8} \times \frac{3}{16}$-inch above the trunnions and $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{4}$-inch below them. The upper end of the tube for a length of 12 inches is made of plating of about $\frac{1}{10}$-inch thick, and cylindrical in order that the corners may not interfere with the apparatus attached to the eyepiece. The supports at the back of the speculum continue the same as they have been since it was first mounted, and it has on that account been considered unnecessary to complicate the drawings by representing them.* The pressure on its edge in every position is now borne by a ring of angle iron, suspended from the speculum box by eight rods, linked to it as shown in dotted lines (*l m*, *l m*) in Plate 11, fig. 4, those which happen to be uppermost taking the weight of the speculum, and the ring is free to move parallel to the axis of the tube as much as may be rendered necessary by the elasticity of the back supports. The eight rods pass freely through holes in the framing of the tube and are screwed and fitted with nuts to tighten them, so that the position of the speculum may change as little as possible on passing from one side to the other of the zenith.† The speculum box is of cast-iron, and it has a second bottom of wood, for the purpose of enclosing an air-space to be heated by a very small lamp, so that a small amount of heat may penetrate through the first or cast-iron bottom to the speculum and thereby diminish the chance of deposit of dew upon its surface. The space between the upper edge of the box and the edge of the speculum is closed in with wood and canvas.

The coarse motions both in Right Ascension and in declination are effected by means of worm-wheels of a pitch of two threads to one inch, bolted respectively to the lower end of the fork and to the extremity of one of its branches. The worm geering into the latter is fixed on to an arm turning on the trunnion and held in position by a screw of eight threads to one inch pitch, which gives the fine movement in declination in the usual manner, and is brought within reach of the observer by means of a wire rope passing over pulleys. Each worm is cut across at the centre of its length and one portion is free to slide lengthwise, so that any shake can be removed by tightening the nut and lock-nut at the outer side of one of the bearings.*

---

* Their principle has been fully described in the Phil. Trans. for 1861, Part 3.

† A collimator has been fitted to the interior of the tube to enable the position of the speculum to be corrected.

Gallery.

The construction of a gallery to enable the observer conveniently to reach the eye-piece has been considered as one of the greatest difficulties to be overcome by the designer of an equatorial mounting for a Newtonian, and it seems to have led to the Cassegrain form being preferred for the great Melbourne reflector. However, I was deterred from adopting the latter construction by what appeared to me the greater difficulties and inconveniences inseparable from it, as also by the risk of fracture of the speculum in boring out its centre. The following form of gallery was adopted as being apparently liable to the least objections.

A circular wall of brick and cement, whose centre nearly coincides with the centre of motion of the mounting, carries a wrought-iron rail (L). On this a wooden, iron-bound framework (O P, Q N) provided with wheels at M, N, O, runs and carries an arm (T U) provided with worm-wheel horizontal and altitude motions. From the extremity of this arm is suspended the platform on which the observer stands. It will then appear that the gallery has three motions, one in altitude and two in azimuth, and one only of them, that of the frame upon the railway, has not been brought under the easy control of the observer. By means of them and the power of turning the ring at the mouth of the tube, the observer can reach the eye-piece in comfort in every position of the telescope, except at low altitudes, where a light independent ladder is used. The arm which carries the platform is constructed of four deal rods sprung out, connected by pairs at their upper ends, and firmly held together at each cross strut by hoop iron. A cast-iron counterpoise (W″), weighing about 1650 lbs., is fastened immovable on the prolongation backwards of the arm, which it balances when 400 lbs. weight is placed upon the platform. In observing, equilibrium is restored by keeping between the bottom (which is of sheet copper) of the gallery and the floor just so many lead weights as will, with the weight of the observer or observers, just make up the 400 lbs. Two counterpoises (W′, W′) serve to bring the centre of gravity of the gallery within the wheel-base, as otherwise the wheels at N and O would not rest on the rail.

---

* Since the erection of the instrument it has been found impossible so to adjust the nuts and lock-nuts that the worms shall be quite free from shake and yet turn with sufficient freedom, but one-sixth of a turn of a helical clump, since fitted to each and working between the end of the worm and its bearing, and under the control of the observer, meets the difficulty.

Clock-movement.

The clock-movement is of the ordinary construction with centrifugal ball-governor, bringing into play as a checking force the friction between leather studs, connected with the balls and a fixed brass ring. Usually the governor has been so adjusted as to give unaided slightly too high a velocity, and its controlling force has been supplemented by one brought into play by a bevel-wheel running between one moving with the train and one connected with a scape-wheel and seconds pendulum. Any excess or defect of velocity of the train produces a motion of the intermediate wheel, accompanied by an elevation or depression of a lever in connexion with it, thus bringing it into contact with, or withdrawing it from, a rapidly revolving disc worked by the train. The arrangement is not novel.*

The method by which the motion of the clock is communicated to the telescope is peculiar. On the lower end of the fork is fitted a sector of six feet radius. It has an arc of nearly 40°, giving a run of over two hours. Its face is smooth. The motion of the clock is imparted to a square-threaded screw (V X), and to the nut working on it are attached two straps of sheet brass, which wrap round the face of the sector and are fastened to it at its eastern end. A third brass strap fastened to the western end of the sector and wrapping round it between the other two keeps them in a proper state of tension, and a bow (j k) connected with the nut by a universal joint at p and at q fastened to the strap gives the tension to the straps, so that the sector and nut are practically rigidly connected with one another. A coupling at V connects the screw with a handle for winding back, and a pulley, round which passes an endless rope up to the gallery, and a pinion geering into the spur-wheel enables the observer to move the telescope forward or backward in Right Ascension by turning the nut.

The mounting has now been completed about three years; but as our attention has been up to this specially directed to the clearing up of doubtful points in former observations of the Nebulæ which came to light on preparing them for publication, that the work of the six-foot since its erection might be brought out in as complete a form as possible, comparatively little work has been engaged in as yet with the three-foot; however, it has been fairly tested as regards its steadiness, and no tremor through the action of the wind was noticed. The fine motions both in Right Ascension and Declination, the two motions of the arm of the gallery, and the clamps are within easy reach of the observer when in the gallery. It is intended to add an arrangement for reading the Declination circle from the gallery, and it is hoped that some sufficiently simple means to enable him to read the Right Ascension circle may be devised, and if these additions can be satisfactorily carried out they will, of course, be of immense service to the observer, as now time is lost in descending for the purpose of making the set.

---

* The late Mr. COOKE, of York, and Mr. HOWARD GRUBB, of Dublin, have employed similar devices for the control of equatorial clock-movements.

The Right Ascension circle is represented at Y, and the declination circle is fixed on to the axis at Z.

Plate 11, fig. 4, represents the mounting as seen in a direction parallel to the plane of the equator in the meridian and in a direction at right angles to it, the Declination axis lying east and west, and the tube directed towards the pole.

Plate 12, fig. 5, shows the mounting as seen from the west, the tube being directed towards a point in the meridian a little north of the zenith. The gallery is shown placed in position for observing.* The wall which supports the rail (L), and which has two openings, one on the south side to afford a space for the fork to move in, and the other at the north side for the admission of the speculum after repolishing, and to afford access in working the telescope, has now been omitted from the drawing to avoid complexity.

Plate 13, fig. 6, represents the mounting in plan, the telescope being directed to the zenith and the gallery lowered to the ground.

---

* There is a slight inaccuracy in the position of the speculum arm in fig. 5. It is screwed on to the ring at a point at right angles to that occupied by the eyepiece as in fig. 6, not opposite to the eyepiece.

VII. *Concluding Observations on the Locomotor System of Medusæ.*

By George J. Romanes, *M.A., F.L.S.*

Communicated by Professor T. H. Huxley, *LL.D., Sec. R. S.*

Received December 30, 1878,—Read January 16, 1879.

In my previous papers on this subject the considerations are sufficiently explained which originally induced me to defer any systematic investigation concerning the histology of the Medusæ, until the more important of the physiological observations should have been completed. While preparing the material for these papers, I was not blind to the fact that it served still further to ripen the promise that a careful development of the histology of the Medusæ would prove highly remunerative in morphological results. But I felt that, even if no other workers should enter the field from the side of morphology, it was more desirable that the physiology of these primitive nervous tissues should be speedily worked out than that their structural peculiarities should be so. Whether or not this view was a wise one, in the result it has proved beneficial; for not only was I fortunate enough to procure the co-operation of so competent an histologist as my friend Mr. Schäfer in working out the histology of the covered-eyed Medusæ, but contemporaneously with his researches there was also being carried on by Messrs. O. and R. Hertwig a most painstaking and thorough investigation of the histology of the naked-eyed Medusæ. At the same time, also, Professor Eimer had been continuing his microscopical researches. In this way, therefore, both the physiology and the morphology of the Medusæ were simultaneously worked out in parallel lines, with the result of furnishing a much larger and more complete body of information than could possibly have been obtained by a single observer in double the time.

Of Professor Eimer's microscopical work it would be now premature to speak, for as yet he has only published a somewhat meagre abstract, with the promise, however, of shortly supplementing it with a more complete and detailed exposition. Of Mr. Schäfer's work I have previously spoken in the highest terms, and before the present remarks will be in print the Fellows of the Royal Society will themselves be in a position to appreciate its merits. Of Messrs. Hertwig's work it is difficult to speak without enthusiasm; but as I shall conclude this paper with a brief *résumé* of the present literature on the morphology of the Medusæ, it is needless that I should here dwell on this, the most meritorious of its existing memoirs.

Understanding that Mr. Schäfer is still prosecuting his investigations, and observing that the last-mentioned memoir is of so masterly a character, I have this year thought it desirable to abandon my original intention of concluding this series of papers with the results of a systematic investigation of the histology of the Medusæ. Entertaining not the smallest doubt as to the accuracy of the results which have been obtained by the laborious inquiry of Messrs. HERTWIG, I felt that time would be more profitably employed in extending the lines of my physiological researches than in confirming morphological work which, to my mind, did not stand in need of confirmation. Feeling thus that the histology of the Medusæ in all its leading features had been, relatively speaking, exhausted, I this year devoted myself to continuing and concluding the physiological work with which I began. I may observe, however, that when opportunity again permits, I hope to investigate the histology of those species of naked-eyed Medusæ on which I have mainly experimented; for with the exception of *Sarsia,* these particular species have not as yet been submitted to microscopical examination by any of the more recent observers. Should this investigation tend to show that the nervous tissues of these particular species present any peculiarities which have not been described as occurring in the numerous species of naked-eyed Medusæ which have been so carefully examined by Messrs. HERTWIG, I shall communicate the results of my investigation in a supplementary paper to the Royal Society. But meanwhile I communicate this additional instalment of physiological results under the title "Concluding Observations on the Locomotor System of Medusæ." Possibly enough in future years various questions concerning the physiology of nerve and muscle may arise, for the solution of which the primitive nerve and muscle tissues of the Medusæ will afford valuable material; but for the present, it seems to me, this paper will pretty well exhaust the physiology of these tissues, and for this reason I have given it a title which indicates that the series of communications of which it is a member may now be considered as closed.[*]

I. STIMULATION.

§ 1. *Artificial Rhythm.*—(*a.*) I shall devote the whole of this lengthy section to a full consideration of what in my previous paper I termed "Artificial Rhythm." It will be remembered that in my previous paper this subject was alluded to in a footnote, of which the following is a copy:—"As the present communication was originally accepted by the Royal Society, there here followed a lengthy subsection on 'Artificial Rhythm.' A condensed epitome of the main facts which were detailed in that subsection may be found in the already published abstract of this paper in the 'Proceedings of the Royal Society.' But as the new theory of ganglionic action, which it is the main object of this passage to disclose, appears to me a theory of sufficient importance to demand its final publication in as complete a form as possible, I have sought and obtained the permission of the Royal Society to withdraw this subsection from the proof. . . . . The facts of which it treats form in themselves a connected body distinct from all the other facts which are detailed in the rest of this communication; so that, while their omission from the present paper does not entail any furthur modification of the latter, they will admit of being introduced *en masse* in my next paper. When, therefore, I shall have completed the further experiments which in this connexion I have devised, I shall hope to communicate to the Royal Society, in a single connected series, all the facts and inferences which bear upon this subject."

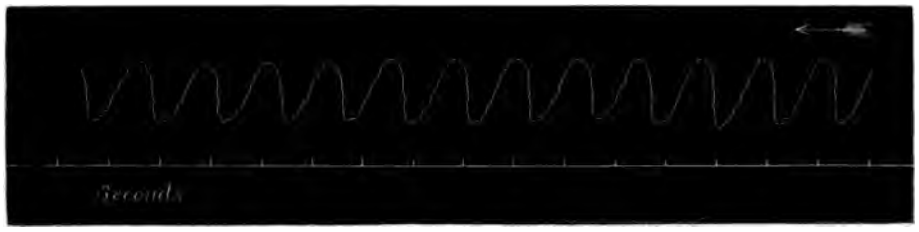
---

[*] The effects of several poisons still remain to be investigated, and I have still in view a number of experiments which have not yet been carried out; but as it is uncertain whether the results of these further observations will be worth publishing, I have thought it desirable thus to indicate that the work, so far as it has now gone, may be regarded as a completed structure.

Although I have not even yet completed all the experiments which it is desirable to make with reference to this subject, I have this year so far added to their number that I feel the research is now sufficiently ripe for publication. As already stated, an epitome of the main facts regarding artificial rhythm has previously appeared in the 'Proceedings of the Royal Society'; but of course it will now be necessary to give a more detailed account of these facts, no less than of those which I have more recently observed. Stating, then, all the facts as concisely as possible, they are as follows.

(*b*.) If the umbrella of *Aurelia aurita* has been paralyzed by the removal of its lithocysts, and if it is then subjected to faradaic stimulation of minimal intensity, the response which it gives is not tetanic, but rhythmic. The rate of this artificial rhythm varies in different specimens, but the limits of variation are always within those which are observed by the natural rhythm of different specimens. The artificial rhythm is not in every case strictly regular; but by carefully adjusting the strength of the current, and by shifting the electrodes from one part of the tissue to another until the most appropriate part is ascertained, the artificial rhythm admits in most cases of being rendered tolerably regular, and in many cases as strictly regular as is the natural rhythm of the animal. To show this, I append a tracing of the artificial rhythm, which may be taken as a fair sample of the most perfect regularity that can be obtained by minimal faradaic stimulation.*

Fig. 1.



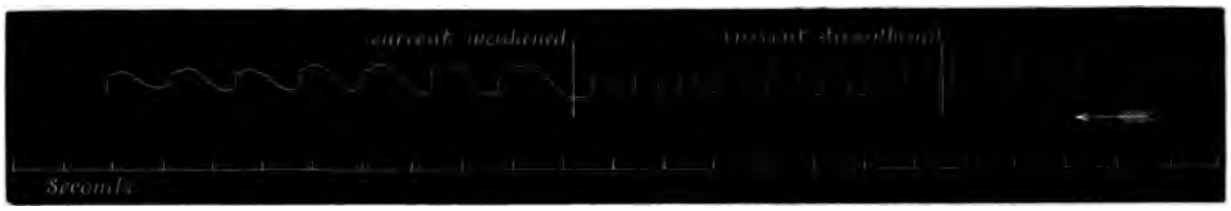
This artificial rhythm may be obtained with a portion of irritable tissue of any size, and whether a large or a small piece of the tissue employed be included between the electrodes.

---

* This and all the subsequent tracings I obtained by the method already described in my previous paper.

(*c.*) As the fact of this wonderfully rhythmic response to faradaic irritation was quite unexpected by me, and as it seemed to be a fact of great significance, I was led to investigate it in as many of its bearings as time permitted. First, I tried the effect on the rhythm of progressively intensifying the strength of the faradaic current. I found that with each increment of the current the rate of the rhythm was increased; and this up to the point at which the rhythm began to pass into tetanus due to summation of the successive contractions. But between the slowest rhythm obtainable by minimal stimulation and the most rapid rhythm obtainable before the appearance of tetanus, there were numerous degrees of rate to be observed. I here append another tracing to show the effect on the rate of the rhythm of alterations in the strength of the current (fig. 2).

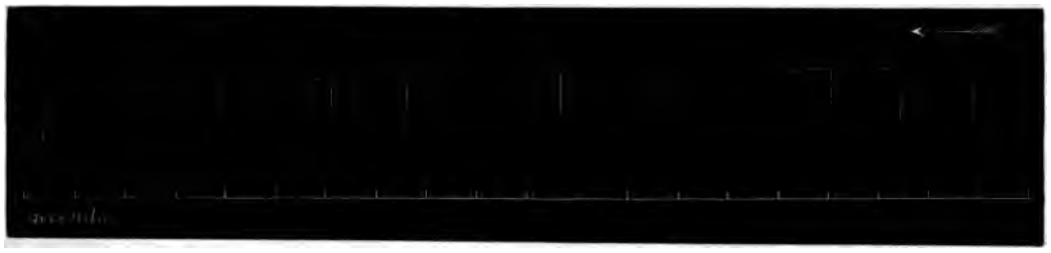
Fig. 2.



It will also be observed from this tracing that, in consequence of the current having been strengthened slightly beyond the limit within which strictly rhythmic response was attainable, the curves in the middle part of the tracing, where the current was strengthened, are slightly irregular. This irregularity is of course due to the first appearance of tumultuous tetanus. If the faradaic stimulation had in this case been progressively made still stronger, the irregularity would have become still more pronounced up to a certain point, when it would gradually have begun to pass into more persistent tetanus. But as in this case, instead of strengthening the current still further, I again weakened it to its original intensity, the rhythm immediately returned to its original rate and regularity.

(*d.*) The next point I investigated was the persistency of any given rate of rhythm under faradaic stimulation of constant intensity. I found this persistency to vary with the vigour of the specimens experimented on, as well as with other conditions which I need not wait to specify; but in all cases the persistency was much greater than I had expected. For the sake of brevity I shall confine myself in this connexion to giving the particulars of one experiment. A detached quadrant of *Aurelia*, whose normal rate of rhythm was 18 per minute, had its lithocysts removed, and then formed the subject of the experiment. The faradaic stimulation supplied to it was of slightly more than minimal intensity, and the resulting rhythm was perfectly regular at the rate of ·38 per minute. This absolutely perfect rhythm continued, without once faltering or changing its rate, for an hour and a quarter (tracing, fig. 1). At the end of this time single omissions in the series of contractions began to occur at long intervals—*i.e.*, after every one or two hundred rhythmic contractions, a short pause would occur of a length equal to the time occupied by a single contraction. These omissions then gradually became more and more frequent, till finally they gave rise to well-marked irregularity (tracing, fig. 3). Shortly afterwards all movements entirely ceased, and failed to be aroused even by strengthening the current. This cessation of response through exhaustion took place rather more than an hour and a half after the first commencement of stimulation. I then allowed the tissue to remain quiescent for fifteen minutes, and again stimulated, but no response was given. After the lapse of a further fifteen minutes response was given to the faradising current at the moment of its closure, but not during its passage. I now gave the tissue another quarter of an hour's rest, and then obtained responses to faradisation of a partly rhythmic nature. Lastly, on waiting for another quarter of an hour—*i.e.*, one hour from the time at which the persistent stimulation for an hour and a half had ceased—the movements in response to faradisation were strictly rhythmic, although for the same strength of current the rate of their rhythm was now slower than before—viz.: 28 per minute. (But on placing another pair of electrodes on another part of the tissue, the original rate of 38 per minute could be obtained.) I now again persistently tetanised for an hour, and throughout that time obtained perfectly regular and sustained rhythm of 28 per minute. Exhaustion had not again supervened when the observation terminated.

Fig. 3.



(*e.*) Such being the facts, the question arises as to their interpretation. At first I was naturally inclined to suppose that the artificial rhythm was due to a periodic variation in the strength of the stimulus, caused by some slight breach of contact between the terminals and the tissue on each contraction of the latter. This supposition, of course, would divest the phenomena in question of all physiological meaning, and I therefore took pains in the first instance to exclude it. This I did in two ways: first, by observing that in many cases (and especially in *Cyanæa capillata*) the rate of the rhythm is so slow that the contractions do not follow one another till a considerable interval of total relaxation has intervened; and second, by placing the terminals close together so as to include only a small piece of tissue between them, and then firmly pinning the tissue all round the electrodes to a piece of wood placed beneath the Medusa. In this way the small portion of tissue which served as the seat of stimulation was itself prevented from moving, and therefore the rhythmic motions which the rest of the Medusa presented cannot have been due to any variations in the quality of the contact between the electrodes and this stationary seat of stimulation.

(*f.*) Any such merely mechanical source of fallacy being thus, I think, excluded, we are compelled to regard the facts of artificial rhythm as of a purely physiological kind. The question, therefore, as to the explanation of these facts becomes one of the highest interest, and the hypothesis which I have framed to answer it is as follows. Every time the tissue contracts it must as a consequence suffer a certain amount of exhaustion, and therefore must become slightly less sensitive to stimulation than it was before. After a time, however, the exhaustion will pass away, and the original degree of sensitiveness will thereupon return. Now the intensity of faradaic stimulation which is alone capable of producing rhythmic response, is either minimal, or but slightly more than minimal, in relation to the sensitiveness of the tissue when fresh. Consequently when the degree of this sensitiveness is somewhat lowered by temporary exhaustion, the intensity of the stimulation becomes somewhat less than minimal in relation to this lower degree of sensitiveness. The tissue, therefore, fails to perceive the presence of the stimulus, and consequently fails to respond. But so soon as the exhaustion is completely recovered from, so soon will the tissue again perceive the presence of the stimulus; it will therefore again respond, again become temporarily exhausted, again fail to perceive the presence of the stimulus, and again become temporarily quiescent. Now it is obvious that, if this process occurs once, it may occur an indefinite number of times; and as the conditions of nutrition, as well as those of stimulation, remain constant, it is manifest that the responses may thus become periodic.

(*g.*) In order to test the truth of this hypothesis, I made the following experiments. Having first noted the rate of the rhythm under faradaic stimulation of minimal intensity, without shifting the electrodes or altering the intensity of the current, I discarded the faradaic stimulation, and substituted for it single induction shocks thrown in with a key. I found, as I had hoped, that the maximum number of these single shocks which I could thus throw in in a given time, *so as to procure a response to every shock*, corresponded with the number of contractions which the tissue had previously given during a similar interval of time when under the influence of the faradaic current of similar intensity. To make this quite clear I shall describe the whole course of one such experiment. The deganglionated tissue under the influence of minimal faradaic stimulation manifested a perfectly regular rhythm of thirty contractions per minute, or one contraction in every two seconds. While the position of the platinum electrodes and the intensity of the current remained unchanged, single induction shocks were now administered with a key at any intervals which might be desired. It was found that if these single induction stimuli were administered at regular intervals of two seconds or more, the tissue responded to every stimulus; but that if the stimuli were thrown in more rapidly than this, the tissue did not respond to every stimulus, but only to those that were separated from one another by an interval of at least two seconds' duration. Thus, for instance, if the shocks were thrown in at the rate of one a second, the tissue only, but always, responded to every alternate shock. And similarly, as just stated, if any number of shocks were thrown in, the tissue only responded once in every two seconds. Now, as this rate of response precisely coincided with the rate of rhythm previously shown by the same tissue under the influence of faradaic stimulation of the same intensity, the experiment tended to verify the hypothesis which it was designed to test.

(*h.*) But in order to test this hypothesis still more effectually, I conducted a number of experiments on a slightly different plan. As already stated, the intensity of the faradaic stimulation has a marked influence on the rate of the rhythm, up to the point at which the rhythmic effect of such stimulation begins to become lost in the tetanic effect. Well, by choosing at random any strength of faradaic stimulation between the limits where rhythmic response occurred, and by noting the rate of the rhythm under that strength of stimulation, I found that I was generally able to predict the precise number of single induction-shocks which I could afterwards afford to throw in with the same strength of current, so as to procure a response to every shock—this number, of course, corresponding with the rate of the rhythm previously manifested under the faradaic stimulation. I say "generally," because this experiment was not invariably successful—any more, I may add, than was the first-mentioned and almost identical experiment in which minimal stimulation was employed (see paragraph *g*). Nevertheless, the exceptions were but slight, and always on the side of somewhat fewer contractions occurring in a given time in response to the single shocks than in response to faradaic stimulation. And as this is the direction in which we should expect, from the principle of the summation of stimuli, that slight deviations from the ordinary rule would occur, if they occurred at all, I do not think that the fact of their occurrence tends to impair the confirmation which these experiments certainly afford to the hypothesis we are considering.

(*i.*) Indeed, it seems to me that this hypothesis is so fully substantiated by these experiments, that I feel it is almost superfluous to adduce another experiment having the same tendency. Nevertheless, as the facts which the hypothesis is intended to explain are facts of considerable importance in themselves, and as the hypothesis in question will be shortly employed as tending to further the theory of ganglionic action in general, it is desirable that I should state all the experiments by which I have hitherto endeavoured to test it.

The experiment which I am about to describe cannot be understood without a preliminary account of certain properties of the contractile tissues of *Aurelia*, which as yet I have not described. The properties in question all appear to arise from the following fact—namely, that the contractile tissue admits of great variations in the vigour of its contractions. In my last paper, while treating of the summation of stimuli, it was fully explained that the force of responsive contraction in *Aurelia* varies greatly in the different parts of a "staircase" series; but quite apart from this principle of the summation of stimuli, enormous variations in the force of contraction may often be observed under the influence of stimuli of varying intensities. This relation, within certain limits, between the intensity of a stimulus and the force of the responding contraction, is, of course, to be observed more or less in the case of all excitable tissues; but in the case of the excitable tissues of *Aurelia* the limits of variation in this respect are extreme. As it will be necessary to discuss this subject at length in the next division of the present paper, it will be sufficient in this place merely to state the principal fact, viz.: that the excitable tissues of *Aurelia* admit of giving either a very strong or a very weak response according to the intensity of the stimulus in relation to the excitability of the tissue. In view of this fact it occurred to me that a valuable means was afforded of testing the hypothesis whereby I had sought to explain the phenomena of artificial rhythm. For, if this hypothesis were sound, we might expect that by employing single stimuli of uniform and slightly more than minimal intensity, and by throwing them in at a rate slightly more rapid than is required to produce a strong response to every stimulus, the strong responses would alternate with weak responses. For after the occurrence of each strong response the restoration of excitability (or recovery from exhaustion) must be a gradual process, so that if the stimulus is thrown in when this restoration has proceeded to a certain, though not to its full extent, the stimulus ought to be strong enough to evoke a feeble response. This feeble response, however, would not greatly exhaust the tissue, so that when the next stimulus is thrown in it would encounter a tissue more excitable than did its predecessor, and consequently would give rise not to a weak contraction, but to a strong one. In order to test this idea I selected a specimen of *Aurelia* which exhibited marked differences in the strength of its contractile waves under the influence of stimuli of different intensities. Employing single induction shocks of slightly more than minimal intensity, and throwing them in at twice the rate that was required to produce a strong response to every shock, I found that midway between every two strong responses there was a weak response. In other words, a stimulus of uniform intensity gives rise alternately to a strong and to a weak contraction, as shown in the appended tracing. It will be observed that in this tracing each large curve represents

Fig. 4.

the whole time occupied by the strong contraction, the latter beginning at the highest point of the curve on the left hand side in each case. The effect of the weak contraction is that of momentarily interrupting the even sweep of diastole after the strong contraction, and therefore the result on the tracing is a slight depression in the otherwise even curve of ascent. Lest any doubt should arise from the smallness of the curves representing the weak contractions that the former are in some way accidental, I may draw attention to the fact that the period of latent stimulation is the same in the case of all the curves. To render this apparent I have placed marks below the smaller curves, which show in each case the exact point where the depressing effect of these smaller curves on the ascending sweeps of the larger curves first become apparent, *i.e.*, the point at which the feeble contraction begins. Now, what I wish to be gathered from the whole tracing is this: If the strength of the induction shocks had been much greater than it was, *all* the contractions would have become strong contractions, and tetanus would have been the result. But, as the strength of the induction shocks was only slightly more than minimal, the exhaustion consequent on every strong contraction so far diminished the irritability of the tissue that when, during the process of relaxation, another shock *of the same intensity* was thrown in, the stimulus was only strong enough, in relation to the diminished irritability of the partly recovered tissue, to cause a feeble contraction. And these facts tend still further to substantiate the hypothesis whereby I have sought to explain the phenomena of artificial rhythm.

(*j.*) Upon the whole, then, I think that this hypothesis may be properly accepted as a full explanation of the artificial rhythm; but whether or not the hypothesis is thus accepted, the facts which it is adduced to account for remain as remarkable and significant as they were before. That these facts are remarkable no one will dispute; the degree of their significance will depend on the degree in which the inferences I draw from them appear to other physiologists of the same legitimate character as they appear to myself. These inferences will perhaps be considered somewhat bold, as they certainly must be considered premature; and were it not that I think physiology may gain, while it cannot lose, from healthy speculation on such facts as those which are now before us, I should prefer to postpone for the present any further treatment of this subject. Nevertheless, as all the experiments I have hitherto made tend, without exception, to substantiate my views, it will at least be interesting to describe those experiments in connexion with the theory which led to their being tried. The theory in question is a theory of ganglionic action in general.

I think every one will feel it to be obviously true that if ganglionic action is ever to receive any considerable elucidation, the Medusæ are by far the most promising structures to yield it. In them we have the first observed appearance, in the ascending scale of animal life, both of nervous and of muscular elements. These elements are, therefore, presumably here present in their simplest or most primitive forms. Again, the organisms are of all sizes, from less than that of a pea to more than that of a parasol; their endurance under experimentation is as great as can be desired; in their different species they present countless modifications of structure, and, in general, they present what one may regard as a typically perfect class of tissues for the purposes of that special division of physiological inquiry to which I have referred. If, therefore, I have been able to detect any peculiarities in the functions of these primitive ganglionic tissues which have not hitherto been detected in the ganglionic tissues of higher animals, I cannot but consider it legitimate to attach a high degree of theoretical importance to such peculiarities; for I think it then becomes legitimate to feel that such peculiarities ground a strong analogy in favour of the belief, that whether or not similar functions admit of being experimentally proved in ganglionic tissues elsewhere, such functions are probably present wherever ganglionic action can be shown to occur, and that however hopelessly these functions may be obscured by the new and complex relations that in other cases are involved, they nevertheless constitute in every case some among the fundamental properties of ganglionic tissue.

Now, viewed in this manner, I think that the strictly rhythmic action of the paralyzed swimming-bell of *Aurelia* in answer to constant stimulation is a fact of the highest significance; for here we have a tissue wholly, or almost wholly, deprived of its centres of spontaneity, yet pulsating as rhythmically in answer to artificial stimulation as it previously did in answer to ganglionic stimulation.* Does not this tend to show that for the production of the natural rhythm the presence of the ganglionic element is non-essential; that if we merely suppose the function of this element to be that of supplying a constant stimulus of a low intensity without in addition supposing the presence of any special resistance mechanism to regulate the discharges, the periodic sequence of systole and diastole would assuredly result; and, therefore, that the *rhythmical* character of the natural swimming motions is dependent, not on the peculiar relations of the ganglionic, but on the primary qualities of the contractile tissue? Or, if we do not go so far as this (and, as I shall conclude by explaining, I am not myself inclined to go so far), must we not at least conclude that the natural rhythm of these tissues is not *exclusively* due to any mechanism whereby the discharges of the ganglia are interrupted at regular intervals; but that whether these discharges are supposed to be interrupted or continuous, the natural rhythm is probably in a large measure due to the same cause as the artificial rhythm—viz.: in accordance with our previous hypothesis, to the alternate exhaustion and recovery of the excitable tissues? This much, at least, must be allowed even by the most cautious of critics, viz.: that if, as current views respecting the theory of rhythm would suppose, it is exclusively the ganglionic element which in the unmutilated *Aurelia* causes the rhythm of the swimming-motions by intermittent stimulation, surely it becomes a most unexpected and unaccountable fact, that after the removal of this element the contractile tissues should still persist in their display of rhythm under the influence of constant stimulation. At any rate, no one, I think, will dispute that the facts which I have adduced justify us in reconsidering the whole theory of rhythm as due to ganglia.

---

* It will not be forgotten that there are a multitude of ganglion-cells distributed throughout the contractile tissues of the Medusæ, but forasmuch as these are comparatively rarely instrumental in originating stimulation, I think it is probable that artificial stimulation acts directly on the contractile tissues, and not through the medium of these scattered cells.

(*k.*) As I have already said, I am not inclined to deny that there is probably some truth in the current theory of rhythm as due to ganglia; I merely wish to point out distinctly that this theory is inadequate, and that in order to cover all the facts, it will require to be supplemented by the theory which I now propose. The current theory of rhythm as due to ganglia attributes the whole of the effect to the ganglionic element, and thus fails to meet the facts of a rhythm which is artificially produced after the ganglionic element has been removed. It also fails to meet a number of other facts of the first importance. For it is beyond all doubt that rhythmic action of the strictest kind occurs in an innumerable multitude of cases where it is quite impossible to suppose anything resembling ganglia to be present. Not to mention such cases as the Snail's heart, where the most careful scrutiny has failed to detect the least vestige of ganglia, but to descend at once to the lowest forms of animal and vegetable life, rhythmic action may here be said to be the rule rather than the exception. The beautifully regular motions observable in some Algæ, Diatomaceæ, and Ocillatoriæ, in countless numbers of Infusoria, antherozoids, and spermatozoa, in ciliary action, and even in the petioles of *Hedysarum gyrans*, are all instances (to which many others might be added) of rhythmical action where the presence of ganglia is out of the question. Again, in a general way, is it not just as we recede from these primitive forms of contractile tissue that we find rhythmic action to become less usual? And, if this is so, may it not be that those contractile tissues which in the higher animals manifest rhythmic action are the contractile tissues which have longest retained their primitive endowment of rhythmicality? To my mind it seems hard to decide in what respect the beating of a Snail's heart differs from that of the pulsatile vesicles of the Infusoria; and I do not think it would be much easier to decide in what essential respect it differs from the beating of the Mammalian heart. The mere fact that the presence of ganglia can be proved in the one case and not in the other, seems to me scarcely to justify the conclusion that the rhythm is in the one case wholly dependent, and in the other as wholly independent, of the ganglia. At any rate, this fact, if it is a fact, is not of so self-evident a character as to recommend to us the current theory of ganglionic action on *à priori* grounds.

(*l.*) Coming, then, to experimental tests, we have already seen that in the deganglionated swimming organ of *Aurelia aurita*, rhythmic response is yielded to constant faradaic stimulation of low intensity. The next question, therefore, which presents itself in relation to our subject, is as to whether other modes of constant stimulation elicit a similar response. Now, in a general way, I may say that such is the case, although I have chosen faradaic stimulation for special mention, because, in the first place, its effect in producing rhythmic action is the most certain and precise, and, in the next place, the effects of administering instantaneous shocks at given intervals admit of being compared with the effects of constant faradaic stimulation better than with any other kind of constant stimulation. Nevertheless, as just stated, other modes of constant stimulation certainly have a more or less marked effect in producing rhythmic response. As mentioned incidentally in my first paper, the constant current, during the whole time of its passage, frequently has this effect in the case of the paralyzed nectocalyx of *Sarsia* (p. 282). At the time I published that paper I had not observed the rhythm due to faradaic stimulation, and so suggested that the response in this case was "perhaps due to the hydrogen bubbles acting as stimulants to contraction." Now, however, I am inclined to think, from the analogy supplied by the effects of the faradaic current, that the rhythmic response to the constant current is not due to any such accidental cause.

With regard to other modes of stimulation, I may in the first instance refer to Division III., §§ 1 and 2, of my second paper. It is there stated: "When the paralyzed swimming organ of *Aurelia aurita* is stimulated by a single mechanical irritation, it often responds with two, and more rarely with three contractions, which are separated from one another by an interval of about the same length as the normal diastole of the unmutilated animal. . . . . Dilute spirit, or other irritant, when dropped on the paralyzed swimming organ of *Aurelia aurita,* often gives rise to a whole series of rhythmical pulsations, the systoles and diastoles following one another at about the same rate as is observable in the normal swimming motions of the unmutilated animal."

From this it will be seen that both in the case of mechanical and of chemical stimulation the same tendency to the production of rhythmic response on the part of the paralyzed tissues of *Aurelia* may be observed, as in the case of electrical stimulation. The principal differences consist in the rhythm being much less sustained in the former than in the latter case. But by experimenting on other species of Medusæ I have been able to obtain artificial rhythm in response to mechanical and chemical stimulation of a much more sustained character than that which, under such modes of stimulation, occurs in *Aurelia.* I have no explanation to offer why it is that some species or some tissues present so much more readiness to manifest sustained rhythm under certain modes of stimulation, and less readiness to manifest it under other modes, than do other species or tissues. Probably these differences depend on some peculiarities in the irritability of the tissues which it is hopeless to ascertain; but, in any case, the fact remains that while *Aurelia, Cyanæa,* and the covered-eyed Medusæ generally, are the best species for obtaining artificial rhythm under the influence of faradaic stimulation, some of the naked-eyed Medusæ are the best species for obtaining it under the influence of the constant current, and also under that of mechanical and chemical stimulation. I have already spoken of this effect of the constant current in the case of *Sarsia;* I shall now proceed to describe the effects of mechanical and chemical stimulation on the same species.

It is but rarely that artificial rhythm can be produced in the paralyzed nectocalyx of *Sarsia* by means of mechanical stimulation, but in the case of the polypite, a very decided, peculiar, and persistent rhythm admits of being produced by this means.

In this particular species the polypite never exhibits any spontaneous motion after the ganglia of the nectocalyx have been removed. But if it be nipped with the forceps, or otherwise irritated, it contracts strongly and suddenly; it then very slowly and gradually relaxes until it has regained its original length. After a considerable interval, and without the application of any additional stimulus, it gives another single, sudden, though slight contraction, to be again followed by gradual relaxation and a prolonged interval of repose, which is followed in turn by another contraction, and so on. These sudden and well-marked contractions occur at intervals of many seconds, and show a decided tendency to rhythmic periodicity, though the rhythm is not always perfectly exact. This intensely slow rhythm as the result of injury may continue for a long time, particularly if the injury has been of a severe character. There can be no doubt, therefore, that the mechanical (or other) injury in this case acts as a source of constant irritation; so that here again we have evidence of rhythmic action independent of ganglia and caused by the alternate exhaustion and recovery of contractile tissues.*

With regard to artificial rhythm caused by chemical stimuli, by far the most conspicuous instance that I have observed is that of the paralyzed nectocalyx of *Sarsia*. It will be remembered that in my first paper, under the heading "STIMULATION," I drew prominent attention to "a highly peculiar motion of a flurried, shivering character," which is manifested by this organ when its marginal ganglia have been removed and it is exposed to the influence of faintly acidulated water. As previously remarked, at the time when this description was published I had not observed the phenomena of artificial rhythm in any of the other cases in which I subsequently observed it, and I was therefore completely at a loss to interpret the facts in the case which I described. I was thus under the necessity of stating the fact merely as "a highly remarkable phenomenon, and one which I was quite unable to explain, though I was persuaded that it was a phenomenon well meriting the attention of physiologists." Now, however, when read in the light of the foregoing facts, there can be no doubt that the present one falls into its place very satisfactorily; it is an additional and very valuable instance of the display of artificial rhythm under the influence of a constant stimulus of low intensity. For the shivering motions of the mutilated nectocalyx under these circumstances are most unmistakably of a rhythmic nature. Viewed from a little distance, indeed, these motions are not distinguishable from the natural swimming motions of the unmutilated animal, except that, not being of quite such a powerful character, they are not so effective for locomotion. Viewed more closely, however, it may frequently be seen that the whole bell does not contract simultaneously, but that, as it were, clouds of contraction pass now over one part and now over another. Still, whether the contractions are partial or universal, they are always more or less rhythmical. As this is the only case that has ever been observed of rhythm as due to a constant chemical stimulus, I have studied it with much care, and shall now give a full description of what appears to me an important body of physiological facts.

---

* We may pretty safely conclude that ganglia are altogether absent in the polypite of *Sarsia*, not only because SCHULTZ has failed to detect them in this organ microscopically, but also because of the complete absence of spontaneity which it manifests. I may here mention that this case of the polypite of *Sarsia* is precisely analagous to another which I have observed in a widely different tissue, namely, the tongue of the Frog. Here, too, the presence of ganglion cells has never been observed microscopically, though specially sought for by Dr. KLEIN and others. Yet under the influence of mechanical and other modes of stimulation, I find that I am able to make the excised organ pulsate as rhythmically as a heart. As however I intend to devote a separate paper to the subject of artificial rhythm in tissues other than those of the Medusæ, I shall not here enter into details with regard to the singularly beautiful instance which I have just mentioned; but I have felt that it is desirable to point out in this place the analogy which it presents to the case of the polypite of *Sarsia*.

Ten to twenty drops of acetic acid having been added to 1000 c.c. of sea-water, and the paralyzed bell of *Sarsia* having been placed in the mixture, an interval of about half a minute will elapse before any movement begins. Sooner or later, however, the artificial rhythm is sure to be induced, and it will then continue for a variable time—occasionally as long as an hour, and generally for a considerable number of minutes. After it ceases it may often be made to recommence, either by adding a few more drops of acid to the sea-water, or by supplying an additional stimulus to the bell by nipping it with the forceps. Eventually, however, all movement ceases, owing to the destruction of irritability by the action of the acid. By this time the whole inner surface of the bell has become strongly opalescent, owing to the destructive influence of the acid on the epithelial cells which overspread the irritable tissues. The latter fact is worth mentioning, because in no case does the artificial rhythm set in until this opalescence has begun to show itself; and as this opalescence is but an optical expression of the damage which the epithelial coat is undergoing, the explanation of the time which elapses after the first immersion of the bell in the acidulated water and the commencement of the artificial rhythm, no doubt is that during this time the acid has not obtained sufficient access to the excitable tissues to serve as an adequate stimulus.

During the soaking stage of the experiment—*i.e.*, before the artificial rhythm begins—the excitability of the tissues may be observed progressively and abnormally to increase; for soon after the soaking stage begins, in response to a single nip with the forceps the bell may give two or three locomotor contractions, instead of a single one, as is *invariably* the case with a paralyzed bell of *Sarsia* in normal water. Later on during the soaking stage four or five successive contractions may be yielded in response to a single mechanical stimulus, and shortly after this a whole bout of rhythmic contractions may be started by the same means. Indeed, in some cases the artificial rhythm in acidulated water requires such a single additional stimulus for its inauguration—the shivering movement failing to begin spontaneously, but beginning immediately upon the application of the additional stimulus. Similarly, after the shivering movements have ceased, a fresh bout may very often be started by again giving the motionless nectocalyx a single stimulation. The interpretation of these facts would seem to be that the general irritability of the excitable tissues is exalted by the universal and constant stimulus supplied by the acid to an extent that is just bordering on that which gives rise to rhythmic movement, so that when the violent contraction is given in response to the mechanical stimulus, the disturbance serves to start the rhythmic movement.

If a paralyzed nectocalyx, while manifesting its artificial rhythm in acidulated sea-water, be suddenly transferred to normal sea-water, the movements do not cease immediately, but continue for a considerable time. This fact can easily be explained by the very probable, and indeed almost necessary, supposition that it takes some time after the transference to the normal sea-water for the acid to be washed out from contact with the excitable tissues. Sooner or later, however, as we should expect, in the normal sea-water the rhythmic movements entirely cease, and the bell becomes quiescent, with a normal irritability as regards single stimuli. If it be now again transferred to the acidulated water, after a short interval the rhythmic movements will again commence—and so on during several repetitions of this experiment, until the irritability of the tissues has finally become destroyed by the influence of the acid.

Other chemical irritants which I have tried produce substantially similar effects on the paralyzed bells of *Sarsia*. I shall therefore only wait to describe the influence of one of these irritants, the action of which in some respects differs from that of acids, and which I have found to be one of the most unfailing in its power to produce the rhythmic movements in question. This irritant is glycerine, and in order to produce its full effect it requires to be added to the sea-water in about the proportion of 5 per cent. The manifestation of artificial rhythm in solutions of this kind is quite unfailing. It begins after an exposure of from 15 to 30 seconds, and continues for a variable number of seconds. It generally begins with powerful contractions, of a less shivering character than those which are produced by acids, and therefore still more closely resembling the normal swimming motions of the unmutilated animal. Sometimes, however, the first manifestation of the artificial rhythm is in the form of a few gentle rhythmic contractions, to be followed by a few seconds of quiescence, and then by the commencement of the sustained bout of strong contractions. In either case, when the bout of strong contractions sets in, the rate of the rhythm becomes progressively and rapidly increased, until in a few seconds it runs up into incipient tetanus. The rate of the rhythm still quickening, the tetanus rapidly becomes more and more pronounced, till at last the bell becomes quiescent in tonic spasm.*

If the bell is still left in the glycerine solution nothing further happens; the tissues die in this condition of strong systole. But if the bell be transferred to normal sea-water immediately after, or still better, slightly before the tonic spasm has become complete, an interesting series of phenomena present themselves. The spasm persists for a long time after the transference without undergoing any change—the length of this time depending on the stage in the severity of the spasm at which the transference is made. After this time is passed, the spasm becomes less pronounced than it was at the moment of transference, and a reversion takes place to the rhythmic contractions. The spasm may next pass off entirely, leaving only the rhythmic contractions behind. Eventually these too fade away into quiescence; but it is remarkable that they leave behind them a much more persistent exaltation of irritability than is the case with acid. For in the case of glycerine the paralyzed bell which has been exposed to the influence of the irritant and afterwards become quiescent in normal sea-water, will often continue for hours to respond to single stimuli with a bout of rhythmic contractions. This effect of glycerine in producing an extreme condition of exalted irritability is also rendered apparent in another way. For if, during the soaking stage of the experiment—*i.e.*, before the first of the rhythmic contractions has occurred—the bell be nipped with the forceps, the effect may be that of so precipitating events that the whole of the rhythmic stages are omitted, and the previously quiescent bell enters at once into a state of rigid tonic spasm. This effect is particularly liable to occur after prolonged soaking in weak solutions of glycerine.

---

* Sometimes, however, the order of events is slightly different, the advent of the spasm being more sudden, and followed by a mitigation of its severity, the bell then exhibiting what is more usually the first phase of the series—namely, the occurrence of the locomotor-like contractions. Occasionally, also, rhythmical shivering contractions may be seen superimposed on the general tonic contraction, either in a part or over the whole of the contractile tissues.

As in the case of stimulation by acid, so in that of stimulation by glycerine, the artificial rhythm never begins in any strength of solution until the epithelial surface has become opalescent to a considerable degree.

In the case of stimulation by glycerine the behaviour of the polypite is more unequivocal than it is in the case of stimulation by acid. I have therefore reserved till now my description of the behaviour of this organ under the influence of constant chemical stimulation. This behaviour is of a very marked though simple character. The polypite is always the first to respond to the stimulation—its retraction preceding the first movements of the bell by an interval of several seconds, so that by the time the bell begins its rhythmic response the polypite is usually retracted to its utmost. The initial response of the polypite is also rhythmic; and the rhythm which it manifests—especially if the glycerine solution be not over strong—is of the same slow character which has already been described as manifested by this organ when under the influence of mechanical stimulation. The rhythm, however, is decidedly quicker in the former than in the latter case.

Lastly, with regard to the marginal ganglia, it is to be observed that in the case of all the chemical irritants I have tried, if unmutilated specimens of *Sarsia* be immersed in a sea-water solution of the irritant, which is of a sufficient strength to evoke artificial rhythm in paralyzed specimens, the spontaneity of the ganglia is destroyed in a few seconds after the immersion of the animals—*i.e.*, in a shorter time than is required for the first appearance of artificial rhythm. Consequently, whether the specimens experimented upon be entire or paralyzed by removal of their margins, the phenomena of artificial rhythm under the influence of chemical stimulation are the same. But although the spontaneity of the ganglia disappears before the artificial rhythm sets in, such is not the case with the reflex activity of the ganglia; for on nipping a tentacle of the quiescent bell before the artificial rhythm has set in, the bell will give a single normal response to the stimulation.

Hence, in historical order, on dropping an unmutilated specimen into a solution of glycerine of the strength named, the usual succession of events to be observed is as follows:—First, increased activity of the normal swimming-motions, to be quickly followed by a rapid and progressive decrease of such activity, till in about fifteen seconds after the immersion total quiescence supervenes. Four or five seconds later the polypite begins to retract by rhythmical twitches, the rate of this rhythm rapidly increasing until it ends in tonic contraction. When the polypite has just become fully retracted—or very often a little earlier—the bell suddenly begins its forcible and well-pronounced rhythmic contractions, which rapidly increase in their rate of rhythm, until they coalesce into a vigorous and persistent spasm. If the animal be now restored to normal sea-water, spontaneity will return in a feeble manner; but there is always afterwards a great tendency displayed by the bell to exhibit shivering spasms instead of normal swimming movements in response to natural or ganglionic stimulation. And, as already observed, this peculiarity of the excitable tissues is also well marked in the case of the artificial stimulation of deganglionated specimens under otherwise similar circumstances.

The only facts which still remain to be considered regarding these shivering movements of paralyzed *Sarsia* are those which have reference to electrical stimulation. It will be remembered that in my first paper (*loc. cit.*) I described these shivering movements as ensuing, not only when the paralyzed bells are immersed in chemical stimulants, but also when they are immersed in normal sea-water immediately after having been submitted to electrical stimulation. As the passage in my former paper which describes this effect of electrical stimulation is a short one, I will here transcribe it:—

"When the swimming-bell of *Sarsia* has had its margin removed, and so, as proved by hundreds of similar experiments, has been entirely deprived of its locomotor centres, nevertheless, in response to electrical stimulation, instead of giving a single contraction to make or break, it may begin a highly peculiar motion of a flurried, shivering character, which lasts without intermission for periods varying from a few seconds to half an hour. I never but once saw a similar motion in the perfect animal, and this was in the case of a specimen which was dying from having been poisoned with iron rust. The motion, I think, can be explained by supposing that the various parts of the muscle-layer are contracting without co-ordination; but why they should sometimes do this in response to stimulation, and why when they do this they should continue the action so long, these questions I cannot answer. In the case of so peculiar a phenomenon, however, it is necessary that I should detail all the facts I have been able to collect. I have never seen any similar or corresponding action performed by the bells of other medusids, and even in the case of *Sarsia* its occurrence is comparatively rare. When it does occur, however, it is always continuous ; that is to say, it never spontaneously recommences after having once ceased. As already stated, the period of its duration is extremely variable ; but when this period is long, it is observable that the shivering motions become feebler and feebler, until they eventually fade away into quiescence. The animal is then quite dead to all further stimulation. Beyond saying that the peculiar motions in question never originate independently of stimulation, I cannot give much further account of the conditions which determine their commencement. The following instances are quoted from my notes *in extenso :* 'A healthy individual with centres removed, after failing to respond to either make or break of direct current, and after about a quarter of a minute's rest from a series of rapidly alternating makes and breaks of this current, began to shiver, and continued to do so for five minutes. Afterwards quite dead.'

"'Another healthy individual, after refusing to contract either on make or break of induced current for two or three times, began to shiver, and continued to do so for twenty minutes. Afterwards quite dead. This individual had been used five minutes before for experiments with the direct current, to which it responded well and without shivering.'

"'Another healthy individual was left for some time after excision of margin, and then put into the well and submitted to induction-shocks. No contraction either on make or break with coil pushed to zero. On now trying direct current, without altering position of electrodes, violent contraction on make and also a decided one on break. On again trying induced current, no contraction. On again trying direct current, strong contractions [thus far, of course, the behavior of the tissue was normal, see above § 2 (A)], and after four or five of these, shivering began ; this lasted for four minutes, and, when again quiescent, the bell again responded to make and break of direct current in the ordinary way.'

"Now all that can be gathered from these and similar notes is, that the shivering motion in question may be started either by the direct or by the induced current, and this in some cases when the bell has repeatedly refused to answer stimuli in the ordinary way. It may further be gathered from these notes that shivering is most likely to begin after the bell has received a number of shocks in succession."

The passage concludes by observing that the shivering movements in question are much more certain to be produced by means of chemical stimulation, as just described in the previous paragraphs of the present paper. Now, when we consider these results of electrical stimulation in the light which has since been shed by the results of all the other experiments on artificial rhythm, there can be no doubt that in some way or other the injury which the electrical stimulation has previously supplied to the tissue continues for a long time afterwards to act as a constant stimulus to these tissues, and thus occasionally throws them into artificial rhythm. What the precise nature of this injury may be is a matter of subordinate interest; but I may state that I have never observed artificial rhythm to occur as a result of electrical stimulation until the latter has acted for a sufficient length of time to cause an opalescence of the tissues in the neighbourhood of the electrodes. This opalescence, in the case of *Sarsia*, is always certain to manifest itself under the influence of an electrical stimulus of sufficient duration, and when it does manifest itself it is indistinguishable from that which has already been described as the result of chemical action. There can then be no doubt that artificial rhythm as an after-effect of electrical stimulation never asserts itself until after that stimulation has inflicted a conspicuous injury upon the tissues. But in order to assure myself still more effectually that the artificial rhythm in this case is due to a constant stimulus supplied by a local injury in the region where the electrodes had rested, I tried the following experiment. Having submitted a paralyzed bell of *Sarsia* to a number of successive electrical shocks in the same area of its excitable surface, and having then observed, on removing the electrodes and allowing the mutilated bell to float freely in the water, that the artificial rhythm was in vigorous progress, I suddenly with two snips of the scissors excised the area of tissue which had previously been the seat of electrical stimulation. Instantly all movement ceased. This experiment, therefore, proved that the rhythmic movements can only have been due to a stimulus emanating from the seat of injury, and continuously spreading over all the other parts of the excitable surface of the bell. And, such being the case, we can have no further doubt that the case of artificial rhythm due to previous electrical stimulation belongs to the same category of physiological facts as the case of artificial rhythm due to present or previous chemical stimulation. For now, in conclusion, I may observe that in order to render the experimental parallel complete, I imitated in the case of chemical stimulation the proof of the fact which I have just narrated in the case of electrical stimulation, viz.: that the artificial rhythm depends on the continuous reception by the excitable tissues of a stimulus which is continuously supplied from a seat, or seats, of injury. The method which I employed to prove this fact in the case of chemical stimulation was as follows. Having slit open the paralyzed bell of *Sarsia* along the whole of one side from base to apex of the cone, I suspended the now sheet-like mass of tissue by one corner in the air, leaving the rest of the sheet to hang vertically downwards. By means of a rack-work support I now lowered the sheet of tissue till one portion of it dipped into a beaker filled with a solution of glycerine of appropriate strength. After allowing this portion to soak in the solution of glycerine until it became slightly opalescent, I dropped the entire mutilated bell, or sheet of tissue, into another beaker containing ordinary sea-water. If the exposure to the glycerine solution had been of sufficient duration, I invariably found that in the normal sea-water the rhythmic movements were performed by the whole tissue mass quite as efficiently as was the case in my other experiments, where the whole tissue-mass, and not merely a portion, had been submitted to the influence of the irritant. But on now suddenly snipping off the opalescent portion of the tissue-mass—*i.e.*, the portion which had been previously alone submitted to the influence of the irritant—all movement in the remainder of the tissue-mass instantly ceased. This experiment I performed repeatedly, sometimes exposing a large and sometimes a small portion of the tissue to the influence of the irritant. As I invariably obtained the same result, there can be no doubt that in the case of chemical, as in that of the after-effects of electrical stimulation, the artificial rhythm depends for its manifestation on the presence of a constant stimulus, and is not merely some kind of obscure fluttering motion which, having been started by a stimulus, is afterwards kept up independently of any stimulus.

Such being the case, I naturally expected that if I were to supply a constant stimulus of a thermal kind I should also obtain the phenomena of artificial rhythm. In this, however, my expectations have not been realised. With no species of Medusa have I been able to obtain the slightest indication of artificial rhythm by immersing the paralyzed animals in heated water. I can only explain this fact by supposing that the stimulus which is supplied by the heated medium is of too uniform a character over the whole extent of the excitable tissues; it would seem that in order to produce artificial rhythm there must be a differential intensity of stimulation in different parts of the responding tissue, for no doubt even the excitatory influence of acidulated water is not of nearly so uniform an intensity over the whole of the tissue area as is that of heated water.

In now quitting the subject of artificial rhythm as it is manifested by the paralyzed bells of *Sarsia*, it is desirable again to observe that sustained artificial rhythm cannot be produced either by means of chemical irritation or as the after-effect of electrical stimulation in the case of any of the species of covered-eyed Medusæ that I have met with. In order to evoke any response at all, stronger solutions of the irritants require to be employed in the case of the covered- than in that of the naked-eyed Medusæ, and when the responses do occur they are not of so suggestive a character as those which I have thought it worth while so fully to describe. Nevertheless, even in the covered-eyed Medusæ well marked, though comparatively brief, displays of artificial rhythm may often be observed as the result of constant chemical stimulation. Thus, for instance, in the case of *Aurelia*, if the paralyzed umbrella be immersed in a solution of glycerine (10 to 20 per cent.) a few rhythmic pulsations of normal rate are usually given, but shortly after these pulsations occur the tissue begins to go into a tetanus, which progressively and rapidly becomes more and more pronounced until it ends in violent tonic spasm. So that the history of events really resembles that of *Sarsia* under similar circumstances, except that the stage of artificial rhythm which inaugurates the spasm is of a character comparatively less pronounced.

(*m.*) Thus far, then, I have detailed all the facts which I have been able to collect with reference to the phenomena of artificial rhythm as produced by different kinds of constant stimulation. It will not be forgotten that the interest attaching to these facts arises from the bearing which they have on the theory of natural rhythm. My belief is that hitherto the theory of rhythm as due to ganglia has attributed far too much importance to the ganglionic as distinguished from the contractile tissues, and I have founded this belief principally on the facts which have been already stated and which certainly prove, at least, this much: that after the removal of their centres of spontaneity the contractile tissues of the Medusæ display a marked and persistent tendency to break into rhythmic action whenever they are supplied with a constant stimulus of feeble intensity. Without waiting again to indicate how this fact tends to suggest that the natural rhythm of the unmutilated organisms is probably in large part due to that alternate process of exhaustion and restoration of excitability on the part of the contractile tissues, whereby alone the phenomena of artificial rhythm can be explained,[*] I shall go on to describe some further experiments which were designed to test the question whether the influences which affect the character of the natural rhythm likewise, and in the same manner, affect the character of the artificial rhythm. I took the trouble to perform these experiments because I felt that if they should result in answering this question in the affirmative, they would tend still further to substantiate the view I am endeavouring to uphold, viz.: that the natural rhythm may be a function of the contractile as distinguished from the ganglionic tissue. Of the modifying causes in question, the first that I tried was temperature.

(*n.*) Having already, in my former paper, treated of the effects of temperature on the natural rhythm, it will now be sufficient to say that we have seen these effects to be similar to those which temperature exerts on the rhythm of ganglionic tissues in general. Now I find that temperature exerts precisely the same influence on the artificial rhythm of deganglionated tissue as it does on the natural rhythm of the unmutilated animal. To economise space I shall only quote one of my observations in a table which explains itself. I also append tracings of another observation to render the difference in the rate of the rhythm more apparent to the eye.[†]

Temperature of water (Fahr.).	Number of contractions per minute.
25°	24
45°	40
75°	60

---

[*] It is of importance to point out the fact that some of my previously published experiments appear conclusively to prove that the natural stimulation which is supplied by the marginal ganglia of the Medusæ resembles all the modes of artificial stimulation which are competent to produce artificial rhythm in one important particular; the *intensity* of the stimulation which the marginal ganglia supply is shown by these experiments to be about the same as that which is required to produce artificial rhythm in the case of artificial stimulation. In proof of this point I may allude particularly to the observations which are detailed in V., § 1 (*a*), (*b*), and (*c*) of my second paper.

[†] As the effects of temperature in modifying the rate of artificial rhythm no doubt arise from the effects of temperature on modifying the excitability of the contractile tissues, I think it is desirable here again to mention a fact which was briefly stated in my second paper (p. 691), viz.: that in the excitable tissues of the Medusæ, temperature exerts an immense influence, both on the latent period and on the activity

Fig. 5.

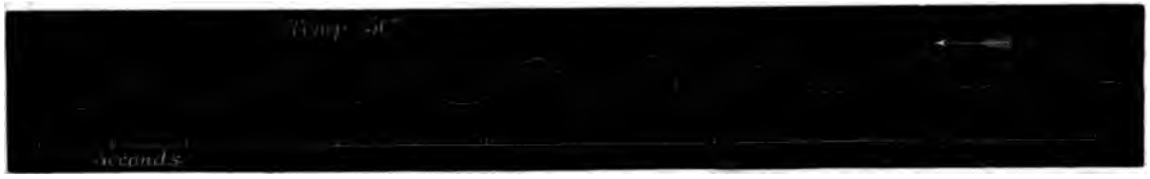
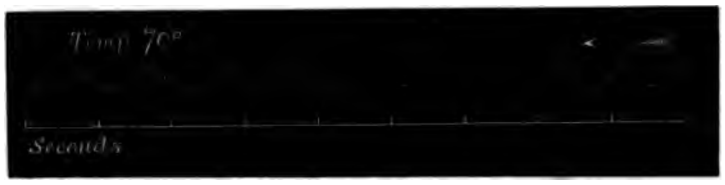


Fig. 6.



During the whole progress of such experiments the faradaic stimulation was, of course, kept of uniform intensity; so that the progressive acceleration is undoubtedly due to the increase of temperature alone. With each increment of temperature the rate of the artificial rhythm increases suddenly, just as it does in the case of the natural rhythm. Moreover, there seems to be a sort of rough correspondence between the amount of influence that any given degree of temperature exerts on the rate of the natural and of the artificial rhythm respectively; for it will be remembered that in warm water the natural rhythm, besides being quicker, is not so regular as it is in cold water; thus also it is with the artificial rhythm. Again, water below 20° or above 85° suspends the natural rhythm, *i.e.*, stops the contractions; and the artificial rhythm is suspended at about the same degrees. Lastly, just as there are considerable individual variations in the extent to which the natural rhythm is affected by temperature, so the artificial rhythm is in some cases more affected by this cause than in others, though in all cases it further resembles the natural rhythm in showing some considerable degree of modification under the influence of this cause.

On the whole, then, it would be impossible to imagine two cases more completely parallel than are these of the effects of temperature on natural and on artificial rhythm of contraction. To render apparent the degree in which these effects are produced, I here append a pair of tracings which were procured from the same piece of tissue when exposed to the different temperatures named. (N.B.—The seconds are wrongly marked in fig. 8, they ought to be the same as in fig. 7.)

Fig. 7.

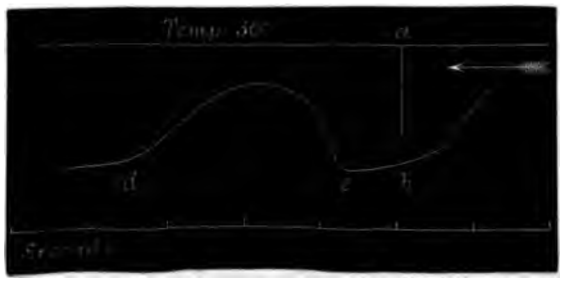
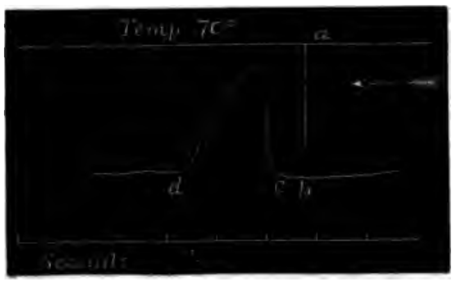


Fig. 8.



respectively; and as it must be considered in the last degree improbable that all these coincidences are accidental, I conclude that the effects of temperature on the natural rhythm of Medusæ (and so, in all probability, on the natural rhythm of other gangliomuscular tissues) are for the most part exerted, not on the ganglionic, but on the contractile element.

(*o.*) In order to test the effects of gases on the artificial rhythm, I took a severed quadrant of *Aurelia*, and floated it in sea-water with its muscular surface just above the level of the water. Over the tissue I lowered an inverted beaker filled with the gas, the effects of which I desired to ascertain, and by progressively forcing the rim of the beaker into the water I could submit the tissue to various pressures of the atmosphere of the gas I was using. By an appropriate arrangement the electrodes passed into the interior of the beaker, and could then be manipulated from the outside, so as to be properly adjusted on the tissue. In this way I was able to observe that different gases exerted a marked influence on the rate of the artificial rhythm. My experiments, however, in this connexion are not as yet complete; so I shall now confine myself to saying that an atmosphere of oxygen appears to accelerate the artificial rhythm, while an atmosphere of carbonic acid certainly retards it—the rate of the rhythm in air being in both cases taken as the standard of comparison, and the intensity of the faradaic stimulation remaining constant throughout the three observations. The following table gives the ratios in the case of one experiment:—

Rate of artificial rhythm, in air.	In oxygen.	In carbonic acid.
36 per minute.	50 per minute.	25 per minute.

It may here be observed that, to produce these results, both carbonic acid and oxygen must be considerably diluted with air; for otherwise they have the effect of instantaneously inhibiting all response, even to the strongest stimulation. When this is the case, however, irritability returns very soon after the tissue is again exposed to air or to ordinary sea-water. But I desire it to be distinctly understood that the results of my experiments on the influence of oxygen, both on the natural and on the artificial rhythm, have proved singularly equivocal; so that as far as this gas is concerned further observations are required before the above results can be accepted as certain.

(*p.*) I have still one other observation of a very interesting character to describe, which is closely connected with the current views respecting ganglionic action, and may therefore be more conveniently considered here than in any other part of this paper. In my first paper I stated that in no case had I observed the manubrium, or polypite, of a Medusa to be affected, "as to its natural motions," by removal of the periphery of the swimming-bell. This statement still remains true; but in the case of *Sarsia* a very interesting change occurs in the polypite soon after the nectocalyx has been paralyzed by excision of its margin. Unlike the polypites of most of the other

Medusæ, this organ, in the case of *Sarsia*, is very highly retractile. In fresh and lively specimens the appendage in question is carried in its retracted state; but when the animals become less vigorous—from the warmth or impurity of the water in which they are confined, or from any other cause—their polypites usually become relaxed. The relaxation may show itself in various degrees in different specimens subjected to the same conditions; but in no case is the degree of relaxation so remarkable as that which may be caused by removing the periphery of the nectocalyx. For the purpose of showing this effect, it does not signify in what condition as to vigour, &c., the specimen chosen happens to be in; for whether the polypite prior to the operation be contracted or partially relaxed, within half an hour after the operation it is sure to become lengthened to a considerable extent.

In order to show the surprising degree to which this extention may proceed, I insert a sketch of a specimen both before and after the operation. The sketches are of life size, and drawn to accurate measurement (figs. 9 and 10).*

Fig. 9.

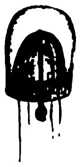
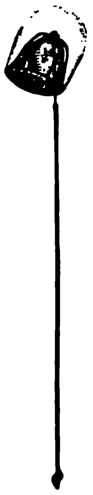


Fig. 10.



With regard to this remarkable effect on the polypite of removing the margin of the nectocalyx, it is now to be observed that in it we appear to have very unexceptionable evidence of such a relation subsisting between the ganglia of the nectocalyx and the muscular fibres of the polypite, as elsewhere gives rise to what is known as muscular tonus. This interpretation of the facts cannot, I think, be disputed; and it fully explains why, in the unmutilated animal, the degree of elongation on the part of the polypite usually exhibits an inverse proportion to the degree of locomotor activity displayed by the bell. I may here state that I have also observed indications of muscular tonus in some of the other Medusæ, but for the sake of brevity I shall here restrict myself to the consideration of this one case.

---

* I may here mention that the fact of the polypite of *Sarsia* undergoing this extreme elongation after the removal of the marginal ganglia, serves to render the artificial rhythm of the organ under the influence of injury, as previously described, all the more conspicuous.

To my mind, then, it is an interesting fact that ganglionic tissue, where it can first be shown to occur in the animal kingdom, has for one of its functions the maintenance of muscular tonus; but it is not on this account that I now wish to draw prominent attention to the fact before us. Physiologists are almost unanimous in regarding muscular tonus as a kind of gentle tetanus due to a persistent ganglionic stimulation; and against this opinion it seems impossible to urge any valid objection. But, in accordance with the accepted theory of ganglionic action, physiologists further suppose that the only reason why some muscles are thrown into a state of tonus by ganglionic stimulation, while other muscles are thrown into a state of rhythmic action by the same means, is because the resistance to the passage of the stimulation from the ganglion to the muscle is less in the former than in the latter case. Here, be it remembered, we are in the domain of pure speculation; there is no experimental evidence to show that such a state of differential resistance as the theory requires really obtains. Hence we are quite at liberty to suppose any other kind of difference to obtain, either to the exclusion of this one or in company with it. Such a supposition I now wish to suggest, and it is this: That all rhythmical action being regarded as due (at any rate in large part) to the alternate exhaustion and restoration of excitability on the part of contractile tissues, the reason why continuous ganglionic stimulation produces incipient tetanus in the case of some muscles and rhythmic action in the case of others, is either wholly or partly because the irritability of the muscles in relation to the intensity of the stimulation is greater in the former than in the latter case. If this supposition as to differential irritability be granted, my experiments on *Aurelia* prove that tetanus would result in the one case and rhythmic action in the other; for it will be remembered that in these experiments, if the continuous faradaic stimulation were of somewhat more than minimal intensity, tetanus was the result, while if such stimulation were but of minimal intensity, the result was rhythmic action. Now, that in the particular case of *Sarsia* the irritability of the tonically contracting polypite is higher than that of the rhythmically contracting bell is a matter, not of supposition, but of observable fact; for not only is the polypite more irritable than the bell in response to direct stimulation of its own substance, but it is generally more so even when the stimuli are applied anywhere over the excitable tissues of the bell. And from this it is evident that the phenomena of muscular tonus, as they occur in *Sarsia*, tend more in favour of the exhaustion than of the resistance theory.*

---

* The evidence, however, is not altogether exclusive of the resistance theory, for it is quite possible that in addition to the high irritability of the polypite there may be conductile lines of low resistance connecting this organ with the marginal ganglia. I entertain this supposition because, as afterwards explained in the text, I see reason to believe that the natural swimming movements of *Sarsia* are probably in part due to an intermittent discharge of the ganglia. I think, therefore, that in this particular case the ganglia supply a tolerably constant stimulation to the polypite, while it is only at intervals that their energy overflows into the bell, and that the higher degree of irritability on the part of the polypite ensures the tonic response

(*q*.) I will now sum up this rather lengthy discussion. The two theories of ganglionic action may be stated antithetically thus: In both theories the accumulation of energy by ganglia is supposed to be a continuous process; but while the resistance theory supposes the rhythm to be exclusively due to an intermittent and periodic discharge of this accumulated energy on the part of the ganglionic tissues, the exhaustion theory supposes that the rhythm is largely due to a periodic process of exhaustion and recovery on the part of the responding tissues. Now, I submit that my experiments have proved the former of these two theories inadequate to explain all the phenomena of rhythm as it occurs in the Medusæ. For these experiments have shown that even after the removal of the only ganglia which serve as centres of natural stimulation, the excitable tissues still continue to manifest a very perfect rhythm under the influence of any mode of artificial stimulation (except heat), which is of a constant character and of an intensity sufficiently low not to produce tetanus. And as I have proved that the rhythm thus artificially produced is almost certainly due to the alternate process of exhaustion and recovery which I have explained, there can scarcely be any doubt that in the natural rhythm this process plays an important part, particularly as we find that temperature and gases exert the same influences on the one rhythm as they do on the other. And, as an additional reason for recognising the part which the contractile tissues probably play in the production of rhythm, I have pointed to the fact that in the great majority of cases in which rhythmic action occurs, the presence of ganglia cannot be suspected. For it is among the lower forms of life, where ganglia are certainly absent, and where the functions of stimulation and contraction appear to be blended and diffused, that rhythmic action is of the most frequent occurrence; and it is obvious with how much greater difficulty the resistance theory is here beset than is the one which I now propose. Granted a diffused power of stimulation with a diffused power of response, and I see no essential difference between the rhythmic motions of the simplest organism and those of a deganglionated Medusa in acidulated water.

But now, in conclusion, I wish it to be distinctly understood that I am not attempting to overturn, but merely to supplement, the current theory of ganglionic action. My belief is that this theory is to a large extent a true one, but that in order to become a complete theory it must incorporate the facts and inferences which have now been fully detailed. By a complete theory I mean, of course, a theory which will cover all the facts; and forasmuch as the facts on which my inferences are founded

---

of this organ at a small cost of nervous energy. How far the rhythm of the nectocalyx is to be attributed to the resistance mechanism of the ganglia, and how far to the alternate exhaustion and recovery of the contractile tissues, I think it is impossible to determine, seeing that it is impossible exactly to imitate the natural ganglionic stimulation by artificial means. But it is, I think, of importance to have ascertained at least this much: that in *Sarsia* the tonus of one organ and the rhythm of another, which apparently both received their stimulation from the same ganglia, must at any rate in part be attributed to a differential irritability of these organs as distinguished from their differential stimulation.

are to a large extent novel, I have thought it desirable to insist on their theoretical importance rather than on that of facts which are better known, and which serve to uphold existing views respecting the action of ganglia. Nevertheless, even within the limits of the Medusæ, I have found only too much reason for concluding that the full explanation of natural rhythm must be more complex than that which I believe to apply to artificial rhythm. In particular I may allude to the case of *Sarsia.* The artificial rhythm which may be produced in the paralyzed nectocalyx of this organism by means of a constant stimulation is of a widely different character from that which occurs in the unmutilated and healthy animal. The comparatively feeble, fluttering, and ineffectual sequence of contraction and relaxation in the former case contrast strongly with the powerful, determined, and pumping-like movement in the latter case. Indeed, striking and suggestive as are the phenomena of artificial rhythm in *Sarsia,* none but the most prejudiced of observers could fail to perceive that it is but a sorry substitute for the natural rhythm of vigorous specimens. In feeble specimens, no doubt, the natural rhythm resembles much more closely the artificial rhythm; but so long as this is not the case with specimens whose ganglionic function is in a state of normal activity, we must conclude that this function presents some quality which a merely constant stimulation is not able to supply. And this function doubtless consists in supplying a stimulation that is periodic. So that, on the whole, my belief is that the natural rhythm of these tissues—and so, from analogy, of ganglio-muscular tissues in general—is probably due to a double process, of which one part consists in the periodic discharge of the ganglia, and the other in the alternate exhaustion and restoration of excitability of the muscles. No doubt in each species of Medusæ the periods of ganglionic discharge are, as it were, timed to coincide with those during which the exhaustion of the responding tissue remains; so that the ganglionic impulse is always thrown in at the moment when the excitability of the responding tissue is at its climax. In this way nervous energy is doubtless economised, and, if so, the natural rhythm becomes analogous, not to the artificial rhythm as produced by the faradaic current, but to the artificial rhythm as produced by single shocks of minimal intensity thrown in at appropriate periods.

Thus the theory of rhythm which I now propose is not subversive of, but complementary to, the current theory; I merely claim to have proved that over and above the already recognised factor in the production of rhythmic movement of ganglio-muscular tissues there is an additional factor, which, although not hitherto recognised, is probably of at least as much importance.

(*r.*) I may fitly conclude this exposition with a brief reference to the only literature bearing on the subject of artificial rhythm with which I am acquainted. In the 'Journal of Anatomy and Physiology' for July, 1876, there is a paper by Dr. M. FOSTER and Mr. DEW SMITH which details results confirmatory of those which had been previously obtained by ECKHARD and HEIDENHAIN, and which are strikingly analogous to some of those which I have just described. The tissue on which these four observers experimented was the heart-apex of the Frog. This, in being a muscular tissue deprived of its centres of spontaneity, constitutes a very fair physiological parallel to the paralyzed bell of a Medusa. In the paper alluded to, the authors confine themselves to describing the effects of the constant current in producing rhythmic response on the part of the heart-apex; but in a note previously published by Dr. FOSTER in the same journal (vol. iii., p. 400), it is shown that minimal faradaic stimulation also has the effect of throwing the heart-apex into rhythmic action. The artificial rhythm is not, indeed, so perfect as that which I obtained in the case of *Aurelia;* but it is nevertheless of an unmistakable character. With regard to it Dr. FOSTER makes the following suggestive remark:—"We may infer that the cardiac muscular tissue differs for some reason from ordinary muscular tissue in a disposition towards rhythmic rather than continuous contraction; and that the influence of the ganglia is probably not rhythmic but continuous, whatever the exact nature of that influence may be." And, as already observed, Dr. FOSTER and Mr. DEW SMITH have also obtained rhythmic response to constant stimulation in the case of the Snail's heart.

The only other observation bearing on this subject with which I have met, is one that was published by Dr. BURDEN SANDERSON and Mr. PAGE in the 'Proceedings of the Royal Society.' The tissue on which these observers experimented was the excitable leaf of *Dionæa,* and they were able to show that slightly less than minimal stimulation produces rhythmic response in these tissues. The response in this case was estimated, not by the occurrence of a contraction—the stimulation employed being too feeble to cause any contraction—but by the occurrence of the electrical disturbance which, as these authors have shown, always constitutes the first result of stimulation, and therefore takes place even in cases where the stimulus is not strong enough to cause contraction. The following are the words in which the fact is stated by its observers:—"When a leaf is excited at regular intervals by single shocks of such intensity as to be just beyond the limit of adequacy, so that the slightest diminution would render them futile, it is sometimes observed that the effects become rhythmical. Thus, in a series of 54 successive excitations, we obtained the following results:—Excitations 1, 2, 3, and 4, were effectual; but of the 16 excitations following every other was futile, the alternate ones only being followed by excursions (*i.e.,* of the electrometer); then followed during eight minutes a series of futile excitations, after which the leaf was allowed to rest for two minutes. On resuming, the alternate rhythm again appeared for six excitations, then becoming modified, so that an excursion followed every fourth instead of every third excitation, a state of things which continued for a quarter of an hour."

Thus, then, as a final result, we see that, not only in the case of the lowest organisms, and not only in the case of the paralyzed tissues of the Medusæ, and of the Frog's tongue, but also in that of the Frog's heart, the Snail's heart, and of the excitable tissues of the *Dionæa,* there is observable a tendency to a more or less marked exhibition of rhythmic response under the influence of constant stimulation of low intensity.

§ 2. *Luminous stimulation of the covered-eyed Medusæ.*—In my previous papers I have detailed the results of various experiments on the luminous stimulation of the naked-eyed Medusæ. Since the publication of these papers I have tried whether the covered-eyed Medusæ are likewise susceptible to this mode of stimulation, and I have found that, although the results are not of so marked a character as they are in the case of the naked-eyed Medusæ, nevertheless, by a little care, it does admit of being proved that the lithocysts of the covered-eyed Medusæ resemble the marginal bodies of the naked-eyed Medusæ in being rudimentary organs of vision.

The best way that I have found of proving this fact is to divide an *Aurelia* into segments, leaving one or more lithocysts in each segment. On then choosing a segment which is not very active, and leaving it for some time in the dark, it may be observed, on approaching it with a candle, that its activity is much less pronounced than it was in the daylight. But after the candle has been allowed to shine upon it for a few seconds, the segment begins to become gradually aroused into a state of greater activity. If the lithocyst, or lithocysts, be now removed, the approach of the candle, or the access of daylight, produces no stimulating effect.

§ 3. *Localising movements of the polypite of* Aurelia aurita.—In my previous paper I have fully described certain movements of localisation which are performed by the polypite of a certain naked-eyed Medusa (*Tiaropsis indicans*), whenever any portion either of the polypite itself or of the concave surface of the nectocalyx is stimulated. I have now ascertained that somewhat similar movements are performed, under similar circumstances, by the polypite of *Aurelia aurita.* For if an *Aurelia* be suspended in a large bell-jar of sea-water, so as to admit of the free movement of the polypite in all directions, and if any part of the concave surface of the umbrella be then touched, or otherwise irritated, the two lobes of the polypite which are nearest to the seat of irritation will slowly approximate and close over that point, as if attempting to embrace the irritating body. Sometimes only one lobe will move over to the seat of irritation, but it is more usual for two, or even more, of the four lobes to take part in these localising movements. These movements, however, are not performed with nearly so much activity and precision as they are in the case of *Tiaropsis indicans;* they are, comparatively speaking, of a very sluggish and somewhat uncertain character, never beginning till a number of seconds, perhaps even half a minute, after the application of the stimulus, and the lobes of the polypite then sweeping slowly backwards and forwards over a large area of the umbrella, instead of being quickly and invariably applied to the precise spot that was stimulated.

§ 4. *Effects of alternating the direction of the constant current on the excitable tissues of the Medusæ.*—It is a fact known to physiologists that, in the muscular tissues of the higher animals, the stimulus which is supplied by make and break of the constant current becomes less and less effective as a stimulus the more frequently the current is passed in the same direction, and that on reversing the direction of the current the stimulations which are supplied by make and break again rise to their original value. (See especially my papers "On the Modification of Excitability produced by Injury," 'Journal of Anatomy and Physiology,' vol. x., and in 'Proceedings of Royal Society,' Nos. 171 and 811.) I have therefore tried whether these effects admit of being produced in the case of the excitable tissues of the Medusæ, and I find that they are so. I find, further, that these effects admit of being equally well produced whether the current be passed in the direction of the length or in that of the thickness of the muscular fibres. In other words, the effects in question occur equally well, whether the electrodes be placed on the same radius of the umbrella—so causing the current to traverse the muscle-fibres for the most part transversely—or placed on different, though not widely separated radii, at points equi-distant from the circumference of the umbrella—so causing the current to traverse the muscle-fibres for the most part longitudinally.

II. SECTION.

§ 1.—Allusion has already been made to the fact that in the contractile tissues of *Aurelia* there are frequently to be observed immense differences in the force of the contractions, according as the latter are originated by a strong or by a weak stimulation; that is to say, while a stimulus which is not below some certain degree of intensity will always start a wave of strong contraction from any point of the tissue area which may be chosen as the seat of stimulation, a weaker stimulus may start a wave of contraction so feeble as to be scarcely perceptible. In some specimens of *Aurelia* these waves of feeble contraction are more easily produced than in other specimens, and occasionally it happens that they are so feeble as to be only rendered perceptible by raising the tissue above the water and inclining the head at such an angle to their level as admits of the eye receiving the reflection of light which falls upon their glancing surface; as a response to each feeble stimulus a slight change in the glancing, indicative of a slight contraction, may then be observed.

It might reasonably be supposed that such feeble waves of contraction would not present much power of radiating through extensive areas of tissue; but, as a matter of fact, they often travel quite as far and well as do the stronger waves. Indeed, it not unfrequently happens that they gain strength as they advance; so that in a long strip of tissue—especially if a large piece of the umbrella be left attached to the end remote from stimulation—it may happen that a contractile-wave, when started by a feeble stimulus at one end of the strip, is barely perceptible at that end, while it grows into a tolerably strong contraction towards the other end.

When I first observed the occurrence of these almost imperceptible waves of contraction it appeared to me that they might possibly serve to explain the occurrence of what in my previous paper I called waves of stimulation. But before I published my previous paper I satisfied myself that such was not the case; and as the establishment of this point entailed a number of experiments, I think it is desirable in this concluding paper to state their general result, lest other observers should suppose that the evidence as to the occurrence of waves of stimulation is vitiated by the occurrence of these almost imperceptible waves of contraction.

That there is here a very probable source of error opened up is, of course, manifest. As stated in my previous paper, the proof as to the occurrence of waves of stimulation is two-fold: First, by stimulating a part of the irritable tissue at a distance from a lithocyst, too gently to start a visible wave of contraction from that part, a wave of stimulation may nevertheless be proved to have been started; for, shortly after applying the stimulus, the distant lithocyst will discharge its ganglionic influence, so giving rise to a visible wave of contraction, which starts, not from the seat of stimulation, but from that of the lithocyst. Secondly, the invisible wave of stimulation through the excitable tissue of the umbrella may often be seen to have its passage visibly recorded by the numerous tentacles which in *Aurelia* fringe the margin of the umbrella—one tentacle after another successively retracting until the "tentacular wave" of retraction which started immediately below the seat of stimulation in the umbrella has passed throughout the whole series of tentacles. In umbrellas with their lithocysts removed these tentacular waves course all the way round the margin without exciting any visible contraction in the umbrella-tissue which was the immediate seat of stimulation; but if any of the lithocysts are left *in situ,* they invariably discharge their ganglionic influence shortly after a tentacular wave reaches them.

These facts, of course, tend to show that in the excitable tissues of *Aurelia* there is some element which performs the essential function of nerve—the function, namely, of conveying stimuli irrespective of the passage of contractile waves. But in view of the fact that in these same tissues contractile waves may occur of so feeble an intensity as to be almost invisible, it becomes necessary to be quite sure that such waves may not sometimes be so feeble as to be quite invisible, and therefore that what I have called nervous waves of stimulation are not in reality only invisible muscular waves of contraction. The evidence which I have to adduce in order to show that such is not the case is happily unequivocal. I cannot, indeed, prove that muscular waves of contraction may not occur in the excitable tissues of *Aurelia* of too feeble an intensity to admit of becoming visible; but I am able to prove that, whether or not such is the case, other waves of a distinctively nervous kind undoubtedly pass through these tissues. The proof of this consists in the fact that sometimes waves of stimulation will continue to pass from one tissue area to another after the passage of contractile waves between these two areas has been blocked by section; or, reverting to the terms employed in my previous paper, excitational continuity may in some cases remain intact after contractional continuity has been destroyed. And forasmuch as it is impossible to suppose that an invisible, or very feeble, wave of contraction can force a passage where a visible, or very strong, wave of the same kind fails to do so, the following observations must be regarded as eliminating the possible source of error with which I am now dealing; they establish the fact that whether or not wholly invisible waves of contraction occur in *Aurelia*, waves of stimulation undoubtedly occur. For the sake of brevity I shall quote these observations from my notes *verbatim*:—

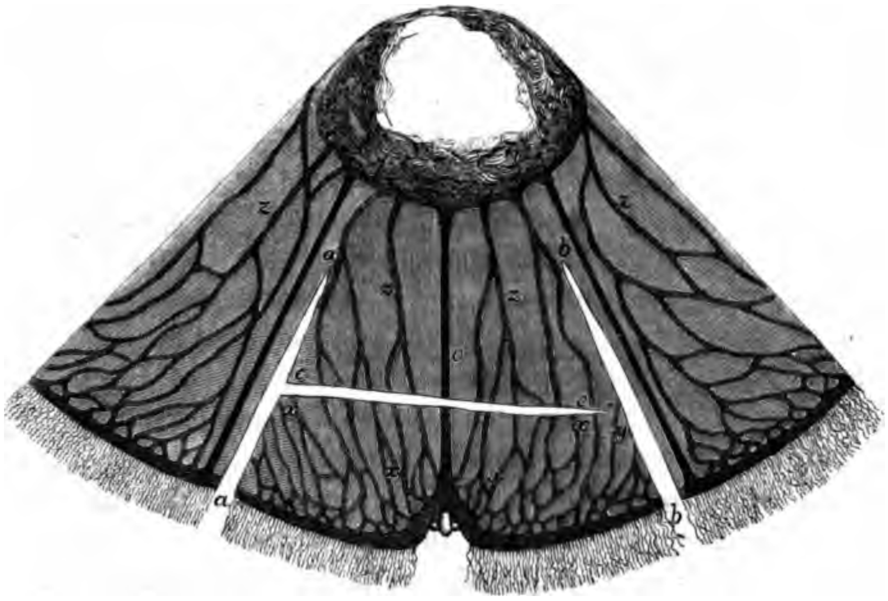
"An *Aurelia* of medium size. One half of the margin removed in the form of a strip two inches broad, the remainder of the umbrella being left attached. After some overlapping sections had completely and permanently destroyed contractional continuity in the strip, excitational continuity clearly remained intact, as shown by the occurrence of a decided tentacular wave at the opposite side of the umbrella whenever the free end of the strip was irritated. That contractional continuity had in this case been effectively destroyed was proved by tetanising the free end of the strip four hours after the section with strong faradaic currents, and also with spirit. Under the influence of such strong stimulation of the free end of the strip, the lobes of the polypite violently writhed, although the umbrella remained quiescent."

This response on the part of the polypite is of conclusive value as proving the occasional persistence of excitational continuity after contractional continuity has been destroyed. My notes contain the record of several other observations confirmatory of this fact, but it is needless to quote them at length. The writhing movements of the polypite under these circumstances are often very decided; but they are usually wonderfully late in beginning. This is probably owing to the prolonged and uncertain period of latency which, as stated in my previous paper, is manifested by the polypite of *Aurelia*; for even in experiments where the contractional continuity of the umbrella tissue has not been destroyed by section, the response of the polypite to strong stimulation of these tissues does not begin till after the violent contractions of the umbrella have been in progress for a considerable time.

§ 2. *Effects of Exhaustion in impeding the passage of contractile waves.*—In my various modes of section of *Aurelia* I have several times observed a fact that is worth recording. It sometimes happens that when the connecting isthmus between two almost severed areas of excitable tissue is very narrow, the passage of contractile waves across the isthmus depends upon the freshness, or freedom from exhaustion, of the tissue which constitutes the isthmus. That is to say, on faradising one of the two tissue areas which the isthmus serves to connect, the resulting contractile waves will at first pass freely across the isthmus to spread all over the other tissue area; but after several such waves have passed in rapid succession across the isthmus, it may happen in some preparations that every now and then a contractile wave fails to pass across the isthmus. When this is the case, if the stimulation is still continued, a greater and greater proportion of waves fail to pass across the isthmus, until perhaps only one in every five or six waves become propagated from the one area to the other. If now single induction shocks be substituted for the faradaic stimulation, it may be found that by leaving an interval of four or five seconds between the successive shocks, every wave that is started in the one area will be propagated across the isthmus to the other area. But if the interval between the successive shocks be reduced to two or three seconds, every now and then a wave will fail to pass across the isthmus. And if the interval be still further reduced to one second, or half a second, comparatively few of the waves will pass across. Now, however, if the tissue be allowed five minutes' rest from stimulation, and the single shocks be thrown in at one second's intervals, all the first six or ten waves will pass across the isthmus, after which they begin to become blocked as before. It may be observed also that when the waves are thus blocked, owing to exhaustion of the connecting isthmus, they may again be made to force a passage by increasing the intensity of the stimulation, and so giving rise to stronger waves having a greater power of penetration. Thus, on re-enforcing the electrical stimulus with the simultaneous application of a drop of spirit, the resulting waves of contraction are almost sure to pass across the isthmus, even though this has been exhausted in the manner just described.

§ 3.—Another fact, which I have several times observed during my sections of *Aurelia*, also deserves to be recorded. I have observed it under several modes of section, but it will be only necessary to describe one observation.

Fig. 11.



In the *Aurelia*, of a portion of which the accompanying woodcut is a representation, seven of the lithocysts were removed, while the remaining one was almost entirely isolated from the general contractile tissue by the incisions *a a*, *b b*, *c c*. The lithocyst continued to animate the tissue area *x x x x*, and through the connecting passage *y* the contractile waves spread over the remainder of the subumbrella tissue *z z z z*. So far, of course, the facts were normal; but very frequently it was observed that the contractile waves did not start from the lithocyst, or from the area *x x x x*, but from the point *o* in the area *z z*. After this origination of the contractile waves from the point *o* had been observed a great number of times, I removed the lithocyst. The effect was not only to prevent the further origination of contractile waves in the area *x x x x*, but also to prevent their further origination from the point *o*—the entire umbrella thus becoming paralyzed. Hence, before the removal of the lithocyst, the contractile waves which originated at the point *o*, no less than those which originated at the lithocyst itself, must in some way or other have been due to the ganglionic influence emanating from the lithocyst and asserting itself at the distant point *o*.

This property which lithocysts sometimes present of asserting their ganglonic influence at a distance from their own locality, can only, I think, be explained by supposing that at the point where, under these circumstances, the contractions originate, there are situated some scattered ganglionic cells of considerable functional power, but yet not of power enough to originate contractile waves unless re-enforced by some stimulating influence which reaches them from the lithocyst through the nervous plexus. But whether or not this is the true interpretation of the facts, I think it is evident that these facts are of considerable importance in relation to the theory of ganglionic action which has already been so fully discussed in the first division of the present paper. For, turning again to the woodcut (fig. 11), if the discharges of the lithocyst are regulated only by resistance, it seems almost unaccountable that these discharges should frequently fail to excite the contiguous area *x x x x*, while they are effectual in exciting the remote area *z z z z*; and this even after *z z z z* has been almost completely severed from *x x x x* by the severity of the incisions. To my mind, the facts seem rather to point to the conclusion that the lithocyst is engaged in emitting some sort of continuous ganglionic impulse which spreads throughout the nervous plexus, and helps to stimulate the ganglionic cells which are scattered throughout this plexus. And this conclusion is not incompatible with the view that the resistance which is offered by the plexus is so adjusted to the accumulation of energy by the lithocyst, that this energy admits of being periodically discharged in greater amount, just at the time when the contractile tissues have completely recovered from their exhaustion due to their previous contraction. In support of this interpretation of the facts, it may be added that after the removal of the lithocyst I tried the effects of gently irritating the tissue in the area *x x x x* with a camel-hair pencil, and I found that by so doing I could again excite the origination of contractile waves from the point *o* in the area *z z z z*.

§ 4. *Regeneration of the excitable tissues of* Aurelia aurita.—The only remaining facts which I have now to communicate have reference to the astonishing rapidity with which the excitable tissues of the Medusæ regenerate themselves after injury. In this connexion I have mainly experimented on *Aurelia aurita*, and shall therefore now confine my remarks to this one species.

If with a sharp scalpel an incision be made through the tenuous contractile sheet of the subumbrella of *Aurelia*, in a marvellously short time the injury is repaired.

Thus, for instance, if such an incision be carried across the whole diameter of the subumbrella, so as entirely to divide the excitable tissues into two parts while the gelatinous tissues are left intact, the result of course is that physiological continuity is destroyed between the one half of the animal and the other, while the form of the whole animal remains unchanged; the much greater thickness of the uninjured gelatinous tissues serving to preserve the shape of the umbrella. But although the contractile sheet which lines the subumbrella is thus completely severed throughout its whole diameter, it again re-unites, or heals up, in from four to eight hours after the operation.

For a more careful study of this process of healing I adopted the following mode of section. With a pair of scissors I cut out the ovaries of the animal, so converting the latter into a broad open ring. This ring I then cut through by means of a single radial cut, thus converting the *Aurelia* into a single broad band. With a sharp scalpel I now carried an incision across this band, taking care only to sever the thin layer of excitable tissue, while leaving the gelatinous tissue uncut. The first effect of the operation was that of making the two edges of the excitable tissue to gape apart from one another, so revealing the clear underlying gelatinous tissue.* Physiological continuity was, of course, immediately destroyed at the line of incision. But after an hour or two a slight tumefaction of the gelatinous tissue caused the gaping in the excitable tissue to become less marked, and eventually the edges of the latter were by this means brought into apposition. Generally in about six hours after the operation some part or parts of the incision through the excitable tissue had become sufficiently healed up to admit of the re-establishment of physiological continuity between the previously discontinuous parts of the strip—contractile waves now coursing all the way along from end to end of the strip. At this stage it might be observed that as each wave of contraction passed across the line of incision, the parts of that line which had not yet adhered together gaped away from each other. But the healing process rapidly continuing, adherence rapidly extended along the whole line of incision, and generally in from twelve to twenty hours this adherence was complete. By observing carefully, however, there might still be seen a cicatrix marking out the line of previous incision, though in the course of next day this too disappeared, leaving no trace behind of the previous injury.

It is an unfortunate circumstance that I found it impossible to procure any microscopical preparations of these lines of incision, owing to the fact that all the hardening and staining reagents immediately caused the newly-healed muscle-fibres again to rupture. After many trials I was therefore at last obliged to satisfy myself with an examination of the newly-healed tissues in their fresh state. I was particularly anxious to ascertain whether the physiological continuity which was thus re-established through narrow necks of tissue depended for its re-establishment upon the regeneration of nerve-fibres or only upon that of muscle-fibres. But owing to the impossibility of procuring a stained preparation of such a regenerated injury, I cannot speak with confidence upon this point. I can only say that the nerves which admit of being seen without staining never pass across the line of incision till long after the contractile waves are able to do so. But, of course, this does not prove that the more delicate nerves which are only brought into view by staining may not have previously passed across the line of incision, and have thus been instrumental in the re-establishment of physiological continuity.

---

* "Underlying," because to perform the operation the umbrella requires to be placed with its concave, or excitable, surface uppermost.

I may here observe that, so far as my experiments in this connexion have gone, it appears to be an essential condition to the regeneration of the excitable tissues that the whole thickness of the gelatinous tissues should not have been severed. That is to say, I have never succeeded, though I have made a number of trials, in obtaining union between any two parts of a Medusa which have been completely severed from one another and are then held in close apposition by stitching. So far as I have seen, it is only when the gelatinous tissues of the umbrella are left intact, or, at least, are not severely injured, that the injuries in the excitable tissues will admit of being repaired.

III. GENERAL SUMMARY.

Artificial rhythm may be produced in various species both of covered- and of naked-eyed Medusæ, though in some species electrical and in other species chemical stimulation is most effective for this purpose. In all cases, however, the stimulation which is supplied must be constant and of not more, or but slightly more, than minimal intensity. In *Aurelia* artificial rhythm is best produced by employing weak faradaic stimulation, when the resulting rhythm is often of a most regular and persistent character. Eventually, however, exhaustion produces irregularity and cessation of the rhythm, and prolonged rest is then required before any perfect rhythm can be again produced by the faradaic stimulation. Increasing the strength of the current within certain limits increases the rate of the rhythm. In *Sarsia* artificial rhythm may be best produced by employing weak chemical stimulation. Soon after the concave surface of the paralyzed bell becomes opalescent, the shivering movements begin, and continue without intermission for a variable time. Shortly after transference of the bell to normal sea-water these movements cease, to be again resumed if the bell is again transferred to acidulated water. The paralyzed bell of *Sarsia* will sometimes manifest these rhythmic movements even in normal sea-water, provided that it has first been injured by the application of either the constant or the induced current. The polypite of *Sarsia* will also manifest a slow and long-continued rhythm as a result of mechanical or chemical injury. The umbrella of *Aurelia* will sometimes give three or four rhythmic contractions in response to a single mechanical stimulation, and will sometimes also respond rhythmically to the constant current. Thus all modes of constant stimulation (except heat) may produce artificial rhythm in one or other of these two species of Medusæ.

The explanation which I have offered to account for the phenomena of artificial rhythm is, that an alternate process of exhaustion and restoration of excitability on the part of the contractile tissues causes the constant stimulation of minimal intensity alternately to fall below and to rise above the limits of adequacy. This explanation has been tested and supported by various experiments, which it is not necessary again to detail. Taking, therefore, this explanation of the artificial rhythm as satisfactory, it follows that it probably has an important bearing on the facts of natural rhythm. For if in the production of artificial rhythm the exhaustion of contractile tissues is found to play so essential a part, it becomes incredible that it should not likewise play some part in the production of natural rhythm. I am therefore led to suppose that in all ganglio-muscular tissues which present rhythmic movement, the rhythm is not exclusively due to any resistance mechanism on the part of the ganglionic tissues, but that it is at any rate in large measure due to this alternate rise and fall of excitability on the part of the muscular tissues. And I have submitted that this view is sustained, *à priori*, by the fact that rhythmic action is of the most frequent occurrence in lowly organized tissues where as yet there has been no segregation of ganglionic structure, and, *à posteriori*, by the fact that in *Sarsia* the same ganglia supply at the same time a stimulus to the rhythmic action of the lowly excitable tissues of the bell, and a stimulus to the tonic action of the more highly excitable tissues of the polypite. And, as additional facts confirmatory of the same view, I have also cited the observations of ECKHARD, HEIDENHAIN, FOSTER, and DEW SMITH on the rhythmic action of the heart-apex under the influence of the constant current; the observations of Dr. FOSTER on the rhythmic action of the same tissue under the influence of the faradaic current; his observations on the phenomena of artificial rhythm as they occur in the Snail's heart; the observations of Dr. BURDON SANDERSON and Mr. PAGE on the rhythmic properties of the excitable tissues of *Dionæa;* and my own observations on the rhythmical motions of the Frog's tongue. But in thus suggesting some modification in the current theory of ganglionic action, I have been careful to state that I do not attempt to overturn, but merely to supplement it. I think I have shown that the theory in question is inadequate to account for all the facts, and that in order to render it a complete theory, we must recognise, not only a periodicity of stimulation on the part of the ganglionic tissues, but also an inherent tendency to rhythmic action on the part of the responding tissues.

Light has been shown to act as a stimulus on the lithocysts of the covered-eyed Medusæ.

The polypite of *Aurelia aurita* has been shown to perform localising movements somewhat similar to those which are performed by the polypite of *Tiaropsis indicans*, though the localising movements are of a much more sluggish and uncertain character in the former than in the latter species.

The effects of alternating the direction of the constant current on the excitable tissues of the Medusæ has been shown similar to those which occur in the muscular tissues of the higher animals, and this whether the current be passed in the direction of the length or in that of the thickness of the muscular fibres.

Waves of contraction in the excitable tissues of *Aurelia*, when started by a very weak stimulus, may be so feeble as to be barely perceptible. This fact in itself raises the presumption that the so-called waves of stimulation, which were described in my former paper, may really be but waves of contraction which are too feeble to be perceptible at all; and hence that the phenomena of reflex action, &c., as they occur in the Medusæ may be due, not to nerve, but to muscle. This presumption, however, is excluded by the additional fact that very often severe section of the excitable tissues has the effect of completely blocking even the strongest waves of contraction while still allowing the waves of stimulation to pass freely, as shown by tentacular waves, reflex action of the lithocyst, and writhing response of the polypite continuing to assert themselves beyond the region in which the contractile waves have been blocked by section.

Exhaustion of a narrow isthmus of junction tissue sometimes has the effect of blocking the passage of contractile waves in that isthmus, their passage, however, being resumed when the junction tissue is allowed time to recover its full excitability.

It has been several times observed that the ganglionic influence of a lithocyst admits of asserting itself at a distance from the seat of the lithocyst itself, and this even when severe forms of section are interposed between the lithocyst and the distant point in the tissue at which the contractile waves are originating. The interpretation of this fact need not again be rendered.

It has been uniformly observed that the contractile tissues of *Aurelia*, after having been completely severed with a sharp scalpel, in from four to eight hours regenerate themselves sufficiently to admit of the re-establishment of physiological continuity across the line of previous severance.

IV. LITERATURE.

I shall terminate this concluding paper on the locomotor system of Medusæ with as brief a reference as possible to the existing literature of the subject. In my first paper I have already had occasion to render an epitome of the memoirs of Professors Agassiz and Hæckel, so that it becomes needless in this place again to state the results at which these observers arrived. Neither is it desirable to discuss in this place the results which were arrived at by the numerous observers prior to the date of Professor Hæckel's work. It is not desirable to do so, because these results, in so far as they professed to be positive, were for the most part erroneous, and in so far as they were combative of error, now possess only an historical interest. Those who may care to consult the literature of this subject on account of any such interest which it may have for them, will require more particularly to consult the following works:—Ehrenberg, 'Die Acalephen des rothen Meeres und der Organismus der Medusen der

Ostsee,' Berlin, 1836; **Kölliker**, 'Ueber die Randkörper der Quallen, Polypen und Strahlthiere,' **Froriep**'s neue Notizen, bd. xxv., 1843; **Von Beneden**, "Mémoire sur les Campanulaires de la côte d'Ostende," 'Mémoires de l'Académie de Bruxelles,' vol. xvii., 1843; **Desor**, "Sur la génération medusipare des Polypes hydraires," 'Annales d. Scienc. Natur. Zool.,' ser. iii., t. xii., p. 204; **Krohn**, "Ueber Podocoryna carnea," 'Archiv. f. Naturgeschichte,' 1851, b. i.; **McCrady**, "Description of Oceania, &c.," 'Proceedings of the Elliot Society of Natural History,' vol. i., 1859; **L. Agassiz**, "Contributions to the Acaliphæ of North America," 'Memoirs of the American Academy of Arts and Sciences,' vol. iii., 1860, vol. iv., 1862; **Leuckart**, 'Archiv. f. Naturgeschichte,' Jahrg. 38, b. ii., 1872; **Hensen**, "Studien über das Gehörorgan der Decapoden," 'Zeitschr. f. wiss. Zool.,' bd. xiii., 1863; **Semper**, "Reisebericht," 'Zeitschr. f. wiss. Zool.,' bd. xiii., u. xiv.; **Claus**, "Bemerkungen über Clenophoren und Medusen," 'Zeitschr. f. wiss. Zool.,' bd. xiv., 1864; **Allman**, "Note on the Structure of Certain Hydroid Medusæ," Brit. Assoc. Rep., 1867; **Fritz Müller**, "Polypen und Quallen von S. Catharina," 'Archiv. f. Naturgesch.,' Jahrg. 25, bd. i., 1859; also "Ueber die Randbläschen der Hydroidquallen," 'Archiv. f. Anatomie und Physiologie,' 1852.

A brief sketch of the contents of these and other memoirs on the histology of the Medusæ is given in Drs. **Hartwig** recently published work on the nervous system and sense organs of the Medusæ, and these authors point to the important fact that before the appearance of **Hæckel**'s memoir, **Leuckart** was the only observer who spoke for the fibrillar character of the so-called marginal ring-nerve; so that in **Hæckel**'s researches on *Gergonia*, whereby both true ganglion cells and true nerve-fibres were first demonstrated as occurring in the Medusæ, we have a most important step in the histology of these animals. **Hæckel**'s results in these respects have since been confirmed by **Claus**, 'Grundzüge der Zoologie,' 1872; **Allman**, 'A Monograph of the Gymnoblastic or Tubularian Hydroids,' 1871; **Harting**, "Notices Zoologiques," Niederlandisches 'Archiv. f. Zool.,' bd. ii., Heft 3, 1873; **F. E. Schulze**, 'Ueber den Bau von Syncorzne Sarsü;' O. and R. **Hertwig**, 'Das Nervensystem und die Sinnesorgane der Medusen;' and possibly also by **Eimer** in his forthcoming publication.

Of these memoirs by far the most important is that of Drs. **Hertwig**. These authors state that their investigations were completed before the physiological investigations of Dr. **Eimer** and myself had fallen under their notice; so that, as they remark, the agreement of their morphological with our physiological results is rendered by this independence the more valuable. In the present paper it will be impossible for me to render a full account of the important work which all scientific naturalists will consult for themselves. I will therefore confine myself to epitomising the lucid epitome which Drs. **Hertwig** give of their own histological results.

There is so great and fundamental a difference between the nervous system of the naked- and of the covered-eyed Medusæ, that a simultaneous description of the nervous system in both groups is not by these authors considered practicable. Beginning, therefore, with the naked-eyed division, they describe the nervous system as here consisting of two parts, a central and a peripheral. The central part is localised in the margin of the swimming-bell, and there forms a "nerve-ring" which is divided by the insertion of the veil into an upper and a lower nerve-ring. In many species the upper nerve-ring is spread out in the form of a flattish layer, which is somewhat thickened where it is in contact with the veil. In these species the nerve-ring is only indistinctly marked off from the surrounding tissues. But in other species the crowding together of the nerve-fibres at the insertion of the veil gives rise to a considerable concentration of nervous structures; while in others, again, this concentration proceeds to the extent of causing a well-defined swelling of nervous tissue against the epithelium of the veil and umbrella. In the Geryonidæ this swelling is still further strengthened by a peculiar modification of the other tissues in the neighbourhood, which has been previously described by Professor HÆCKEL. In all species the upper nerve-ring lies entirely in the ectoderm. Its principal mass is composed of nerve-fibres of wonderful tenuity, among which are to be found sparsely scattered ganglion cells. The latter are for the most part bipolar, more seldom multipolar. The fibres which emanate from them are very delicate, and, becoming mixed with others, do not admit of being further traced. Where the nervous tissue meets the enveloping epithelium it is connected with the latter from within, but differs widely from it; for the nerve-cells contain a longitudinally striated cylindrical or thread-like nucleus which carries on its peripheral end a delicate hair, while its central end is prolonged into a fine nerve-fibre. There are, besides these, two other kinds of cells which form a transition between the ganglion and the epithelium cells. The first kind are of a long and cylindrical form, the free ends of which reach as far as the upper surface of the epithelium. The second kind lie for the most part deep under the upper surface. They are of a large size, and present, coursing towards the upper surface, a long continuation, which at its free extremity supports a hair. In some cases this continuation is smaller, and stops short before reaching the outer surface. Drs. HERTWIG observe that in these peculiar cells we have tissue elements which become more and more like the ordinary ganglion cells of the nerve-ring the more that their long continuation towards the surface epithelium is shortened or lost, and these authors are thus led to conclude that the upper nerve-ring was originally constituted only by such prolongations of the epithelium cells, and that afterwards these prolongations gradually disappeared, leaving only their remnants to develop into the ordinary ganglion cells already described.

Beneath the upper nerve-ring lies the lower nerve-ring. It is inserted between the muscle tissue of the veil and umbrella, in the midst of a broad strand wherein muscle-fibres are entirely absent. It here constitutes a thin, though broad, layer which, like the upper nerve-ring, belongs to the ectoderm. It also consists of the same elements as the upper nerve-ring, viz.: of nerve-fibres and ganglion cells. Yet there is so distinct a difference of character between the elements composing the two nerve-rings that even in an isolated portion it is easy to tell from which ring the portion has been taken. That is to say, in the lower nerve-ring there are numerous nerve-fibres of considerable thickness which contrast in a striking manner with the almost immeasurably slender fibres of the upper nerve-ring. A second point of difference consists in the surprising wealth of ganglion cells in the one ring as compared with the other.

Thus, on the whole, there is no doubt that the lower nerve-ring presents a higher grade of structure than does the upper, as shown not only by the greater multiplicity of nerve-cells and fibres, but also by the relation in which these elements stand to the epithelium. For in the case of the lower nerve-ring the presumably primitive connexions of the nervous elements with the epithelium is well-nigh dissolved—this nerve-ring having thus separated itself from its parent structure, and formed for itself an independent layer beneath the epithelium. The two nerve-rings are separated from one another by a very thin membrane, which, in some species at all events, is bored through by strands of nerve-fibres which serve to connect the two nerve-rings with one another.

The peripheral nervous system is also situated in the ectoderm, and springs from the central nervous system, not by any observable nerve-trunks, but directly as a nervous plexus composed both of cells and fibres. Such a nervous plexus admits of being detected in the subumbrella of all Medusæ, and in some species may be traced also into the tentacles. It invariably lies between the layer of muscle fibre and that of the epithelium. The processes of neighbouring ganglion-cells in the plexus either coalesce or dwindle in their course to small fibres; at the margin of the umbrella these unite themselves with the elements of the nerve-rings. There are also described several peculiar tissue elements, such as, in the umbrella, nerve-fibres which probably stand in connexion with epithelium-cells; nerve-cells which pass into muscle-fibres, similar to those which KLEINENBERG has called neuro-muscular cells; and in the tentacles neuro-muscular cells joined with cells of special sensation (Sinneszellen).

No nervous elements could be detected in the convex surface of the umbrella, and it is doubtful whether they occur in the veil.

In some species the nerve-fibres become aggregated in the region of the generative organs and in that of the radial canals, thus giving rise in these localities to what may be called nerve-trunks. But in other species no such aggregations are apparent, the nervous plexus spreading out in the form of an even trellis-work.

In the covered-eyed Medusæ the central nervous system consists of a series of separate centres which are not connected by any commissures. These nerve-centres are situated in the margin of the umbrella, and are generally eight in number, more rarely twelve, and in some species sixteen. They are thickenings of the ectoderm, which either enclose the bases of the sense-organs, or only cover the ventral side of the same. Histologically they consist of cells of special sensation, together with a thick layer of slender nerve-fibres. Ganglion-cells, however, are absent, so that the nerve-fibres are merely processes of epithelium-cells.

Drs. HERTWIG made no observations on the peripheral nervous system of the covered-eyed Medusæ; but they do not doubt that such a system would admit of being demonstrated, and in this connexion they cite the observations of **Claus**, who describes numerous ganglion-cells as occurring in the subumbrella of *Chrysaora*. It is needless to remark how fully the anticipations of these authors have in this respect been realised by the researches of Mr. **Schäfer**.

Lastly, these authors compare the nervous system of the naked-eyed with that of the covered-eyed Medusæ, with the view of indicating the points which show the latter to be much less completely developed than the former. These points are, that in the nerve-centres of the covered-eyed Medusæ there are no true ganglion-cells, or only very few; that the mass of the central nervous system is very small, so that all the nerve-centres taken together contain far fewer nerve-fibres than do the nerve-rings of the naked-eyed Medusæ; and that the centralization of the nervous system is less complete in the covered-eyed than it is in the naked-eyed Medusæ, as shown by the multiplication of similarly formed nerve-centres which occurs in the latter group.

In this memoir the authors also supply an interesting description of the structure of the sense organs in various species of Medusæ; but it seems scarcely necessary to extend the present *résumé* of their work by entering into this division of their subject.

Within the last few weeks Drs. **Hertwig** have supplemented the work which I have thus briefly epitomised with another valuable contribution to the morphology of the Medusæ. This is entitled 'Der Organismus der Medusen und seine Stellung zur Keimblättertheorie.' As, however, this memoir has no special relation to the locomotor system of the Medusæ, it does not appear to require in this paper any special consideration.

VIII. *Researches on Explosives.* No. II.—*Fired Gunpowder.*

By Captain Noble (*late R.A.*), *F.R.S., F.R.A.S., F.C.S., &c., and* F. A. Abel, *C.B., F.R.S., V.P.C.S., &c., &c.*

Received May 21,—Read May 29, 1879.

Contents.

	Page
Criticisms on the first memoir of the authors by the Commission appointed by the Académie des Sciences	204
Statement of mean results of analysis of the gaseous and solid products of explosion of all the descriptions of powder experimented with	206
Complete tabular statement of results of analysis of products obtained at different densities	207
Discussion of M. Berthelot's views as to the expression of the decomposition of gunpowder	208
Observations relating to the formation of potassium hyposulphite	211
Special experiments relating to this point	213
Further analysis of R. L. G. powder used in the experiments	217
Composition of other gunpowders used in later experiments	218
Results furnished by these	219
General observations on the complete analytical results	220
Further experiments on the determination of heat generated by fired gunpowder	223
Tabulated results of heat determinations	227
Results of calculation of the mean specific heats of the products of explosion	228
Volumes of permanent gases generated by different powders	229
Relation of volumes of permanent gases to the units of heat evolved	230
Pressures given by mining powder and Waltham Abbey powder	231
Actual temperatures of explosion	232
Relation between tension of the gases developed in a gun and the volume which they occupy	232
Table of tensions of the gases (1) in a gun and (2) if allowed to expand without production of work	235
Table showing total work performed in a gun by Waltham Abbey powder	236
Application of this table	239
Effects of retention of the shot in the chamber of a gun upon the tensions developed	242
Estimation of the position of the shot in a gun's bore when complete combustion has been effected	244
Concluding remarks	245
Appended memorandum of proportions of elements found in each product of explosion, and existing in the original powder	246
Abstract of experiments	262
Note with respect to hyposulphite	277
Products of explosion calculated on the supposition that all hyposulphite found originally existed as sulphides	278

During the course of the experiments that we have undertaken in extending our researches on explosives to the investigation of the action and results of fired gun-cotton, we have had occasion to examine some points connected with the subject of our former memoir on "Fired Gunpowder;" and as the attention which the researches described in that memoir have received, especially on the Continent, is an evidence both of the theoretical and practical importance of our subject, we propose, prior to laying before the Society our researches on gun-cotton, to discuss a few points of considerable interest which have arisen out of our former investigations, and to give the results of some further experiments on gunpowder.

The Academy of Sciences of France having done us the honour to appoint a Commission to report on our researches, there have appeared in the 'Comptes Rendus' a joint report[*] by General MORIN and M. BERTHELOT, and two[†] separate memoirs on certain chemical points by the latter savant.

The high appreciation of our labours shown by the Academy has induced us to pay special attention to one or two points mentioned by the distinguished reporters as being open to discussion; we will now proceed to consider them, and to detail some further experiments calculated to throw light upon the different questions raised.

The principal points to which General MORIN and M. BERTHELOT draw attention are—

1. Potassium hyposulphite has been found as one of the products of combustion of gunpowder by every recent investigator. But the question arises, Is this product either wholly or in part primary? Or is it to be considered as secondary, formed from the primary products during the rapid loss of heat to which they are exposed? Or is it, finally, to be considered only as formed from the sulphide by the absorption of oxygen, during the processes of removal from the cylinder and of analysis, and therefore to be regarded as an accidental product?

2. In the memoir in question we stated that, according to our view, "any attempt to express, even in a complicated chemical equation, the nature of the metamorphosis which a gunpowder of average composition may be considered to undergo, would only be calculated to convey an erroneous impression as to the simplicity or definite nature of the chemical results and their uniformity under different conditions, while possessing no important bearing upon an elucidation of the theory of the explosion of gunpowder."

M. BERTHELOT, however, in a memoir upon the explosion of powder, based on our results, proposes to represent these results by a system of simultaneous equations expressing the chemical metamorphosis undergone by powder, at least as far as regards its fundamental products.

3. In the joint report of General MORIN and M. BERTHELOT, and in the separate memoir above referred to of M. BERTHELOT, special attention is called to the heat disengaged by the explosion, and our determination, presented, as the reporters point out, with some reserve, is considered to be too low, partly because the apparatus used did not admit of extreme delicacy, and partly because higher determinations have been made by M. TROMENEC and MM. ROUX and SARRAU.

---

[*] 'Comptes Rendus,' tom. lxxxii., p. 487.

[†] Idem., pp. 400 and 469.

We now proceed to the discussion at length of these points, prefacing our remarks by Tables I. and II. hereto annexed.

The former of these tables, in addition to giving the results of one or two analyses which had not been completed when our first memoir was published (we have not considered it necessary to repeat the portions of this table already published), shows the mean percentage composition, by volume, of the gases, and the mean percentage composition, by weight, of the solid residues for each of the three principal powders examined by us. It also shows the highest and lowest proportions in which, with each powder, any particular product occurs, and gives the results obtained from the examination of the products of combustion of four descriptions of powder differing in many respects from the powders which formed the main subject of our memoir.

Table II. contains the complete results of all our analyses; it shows the proportion by weight of each solid and gaseous product, and includes also the amount of water pre-existent in the various specimens of powder operated on.

TABLE I.[*]—Showing the mean analytical results obtained from an examination of the solid and gaseous products of decomposition of pebble, R. L. G., and F. G. powders; showing also the same particulars with respect to four other powders.

Nature of powder.	Mean density of products of combustion.	Percentage composition by volume of the gas.							Percentage composition by weight of the solid residue.									
		Carbonic anhydride.	Carbonic oxide.	Nitrogen.	Sulphydric acid.	Marsh gas.	Hydrogen.	Oxygen.	Potassium carbonate.	Potassium sulphate.	Potassium hyposulphite.	Potassium monosulphide.	Potassium sulphocyanate.	Potassium nitrate.	Potassium oxide.	Ammonium sesqui-carbonate.	Sulphur.	Charcoal.
Pebble, W. A.	Means	48·92	13·70	32·13	2·60	0·31	2·34	..	57·19	12·47	13·89	10·25	0·27	0·23	..	0·10	5·45	0·15
	Highest	51·75	16·09	32·75	4·23	0·68	3·50	..	64·20	15·02	32·18	19·12	0·57	0·48	..	0·17	8·45	1·35[†]
	Lowest	44·78	10·87	31·31	1·70	..	1·67	..	50·20	9·13	3·71	2·23	0·06	..	..	0·06	0·61	Trace.
R. L. G., W. A.	Means	49·29	12·47	32·91	2·65	0·43	2·19	0·06	59·06	14·59	13·20	6·27	0·22	0·26	..	0·07	6·26	0·07
	Highest	52·65	17·04	35·60	4·29	0·84	3·01	0·57	66·51	24·22	25·33	9·92	0·49	0·56	..	0·18	12·03	0·71
	Lowest	46·29	8·98	30·29	1·56	..	1·27	..	48·68	4·64	3·08	2·02	0·05	..	..	0·04	1·25	Trace.
F. G., W. A.	Means	50·63	10·46	33·20	2·48	0·19	2·96	0·08	48·32	21·11	24·79	2·41	0·12	0·16	1·29	0·05	1·75	..
	Highest	53·34	16·25	34·64	3·76	0·50	4·13	0·28	59·39	24·22	34·61	5·12	0·25	0·26	5·39	0·15	5·72	..
	Lowest	44·76	7·71	32·22	2·00	..	2·04	..	41·88	17·86	5·30	..	0·02	0·08	..	0·01	0·09	..
96. R. L. G., W. A.	60 per cent.	46·29	14·52	32·40	4·29	0·36	2·14	..	65·27	10·71	6·89	7·65	0·28	0·45	..	0·09	8·66	..
78. R. F. G., W. A.	70 „	52·40	8·86	34·51	1·60	0·12	2·51	..	58·94	21·89	8·15	4·22	0·04	0·06	..	0·06	6·65	Trace.
79. Spanish spherical	70 „	53·34	4·62	37·80	2·74	..	1·29	0·21	34·97	47·62	7·60	3·17	0·04	0·93	..	0·04	5·63	..
196. CURTIS and HARVEY'S No. 6	30 „	50·22	7·52	34·46	2·08	2·46	3·26	..	58·51	21·43	3·93	10·02	..	0·29	..	0·09	5·73	..
194. Mining powder	30 „	32·15	33·75	19·03	7·10	2·73	5·24	..	40·78	0·58	5·84	33·20	2·92	0·09	..	1·76	12·93	2·00

---

[*] See Table XII.

[†] Not estimated—insoluble residue chiefly iron pyrites.

natures, and of various gravimetric densities.

No. of experiments.	Nature of powder.	Mean density of products of combustion.*	Proportions by weight of gaseous products.							Proportions by weight of the solid residue.										Water.	Proportion by weight of total gaseous products.	Proportion by weight of total solid products.	Water.
			Carbonic anhydride.	Carbonic oxide.	Nitrogen.	Sulphydric acid.	Marsh gas.	Hydrogen.	Oxygen.	Potassium carbonate.	Potassium sulphate.	Potassium hyposulphite.	Potassium monosulphide.	Potassium sulphocyanate.	Potassium nitrate.	Potassium oxide.	Ammonium sesqui-carbonate.	Sulphur.	Charcoal.				
8	Pebble, W. A.	10	·2553	·0514	·1140	·0133	..	·0007	..	·3084	·0835	·1152	·0412	·0005	·0027	..	·0009	·0034	..	·0095	·4347	·5558	·0095
7		20	·2494	·0570	·1108	·0182	..	·0009	..	·3186	·0760	·0206	·1001	·0003	..	..	·0005	·0381	..	·0095	·4363	·5542	·0095
9		30	·2595	·0545	·1113	·0124	..	·0007	..	·3282	·0696	·0239	·0911	·0012	·0002	..	·0007	·0372	..	·0095	·4384	·5521	·0095
12		40	·2624	·0471	·1085	·0069	·0006	·0006	..	·3115	·0745	·0794	·0548	·0014	·0005	·0077	·0004	·0342	..	·0095	·4261	·5644	·0095
14		50	·2743	·0469	·1128	·0083	·0012	·0005	..	·3069	·0652	·0335	·1045	·0013	·0011	..	·0003	·0337	..	·0095	·4440	·5365	·0095
37		60	·2654	·0470	·1087	·0093	·0011	·0005	..	·3213	·0752	·0556	·0641	·0019	·0017	..	·0003	·0384	..	·0095	·4320	·5585	·0095
38		70	·2604	·0415	·1065	·0128	·0007	·0005	..	·2852	·0726	·1827	·0127	·0022	·0014	..	·0003	·0110	..	·0095	·4224	·5681	·0095
43		80	·2690	·0396	·1084	·0080	·0006	·0004	..	·3322	·0584	·1167	·0220	·0026	·0018	..	·0005	·0303	..	·0095	·4260	·5645	·0095
77		90	·2684	·0359	·1080	·0085	·0013	·0005	..	·3645	·0518	·0754	·0218	·0032	·0025	..	·0007	·0480	..	·0095	·4226	·5679	·0095
		Means	·2627	·0468	·1099	·0109	·0006	·0006	..	·3196	·0696	·0781	·0569	·0016	·0013	·0009	·0005	·0306	..	·0095	·4314	·5601	·0095
		Highest	·2743	·0570	·1140	·0182	·0013	·0009	..	·3645	·0825	·1827	·1045	·0032	·0027	·0077	·0009	·0480	..	·0095	·4440	·5681	·0095
		Lowest	·2494	·0359	·1065	·0069	..	·0004	..	·2852	·0518	·0206	·0127	·0003	·0002	..	·0003	·0034	..	·0095	·4224	·5521	·0095
1	R. L. G., W. A.	10	·2569	·0300	·1188	·0164	·0006	·0005	..	·2972	·1160	·1154	·0228	..	·0032	..	·0003	·0041	·0072	·0106	·4232	·5662	·0106
3		20	·2477	·0389	·1189	·0148	·0001	·0006	·0022	·3094	·1364	·0326	·0541	·0003	·0007	..	·0004	·0323	·0001	·0106	·4232	·5662	·0106
4		30	·2582	·0386	·1096	·0126	..	·0007	..	·2984	·1380	·0732	·0334	·0003	·0002	..	·0002	·0260	..	·0106	·4197	·5697	·0106
11		40	·2595	·0356	·1125	·0077	·0005	·0006	..	·2789	·1310	·1379	·0116	·0009	·0007	..	·0002	·0118	..	·0106	·4164	·5730	·0106
70		50	·2494	·0558	·1011	·0065	·0016	·0007	..	·3482	·0266	·1455	·0204	·0017	·0029	..	·0006	·0284	..	·0106	·4151	·5743	·0106
39		60	·2648	·0467	·1065	·0066	·0007	·0005	..	·3597	·0619	·0365	·0559	·0015	..	..	·0006	·0475	..	·0106	·4258	·5636	·0106
96		60	·2457	·0490	·1090	·0176	·0007	·0005	..	·3702	·0608	·0392	·0434	·0016	·0026	..	..	·0491	..	·0106	·4225	·5669	·0106
41		70	·2576	·0441	·1053	·0114	·0011	·0004	..	·3435	·0600	·1059	·0219	·0028	·0024	..	·0001	·0329	..	·0106	·4199	·5695	·0106
44		80	·2672	·0401	·1060	·0062	·0014	·0004	..	·3777	·0501	·0175	·0514	·0014	·0010	..	·0006	·0684	..	·0106	·4213	·5681	·0106
68		90	·2720	·0352	·1074	·0076	·0015	·0003	..	·3715	·0482	·0486	·0409	·0021	·0011	..	·0009	·0521	..	·0106	·4240	·5654	·0106
		Means	·2580	·0414	·1095	·0106	·0008	·0005	·0002	·3355	·0829	·0752	·0356	·0013	·0015	..	·0004	·0353	·0007	·0106	·4211	·5683	·0106
		Highest	·2720	·0558	·1189	·0176	·0016	·0007	·0022	·3777	·1380	·1455	·0559	·0028	·0032	..	·0009	·0684	·0072	·0106	·4258	·5743	·0106
		Lowest	·2457	·0300	·1011	·0062	..	·0003	..	·2789	·0266	·0175	·0116	..	..	..	..	·0041	..	·0106	·4151	·5636	·0106
16	F. G., W. A.	10	·2423	·0561	·1122	·0095	·0004	·0010	·0006	·2772	·1005	·1338	·0193	..	·0011	·0308	·0004	..	..	·0148	·4221	·5631	·0148
17		20	·2475	·0410	·1074	·0153	..	·0010	..	·3401	·1388	·0364	·0294	·0001	·0005	..	·0009	·0328	..	·0148	·4122	·5730	·0148
18		30	·2586	·0370	·1050	·0088	..	·0008	·0010	·2645	·1302	·1583	·0161	·0004	·0006	..	·0001	·0038	..	·0148	·4112	·5740	·0148
19		40	·2639	·0334	·1055	·0079	..	·0008	..	·2576	·1250	·1640	·0193	·0004	·0005	..	·0002	·0067	..	·0148	·4115	·5737	·0148
75		50	·2611	·0338	·1080	·0087	·0005	·0007	..	·3207	·1186	·0768	·0248	·0004	·0005	..	·0005	·0301	..	·0148	·4128	·5724	·0148
40		60	·2651	·0312	·1080	·0089	·0003	·0006	..	·2391	·1269	·1822	..	·0009	·0010	·0183	·0001	·0026	..	·0148	·4141	·5711	·0148
42		70	·2678	·0253	·1100	·0080	·0009	·0005	·0006	·2461	·1202	·1836	..	·0013	·0011	·0170	·0002	·0026	..	..	·4131	·5721	·0148
47		80	·2598	·0265	·1105	·0101	·0008	·0005	·0007	·2514	·1218	·1995	..	·0014	·0015	..	·0002	·0005	..	·0148	·4089	·5763	·0148
69		90	·2698	·0247	·1088	·0116	·0003	·0005	..	·2880	·1046	·1474	·0152	·0014	·0015	..	·0002	·0112	..	..	·4157	·5695	·0148
		Means	·2596	·0343	·1084	·0099	·0003	·0007	·0008	·2762	·1207	·1418	·0138	·0007	·0009	·0074	·0003	·0100	..	·0148	·4135	·5717	·0148
		Highest	·2698	·0561	·1122	·0153	·0009	·0010	·0010	·3401	·1388	·1995	·0294	·0014	·0015	·0308	·0009	·0328	..	·0148	·4221	·5763	·0148
		Lowest	·2423	·0247	·1050	·0079	..	·0005	..	·2391	·1005	·0304	..	..	·0005	..	·0001	..	..	..	·4089	·5681	·0148
78	R. F. G., W. A.	70	·2652	·0285	·1110	·0063	·0002	·0006	..	·3420	·1268	·0472	·0245	·0002	·0004	..	·0005	·0386	..	·0080	·4118	·5802	·0080
79	Spanish spherical	70	·2424	·0133	·1091	·0096	..	·0003	·0007	·2161	·2943	·0470	·0196	·0003	·0058	..	·0002	·0348	..	·0065	·3754	·6181	·0065
196	Curtis & Harvey's No. 6	30	·2576	·0245	·1124	·0082	·0046	·0008	..	·3395	·1243	·0228	·0582	..	·0017	..	·0005	·0332	..	·0117	·4081	·5802	·0117
194	Mining powder	30	·2254	·1508	·0849	·0385	·0070	·0017	..	·1938	·0028	·0277	·1578	·0138	·0004	..	·0084	·0614	·0095	·0161	·5083	·4756	·0161

* See Table XIII.

A careful examination into the nature and proportions of the products furnished by the explosion of three descriptions of service gunpowder, differing but little in composition from each other, and by one and the same sample of powder under different conditions as regards pressure (or space in which the explosion took place), led us to the conclusion, which it may be as well to repeat in the precise terms used in our former memoir, namely, that "any attempt to express, even in a comparatively complicated chemical equation, the nature of the metamorphosis which a gunpowder of average composition may be considered to undergo when exploded in a confined space, would . . . . only be calculated to convey an erroneous impression as to the simplicity or the definite nature of the chemical results, and their uniformity under different conditions" (p. 85).

In giving expression in the foregoing terms to this conclusion, we certainly did not intend to convey the impression, nor do we consider that our words are at all susceptible of the interpretation, that it was impossible to put into some form of equation (as was done, for instance, by BUNSEN and SCHISCHKOFF in the case of the analytical results arrived at by them), a representation of a variety of reactions, which if *assumed* to take place simultaneously among different proportions of the powder-constituents, might express results approximating to one or other of the analytical results obtained by us, and might thus afford some approximate theoretical representation of the metamorphosis of gunpowder when fired in closed vessels.

But the very great variations in composition (of the solid portion more especially) of the products of explosion of samples of gunpowder presenting only small differences in constitution, afforded, in our opinion, most conclusive proof that the reactions which occur among the powder constituents are susceptible of very considerable variations, regarding the causes of which it appears only possible to form conjectures, and that consequently "no value whatever can be attached to any attempt to give a general chemical expression to the metamorphosis of gunpowder of normal composition."

In one of the series of interesting communications made by M. BERTHELOT to the Académie des Sciences in 1876,[*] as contributions to the "History of Explosive Agents," that chemist gives to our conclusions, as expressed in our former memoir, an interpretation which, as above pointed out, they certainly cannot be considered to bear, when he says we have stated that the variations in the proportions of the principal products of explosion are opposed "to all general chemical representation of the metamorphosis produced by the explosion," an opinion contrary, as he states, to all that is known in chemistry. Starting with the above assumption of the nature of our views, M. BERTHELOT proceeds to demonstrate that, in order to account for the formation of the *chief* products in some particular proportion in which potassium sulphate is so small as to allow of its being neglected, the powder-constituents must be presumed to react upon each other simultaneously, in prescribed proportions, according to *three*, or, if the sulphate amount to 12 or 14 per cent., according to *four* out of *five* different

[*] 'Comptes Rendus,' tom. lxxxii., p. 400.

theoretical reactions, which, if assumed to occur simultaneously, in variable proportion and number, M. BERTHELOT regards as satisfactorily explaining ("and definitely reducing to five simple reactions") the formation of carbon dioxide, carbon oxide, potassium sulphate, sulphide and carbonate, from a powder of what we call normal composition.*

After giving further equations which apply to the extreme results (in regard to the chief products only) assumed to be obtainable from the introduction, on the one hand, of excess of saltpetre, on the other, of excess of charcoal, into the composition of powder, M. BERTHELOT passes to what he terms the accessory products and, excluding from these potassium hyposulphite, which he deals with separately, he first gives two equations to account for the production of sulphocyanide; then two more to explain the existence of ammonium sesquicarbonate (which he believes to be formed by the action of water-vapour on potassium cyanide). The existence of sulphuretted or free hydrogen are explained by two more equations, and marsh gas is assumed to result from "the pyrogenous decomposition of the charcoal in the powder." Lastly, an equation is given to account for the possible formation of traces of hyposulphite, which BERTHELOT however regards entirely as a product formed during the collection and analytical treatment of the solid residue, but which we nevertheless believe we shall conclusively prove[*] to be formed in very notable quantities before the solid residue can have undergone alteration from external causes.

---

* The five simple reactions in question are thus explained:—

1. $NO_6K+S+C_3=K.S.+3CO_2+N.$
2. „ „ „ $=K.C.O_3+CO.+CO_2+N+S.$
3. „ „ „ $=KCO_3+1\frac{1}{2}C\,O._2+N+S+\frac{1}{2}C.$
4. „ „ „ $=KSO_4+2CO.+N+C.$
5. „ „ „ $=KSO_4+CO._2+N+C_2.$

When sulphate is formed in such small quantities that it may be neglected, the simultaneous reactions supposed to occur are 1, 2, and 3, by quantities of the powder proportionate to the numbers $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{6}$, but when the sulphate amounts to 12 or 14 per cent., the simultaneous reactions supposed to occur are Nos. 1, 3, 4, and 5, with quantities of powder corresponding to the numbers $\frac{1}{3}$, about $\frac{1}{2}$, $\frac{1}{8}$, and $\frac{1}{12}$. As there is only one single instance out of twenty-nine analyses of powder residues in which the sulphate was found to amount to as little as 4·6 per cent. of the solid products (the next lowest proportion being nearly double that amount), it can scarcely be assumed that M. BERTHELOT's first arrangement of reactions can represent any but a most exceptional result. Again, the acceptance of his arrangement of four equations in the proportions he indicates as accounting for the formation of the chief products when the sulphate amounts to 12 or 14 per cent. of the total constituents, involves the assumption that a somewhat considerable proportion of charcoal should remain unoxidised; in fact, nearly 2·5 per cent. of carbon should be found in the residue. The detection and determination of such a constituent of powder-residue does not involve any difficulty, yet there were only three instances out of eighteen residues (in which the sulphate was considerable in amount) where the charcoal was present in estimable quantities; in two of these it was below 1 per cent., in the other it was only 0·01 per cent. In a few other residues only traces of charcoal were discovered; the larger number contained none.

These points are referred to in illustration of how imperfectly M. BERTHELOT's not very simple arrangement of theoretical reactions correspond to the results actually obtained, even so far only as the chief products are concerned.

It will be seen from the foregoing outline of M. BERTHELOT'S theoretical explanation of the chemical changes involved in the metamorphosis of gunpowder, that the simplest form of expression which he can give to the formation of the products of explosion consists in the incorporation of nine or ten distinct reactions occurring simultaneously, but in very variable proportions, which have to be supplemented by three or four other chemical equations, by which the formation, during the process of cooling, of certain products believed to be secondary, is explained. Now, although such speculations as the above are unquestionably interesting, and, it may be added, of a nature which must occur to those who desire to give some kind of definite explanation, for purposes of elementary instruction, of the chemical changes involved in the explosion of powder, we fail to see that beyond this they do more than afford the strongest confirmation of the correctness of our conclusion, that "no value whatever can be attached to any attempt to give a general chemical expression to the metamorphosis of a gunpowder of normal composition."

With regard to the *potassium hyposulphite* which is included in our statement of the composition of the solid products of explosion, we have to submit the following considerations.

In the analytical results furnished by the solid residues, as detailed in our first memoir, the hyposulphite ranged in amount from 3 to 35 per cent.; and on comparing the results of different analyses, it is observed that in most instances the proportion of monosulphide was small when the hyposulphite was large in amount, and in a few instances—all of them F. G. powder-residues—in which the proportion of the latter was very high, there was no sulphide at all.

Being fully alive to the possibility of the existence of potassium polysulphide in the solid residue giving rise to the production of some hyposulphite through the agency of atmospheric oxygen, great precautions were taken, especially in the later experiments, in collecting and preserving the residue and in submitting it to treatment for analysis, to guard against this possible source of error.

In the first place, it should be mentioned that the residue consisted in nearly all cases of fused, very hard masses, collected at the bottom of the explosion vessel, the sides of which were, moreover, generally covered with very thin films. The action of atmospheric oxygen upon the fused solid could only be superficial, but would vary in extent with the amount of surface of the residue exposed to the air during removal from the explosion apparatus or subsequent exposure. The latter was avoided as much as possible, as the residues were transferred at once, as they were detached from the surfaces of the explosion-vessel, into small bottles, in which they were carefully sealed up. It was only in one or two instances that, before opening the bottles, an odour of sulphuretted hydrogen, distinctly perceptible at the sealed surfaces of the mouths, indicated a slight imperfection in the sealing of the bottles.

---

[*] See note at end of this memoir.

The difficulties in the way of reducing to a minimum the exposure to air of the residues during their detachment from the explosion-vessel were, however, very much greater. We pointed out in our first memoir that in almost all cases the residues were in the form of exceedingly hard and compact masses, which had to be cut out with steel chisels, and that, although portions of the mass were detached in the form of lumps, a considerable amount of it flew off before the chisel in fine dust. The utmost care was taken to avoid exposure of the detached residues to the air, but it was of course impossible to avoid their being more or less attacked by atmospheric oxygen during the period of their collection. There is no doubt, moreover, that the residues, which differed greatly from each other in structure and in their tendency to absorb moisture and to become heated upon exposure to air, were susceptible in very variable degree to atmospheric oxidation. We, therefore, are quite prepared to admit that, of the large amount of hyposulphite found in a number of the analyses, a proportion, and in some instances possibly a large one, may have been produced by the agency of atmospheric oxygen during the removal of the residue from the apparatus; and the results of some special experiments, which we shall presently quote, appear to favour the conclusion that in those instances where no sulphide was discovered, its absence may have been ascribable to atmospheric oxidation. We regret having neglected to make any reference to this probable source of error in describing the results of our analyses, our belief being at the time that any important alteration of the residue by atmospheric action was sufficiently guarded against; at the same time, it is right we should point out that, in several instances in which the circumstances attending the manipulation of the solid residue and its consequent mechanical condition were apparently most favourable to its accidental oxidation, the proportion of hyposulphite formed was comparatively moderate in amount.

On the other hand, we cannot concur in M. BERTHELOT's view that the existence of hyposulphite among our analytical results is also ascribable in part to accidental oxidation of potassium sulphide during the analytical manipulations. These were carried out with great uniformity so far as certain preliminary operations were concerned, which consisted, firstly, in dissolving the residue in water which had been carefully boiled to expel air, and, secondly, in filtering the solution in closed vessels—both of these being rapidly completed operations. The receiving vessel contained pure ignited cupric oxide, with which, as soon as the filtering operation was completed, the solution was agitated until it became colourless.

The fact that in some of the analyses, all of which, we repeat, were uniformly conducted in regard to the above points, from 3 to 10 per cent. only of hyposulphite were found, while the proportion of monosulphide in these analyses ranged from 7 to 19 per cent. (being above 9 per cent. in eight instances), appears to afford substantial proof that accidental atmospheric oxidation during the collection and analysis of the residues is not sufficient to account for all but the very small quantities of hyposulphite which M. Berthelot considers could only have pre-existed in the residue examined by us. That chemist appears, moreover, to have overlooked the following facts given by us in our first memoir :—

1. Separate examinations (conducted precisely alike) of the upper and lower portions of some of the residues showed that considerably larger proportions of hyposulphite existed in the *upper* portions. In one case quoted by us in our first memoir, the upper portion contained 17·14 per cent. of hyposulphite, while the lower portion only contained 4·34 per cent. At the same time there was only a difference of 1·27 per cent. in the proportions of monosulphide existing in the two portions of the residue (6·03 in the upper part, and 7·3 in the lower), while there was a very great difference in the amount of free sulphur (4·88 in the upper part, and 10·09 in the lower).

2. One of the small buttons of the fired solid products, of which there was generally one found attached to the firing plug in the cylinder, was examined for sulphide and hyposulphite (it having been detached without fracture, and at once sealed up in a small tube). It contained the latter, but none of the former, while the mass of the residue of this particular experiment contained a somewhat considerable proportion of sulphide.

3. The production of high proportions of hyposulphite was but little affected by any variations in the circumstances attending the several explosions (*i.e.*, whether the spaces in which the powder was exploded were great or small), excepting that the amount was high in all three cases when the powder was exploded in the largest space. On the other hand, a great reduction in the size of grain of the gunpowder used appeared to have a great influence upon the production of hyposulphite, as when passing from a very large grain powder (pebble or R. L. G.) to a fine grain powder (F. G.).

Thus the production of hyposulphite exceeded 20 per cent. in—

3 experiments out of 9 with pebble powder (Nos. 8, 38, 43).
3 ,, ,, 10 ,, R. L. G. ,, (Nos. 1, 11, 70).
7 ,, ,, 9 ,, F. G. ,, (Nos. 16, 18, 19, 40, 42, 47, 69).

It was below 10 per cent. in—

4 experiments out of 9 with pebble powder (Nos. 7, 9, 11, 37).
5 ,, ,, 10 ,, R. L. G. ,, (Nos. 3, 39, 44, 68, 96).
1 ,, ,, 9 ,, F. G. ,, (No. 17).

There were no circumstances connected with the carrying out of the explosions, or with the collection and analysis of the residues, to which the above great differences between the results furnished by fine grain powder and by the two large grain powders could be ascribed.

While, however, certain of the great variations in the proportions of hyposulphite and sulphide, which cannot be accounted for by variations of structure of the residue or of manipulations favourable to oxidation by atmospheric agency, appear to us to demonstrate that the hyposulphite is formed in the solid residue before the explosion-vessel is opened, and indeed in such amount that it must be regarded as an important product (whether it be a primary or a secondary one), we have been anxious to obtain, if possible, some more decisive evidence as to the probable proportions of hyposulphite actually existing in the residues furnished by the explosion of gunpowder in closed vessels. We therefore varied the method of collecting and preparing the residues for analysis, in the experiments of which the following is an account:—

1. 5960 grains (386·2 grammes) of the R. L. G. and pebble powders used in these researches were fired in the large cylinder under a density of 0·40.

Immediately on opening the cylinder in each case, the solid products were as nearly as possible divided into two equal portions, consisting of the top and the bottom. Each of these portions was again divided roughly into two equal parts, one of which, in large lumps, was, as rapidly as possible (being but for a few seconds exposed to the air), sealed in dry bottles freed, or nearly so, from oxygen, the other moieties being finely ground and freely exposed to the air for 48 hours.

The only point of difference calling for remark in the appearance of the two residues was the difference in colour, the residue from the pebble being decidedly the lighter in colour, both on the surface and in fracture; but there were material differences in the behaviour of the *ground* portions of the two powder residues.

With both powders, the bottom ground portion heated very decidedly more than the top; but while, in the R. L. G., this tendency was exhibited in a remarkably low degree, with the pebble the tendency to heat was, we think, abnormally high. In the latter case, the ground deposit from the top began to heat immediately on being placed upon paper. The deposit on the apex of the cone and in the interior, where the heat was highest, changed rapidly in colour to a light yellow, tinged with green.

The ground bottom part of the residue darkened considerably during the development of heat, and an orange-coloured deposit was condensed on the surface.

When the heat was highest, a considerable quantity of vapour was given off. Its smell was very peculiar; SH_2 was distinctly perceptible, but was by no means the dominant odour.

The maximum temperature appeared to be reached at about twenty minutes after exposure. A thermometer placed in the centre indicated a temperature of over 600° F. (315° C.), and the paper on which the residue was placed was burnt through. After half an hour's exposure the deposit cooled very rapidly.

It should be observed that the physical characteristics of the ground deposit were altered very materially by the heating. When the residue is taken out of the exploding cylinder, it is difficult to pound in the mortar, being somewhat unctuous; but after the development of heat it becomes crisp, and is readily powdered.

2. In the examination that we have instituted of the products of explosion of a

sample of sporting powder (CURTIS and HARVEY'S No. 6), and of mining powder, the following course of proceeding was adopted for the removal of the solid residue from the explosion-vessel, and its preparation for analysis:—Distilled water which had been freed from air by long-continued boiling, was siphoned into the explosion-vessel when the latter had cooled, so that air was never allowed to come into contact with the solid residue. When the cylinder was thus quite filled with water, it was closed, and set aside for sufficient time to allow the residue to dissolve completely. The solution was then decanted into bottles freed from oxygen, which were quite filled with the liquid, and carefully sealed up until required for analysis, in carrying out which the course already described was pursued.

The products obtained by the first of these modifications of the ordinary course of procedure were submitted to partial examination, the chief object being to see to what extent the proportions of hyposulphite and sulphide varied in the upper and lower portions of the residue, and the extent to which they were affected by the great difference in the mode of treatment sustained by the different portions of one and the same residue. The proportion of hyposulphite was determined in every instance, and the products were also examined in all cases for sulphide. In the first experiment the exact proportion of this latter constituent was ascertained only in one of the three portions of the residue in which it existed; it will be seen that one of the ground portions contained none. The sulphate was determined in all instances, and, in the second experiment, the proportions of carbonate existing in the upper and lower portions of the (unground) residue were ascertained. The analytical results obtained are given in the following table:—

TABLE III.—Illustrating the effects of exposure upon the proportions of hyposulphite and monosulphide in the solid products.

Description of experiment.			Percentage composition by weight of solid residue.				
			Potassium sulphate.	Potassium hyposulphite.	Potassium monosulphide.	Sulphur.	
EXPERIMENT No. 121.							
R. L. G. powder, Waltham Abbey make . . . . Proportion of space occupied by the charge in the chamber about 40 per cent.	Top portion of residue	Unground	12·14	5·83	Large quantity	not determined	
		Ground	11·52	27·52	none		
	Bottom portion of residue	Unground	10·85	8·51	19·76	5·0	
		Ground	10·87	36·34	under 3 per cent.		
EXPERIMENT No. 122.							
Pebble powder, Waltham Abbey make . . . . Proportion of space occupied by the charge in the chamber about 40 per cent.	Top portion of residue	Unground	14·08	5·01	13·26	8·71	Carbonate of potassium 58·9
		Ground	16·22	31·06	none		
	Bottom portion of residue	Unground	13·80	6·46	16·12	7·49	Carbonate of potassium 56·06
		Ground	16·62	32·91	none		

It will be seen from the foregoing numerical results that in both experiments those portions of the residue which were exposed to the air only for a few seconds, and of which only small surfaces *were* thus exposed (as they were collected in large lumps), contained hyposulphite ranging in amount from 5 to 8·5 per cent. Those portions which were specially treated for the purpose of favouring to the utmost the formation of hyposulphite from sulphide through atmospheric agency, contained, as was to be expected, very large proportions of the former, while the latter had entirely disappeared in three out of the four portions of very finely pulverised residue. In the fourth, however, even after its free exposure to air for forty-eight hours, there still remained nearly 3 per cent. of sulphide. Now, as in no single instance in the entire series of our experiments did any accidental circumstances occur which even distantly approached the special conditions favourable to the oxidation of the sulphide which were introduced into these particular experiments, we consider ourselves justified in arriving at the conclusion that the total absence of sulphide in the residues furnished by the fine grain powder in experiments 40, 42, and 47 was not due to accident in the manipulations, and that in those residues in our series of analyses which were found to contain large quantities of hyposulphite (as in six out of the nineteen experiments with pebble and R. L. G. powder, and eight out of the nine with F. G. powder) a great proportion, at any rate, of that hyposulphite existed in those particular residues before their removal from the explosion-vessel. The circumstance that the residues furnished by our two most recent experiments (with sporting powder and mining powder), in the treatment of which the possibility of accidental formation of hyposulphite was altogether guarded against, contained 4, and about 6, per cent. of hyposulphite, demonstrates that even under these conditions a very notable quantity of that substance is found in powder-residue; but it cannot, we consider, be taken to support the view that, in a residue containing a much higher proportion of hyposulphite, the existence of the whole or a large part of that excess is ascribable to accident of manipulation. In the series of experiments with pebble powder there were three, in that with R. L. G. powder four, while in that with F. G. powder there was one, of which the residues contained proportions closely similar to those furnished by the two experiments above quoted, there being no peculiarity in those seven experiments nor any attendant circumstances which could be assigned as a possible reason why the proportions of hyposulphite in those cases should be so much lower than in the other experiments with the same powders, carried out under the same conditions. We therefore cannot but conclude that the production of a small or of a large proportion of hyposulphite (whether as a primary or secondary product, *but before the explosion-vessel is opened*) is determined by some slight modification of the conditions attending the explosion itself.

From an examination of the proportions of potassium sulphate found in the different parts of the two residues, it will be seen that in the case of the R. L. G. there was a variation of only 1·27 per cent. in the sulphate in different parts of the residue, and that in the top and bottom portions the amounts found in the unground and ground parts were almost identical, so that the proportion of sulphate was in no way affected by the prolonged exposure of the finely-ground residue to air. In the pebble powder experiment the sulphate varied in the different parts to the extent of 2·82 per cent., and the higher proportions happened to exist in the ground parts of this residue; but the differences can scarcely be considered sufficiently important to ascribe them to any other cause than a little variation in the composition of different parts of this residue.

Some of the later of our experiments with R. L. G. powder given in the first memoir furnished products which, when calculated upon the results of analysis made at the commencement of these researches, of a sample of the powder taken from the upper part of the contents of the barrel, presented greater discrepancies in some points than was the case with some of the earlier products. It was, therefore, considered desirable to repeat the analysis of this powder, operating upon a sample taken from the *lower* part of the barrel. The results of the two analyses are as follows:—

Components, per cent.		R. L. G. powder.			
		From upper part of barrel.		From lower part of barrel.	
Saltpetre			74·95		74·430
Potassium sulphate			0·15		0·133
Sulphur			10·27		10·093
Charcoal	Carbon	10·86	13·52	12·398	14·286
	Hydrogen	0·42		0·401	
	Oxygen	1·99		1·272	
	Ash, &c.	0·25		0·215	
Water			1·11		1·058

The foregoing numbers, which are in each case the means of two very concordant analyses, present sufficient differences to establish a small but decided variation in the composition of this powder, of which, be it remembered, very considerable quantities were employed in the course of the series of experiments. It appeared to us, therefore, that we were warranted in calculating the earlier results obtained with this powder (produced in 10 to 40 per cent. space) upon the composition as represented by the first analysis, and in recalculating the later ones furnished by the lower part of the contents of the barrel upon the analysis of the sample taken from that part of the powder.

In completing these researches it appeared to us of interest to include two other descriptions of gunpowder in our series of experiments, one of them a representative of the sporting powder class, the other representing the higher qualities of blasting or mining powder, which are well known to differ very materially in composition from the powders used for military purposes. The composition of these two samples of powder, which were obtained from Messrs. CURTIS and HARVEY, is given in the following tabular statement:—

Components, per cent.		Sporting powder. CURTIS and HARVEY'S No. 6.		Mining powder.	
Saltpetre			74·40		61·66
Potassium	Sulphate		0·29		0·12
	Chloride		Trace.		0·14
Sulphur			10·37		15·06
Charcoal	Carbon	10·66	13·78	17·93	21·41
	Hydrogen	0·52		0·66	
	Oxygen	2·29		2·23	
	Ash	0·31		0·59	
Water			1·17		1·61

It will be seen that the sporting powder did not differ very widely in composition from the several military powders of Waltham Abbey manufacture, while the mining powder presents very important differences from any of the other powders experimented with by us, as well as from those used by recent foreign experimenters, to which we have referred in our first memoir.* The proportion of saltpetre is about 11·3 per cent. lower than in the military powders, while the proportions of charcoal and sulphur are each higher by about one-half. The percentage composition of the charcoal contained in these two powders is given below, and a comparison of the numbers with those furnished by analysis of the charcoal from the several military powders shows that the sporting powder charcoal is intermediate between the R. F. G. and F. G. charcoal, and that the mining powder charcoal resembles that contained in the pebble powder. Both these charcoals contained, however, a decidedly higher proportion of mineral constituents than the charcoal manufactured at Waltham Abbey.

PERCENTAGE composition of the charcoals contained in the several descriptions of gunpowder employed.

	Pebble.	R. L. G.	R. F. G.	F. G.	CURTIS and HARVEY.		Spanish.
					Sporting.	Mining.	
Carbon	85·26	80·32	75·72	77·88	77·36	83·74	76·29
Hydrogen	2·98	3·08	3·70	3·37	3·77	3·07	3·31
Oxygen	10·16	14·75	18·84	17·60	16·62	10·45	14·87
Ash	1·60	1·85	1·74	1·15	2·25	2·74	5·53

The results obtained by firing these two gunpowders in 30 per cent. space are included in the statements given in Tables I. and II.†

---

* Phil. Trans., 1875, Part I., p. 72.

† See also Tables XII. and XIII.

It will be remembered that a special method of collecting for analysis the solid products furnished by these powders was adopted, with the object of altogether guarding against the possibility of accidental conversion of sulphide into hyposulphite. The analytical results show that the chief differences between the results of explosion of the sporting powder (in 30 per cent. space) and of the R. F. G. powder (in 70 per cent. space) consisted, as regards the gaseous products, in the slightly higher proportion of carbonic anhydride and in the very decidedly larger amount of marsh gas furnished by the former, and, as regards the solid products, in the higher proportion of sulphide and lower proportion of hyposulphite; the quantities of these constituents of the residue are very similar to those found in the R. L. G. residue exploded in 80 per cent. space.

The gaseous and solid products of explosion of the mining powder differed very greatly, as was anticipated, from those furnished by all the other powders, as regards their proportions. The carbonic oxide was double the highest amount furnished by any of the other powders, while the carbonic anhydride, which in the three series of experiments ranged from 45 to 53 per cent. of the total volume of the gases, amounted only to 32 per cent., the two gases existing in about equal proportions. Marsh gas and hydrogen were present in unusually high proportions, and the sulphuretted hydrogen amounted to 7 per cent., being nearly double the highest proportion found in all the other experiments. The solid residue presented very interesting points of difference. The potassium carbonate was, as might have been anticipated, comparatively small in amount (though some of the experiments with F. G. powder gave similar results in this respect), but there was only 0·5 per cent. of sulphate formed, while the monosulphide amounted to 33 per cent. of the solid residue, and the free sulphur to nearly 13 per cent. FEDEROW's experiments are the only ones in which so high a (and indeed a somewhat higher) percentage of sulphide is recorded, and among the several experiments with R. L. G. powder, in which only small proportions of sulphate were formed, there was only one residue in which the free sulphur was as high in amount as that formed in the mining powder residue. It will be seen that the hyposulphite amounted to nearly 6 per cent.: 2 per cent. more than was furnished by the sporting powder under *precisely* similar conditions of experiment, and *double* the smallest amount formed in any of the series of experiments, conducted without the very special precautions which were applied in dealing with the residue of the powder under discussion. The ammonium sesquicarbonate was considerably higher in amount than in any other experiments (though still much below the amounts found by KAROLYI and LINCK), and the potassium sulphocyanide amounted to 3 per cent., or about five times the amount found in any other experiments excepting that of LINCK.

Lastly, there was a much more considerable amount of charcoal in this experiment than in any other.

It need scarcely be stated that the very distinctive difference between the composition of the solid and gaseous products of this powder are, generally, such as would have been predicted from the comparatively small proportion borne by the oxidising agent to the oxidisable constituents in the mining powder.

It will be seen presently that the experiments with mining powder presented other features of great interest in addition to those elicited by the chemical examination of the products of explosion. In concluding our observations on these, we should point out that fresh confirmation is afforded by this experiment of the fact insisted upon by us, namely, that hyposulphite must be classed among the invariable* products of explosion of gunpowder in closed spaces.

An examination of the three complete series of results obtained by the explosion of pebble, R. L. G., and F. G. powders in closed spaces in which their gravimetric densities varied from 0·1 to 0·9 per cent. of the space, and which are given in detail in Tables I. and II.,† suggests the following observations additional to those included in our first memoir, on the nature and relations to each other of the *solid* products furnished by the several powders.

Comparing the highest, lowest, and mean proportions of the chief solid products furnished by the three powders, which differed but little in composition from each other, the following points are observed :—

1. The proportions of *potassium carbonate* furnished by R. L. G. and by P. powder are very similar, while the highest, lowest, and mean results furnished by F. G. are all decidedly lower than those from the other two powders.

2. The mean proportions of *sulphate* furnished by P. and R. L. G. are not far different, though the highest amount furnished by the *former* is considerably below, and the lowest number considerably above the corresponding numbers furnished by R. L. G. powder. But the mean and the lowest proportions of *sulphate* furnished by F. G. is very considerably higher than the corresponding numbers obtained with the two other powders (the *highest* amounts obtained with F. G. and R. L. G. being identical).

The generally greater extent to which the sulphur has undergone *complete* oxidation in the case of the F. G. powder is certainly not a result which can be in the least ascribable to accidental circumstances attending the experiments, and the fact that it corresponds with the generally higher proportions of hyposulphite furnished by this powder affords additional support to the view which we maintain, that the production of the latter substance, in variable, and sometimes very considerable amounts from the powders experimented with, is not to be explained away by ascribing it to accident of manipulation.

3. It will be seen that, whereas the means of the amounts of hyposulphite obtained in the analyses of the residues from pebble and R. L. G. powders are almost identical, they amount to little more than half that of the mean numbers furnished by the

---

* See note at end of this paper

† See also Tables XII. and XIII.

analyses of the F. G. residues. But the highest result furnished by pebble powder is nearly equal to, and that furnished by R. L. G. powder is not far below the highest number obtained with F. G. powder.

4. As regards potassium monosulphide, the mean result in the case of pebble is very considerably higher than that furnished by R. L. G. powder, and more than four times the mean result given by F. G. powder. It is noteworthy, too, as indicative of the great and apparently uncontrollable irregularity in the amount of this product formed by the explosion of powders of normal composition in closed spaces, that the proportions of sulphide produced from pebble powder at the higher pressures were very small, and similar to those formed from F. G. powder at low as well as high pressures (except where none was found in the residues), and that in several instances very considerable quantities were formed from Pebble at densities ranging from 20 to 60 per cent., some of them being indeed very much higher than that produced in the special experiment with sporting powder, in which extra precautions were adopted to guard against oxidation of sulphide. Then, in the case of R. L. G. powder, the proportions of sulphide found may be said to be intermediate between those produced from pebble and F. G., the amounts ranging from 2 to 10 per cent.; and the higher and lower proportions are indiscriminately distributed through the different residues obtained at low, high, and intermediate pressures. In no instance, either with pebble or R. L. G., was there a complete absence of sulphide, as in *three* instances, at the higher pressures, with F. G. powders. We feel bound again to lay stress upon the fact that there were no accidents of manipulation to account for these remarkable differences in residues furnished by powders of practically the same composition.

5. One or two other points of interest present themselves in connexion with the potassium sulphate found in the residues from the three powders. Both in the pebble and F. G. residues, those obtained at the lowest and the highest densities differed very decidedly from the remainder in regard to the sulphate present, while the proportions in the residues obtained at the densities intermediate between those two extremes present comparatively slight differences in the case of both these powders. But the residues furnished by R. L. G. powder exhibit a very decided difference to the above; the proportions of sulphate in those produced at the four lowest pressures (up to a density of 40) are high, and very similar in amount to those found in the majority of the F. G. powder-residues, while those in the residues produced at the six higher pressures are only from one-fourth to one-half in amount of the others, some being similar to, and others still lower than those formed in the pebble powder-residues. On comparing the proportions of hyposulphite and sulphide in these R. L. G. residues with the sulphate, it will be seen that they vary as much among themselves in the residues where the proportion of sulphate is high as in those where it is small in amount. That the amount of sulphate produced by the explosion of this particular powder, even under the same conditions as regards pressure, was liable to considerable variation, is demonstrated by comparing the results of special examination of the residue

obtained by its explosion in 40 per cent. space (as given in Table III., page 215) with those in the general series (Tables I. and II.) furnished by the same powder, fired in the same space. In the one instance 22·87 per cent. of sulphate were found, in the other it was barely half that amount. With such variations occurring in the proportion of *sulphate* formed under like conditions of explosions, it can hardly be matter for surprise that variations of the same kind should occur in the proportions of sulphide and hyposulphite. In fact, in the two parallel experiments now referred to, the amount of sulphide contained in the residue rich in sulphate was only about one-fifth that contained in the other residue, the proportion in this being higher than any in the three series of analyses. Be it observed, at the same time, that this high amount cannot be ascribed to the adoption of any special precautions to prevent accidental oxidation of the sulphide in the special experiment in which it was found, as there were several residues in the series furnished by *pebble* powder (of the same composition as the R. L. G.) which contained proportions of sulphide not much lower than in this particular case.

Two other parallel experiments (39 and 96) with R. L. G. conducted in 60 per cent. space, exhibit, on the other hand, a remarkably close concordance in regard to the proportions of sulphate and hyposulphite (the numbers being almost identical), though there was a decidedly higher proportion of sulphide in the one than in the other, and a not unimportant difference in the proportions of the gaseous constituents.

With regard to the results of the special experiment with *pebble* powder (p. 215) conducted in 40 per cent. space, and the parallel experiment in the series of pebble powder results, it will be seen that the amounts of sulphate formed in the two experiments were closely similar. The proportions of sulphide and hyposulphite found in the residue of the special experiment correspond very well with the results obtained with this powder at the two lower and the next higher densities, which is not the case with the parallel experiment in the series (No. 12, 40 per cent.), and this may possibly indicate that, in the latter, there may have been an exceptionally considerable conversion of sulphide into hyposulphite during the removal of the residue from the explosion-vessel, it being borne in mind that marked differences in the behaviour and appearance of the residues (even those resulting from the service powder) were frequently noted at the time of collection.

Before leaving this part of our subject we may remark that we have placed in the appendix to this memoir a statement in which we have given for every analysis we have made, the results of the following rather laborious calculations, which have been made in the manner described in our first memoir.*

(1) The amount of gaseous products calculated from the data furnished by the analysis of the solid products.

(2) The amount of solid products calculated from the data furnished by the analysis of the gaseous products.

---

* Phil. Trans., 1875, Part I., p. 90.

(3) A comparison between the weights of the elementary substances found in the products of combustion and the weights of the same elements found in the powder prior to ignition.

(4) The weight of oxygen contained in the total quantity of hyposulphite found.

An examination of this statement will show how closely accordant the various analyses are as a whole; but in estimating the degree of accuracy attained, the following points must carefully be borne in mind.

1. That in the very large quantity of powder used, slight variations in its composition (as indeed have been found) were sure to exist.

2. That we have adopted as one of the data of our calculations the average quantity of gas found to be produced by the explosion. But, as we have elsewhere pointed out, our investigations seem to prove that exceedingly slight and inappreciable variations in the circumstances of explosion give rise to very notable changes in the products, and among others in the amount of gaseous products. Any change in this direction would of course affect the accordance of the analysis in which the abnormal decomposition occurred.

A review of the comparison between the weight of oxygen originally in the powder, and the weight found in the products of explosion, appears to show that there is in the former, on the average, a very appreciable excess of oxygen. Hence it may pretty fairly be concluded that a portion of the hyposulphite found is due to the oxidation of the monosulphide after removal from the explosion-vessel.

On the other hand, a reference to those analyses in which hyposulphite exists in *large* proportion, shows that were we to assume the whole of the hyposulphite as formed after the removal of the products from the cylinders, there would exist a deficiency in oxygen much larger than the existing excess. Hence we may equally fairly, from this line of argument, conclude that it is impossible to attribute to accidental causes the formation of the whole of the hyposulphite, and that a considerable proportion of it *must* be looked on either as a primary or secondary product.

We now pass to the question of the amount of heat generated by the combustion of gunpowder, and in so doing we may remark that we were fully cognisant of the inconveniences inseparable from the form of apparatus used by us for this purpose and described in our first memoir.* In fact, the errors likely to arise from its use were very exactly pointed out by ourselves,† and we were quite aware of the great advantages in regard to saving of time and labour, and to accuracy, that would result from the use of apparatus similar to that which we shall presently describe.

But the apparatus, such as it was, was deliberately adopted, because at the time when these experiments were made we could not be sure that the decomposition experienced by gunpowder in its explosion when fired in considerable quantities and under tensions similar to those existing in the bores of guns was by any means the same as that occurring when it is fired in small quantities and under feeble tensions.

* Phil. Trans. 1875, Part I., p. 63.

† *Loc. cit.*, same page.

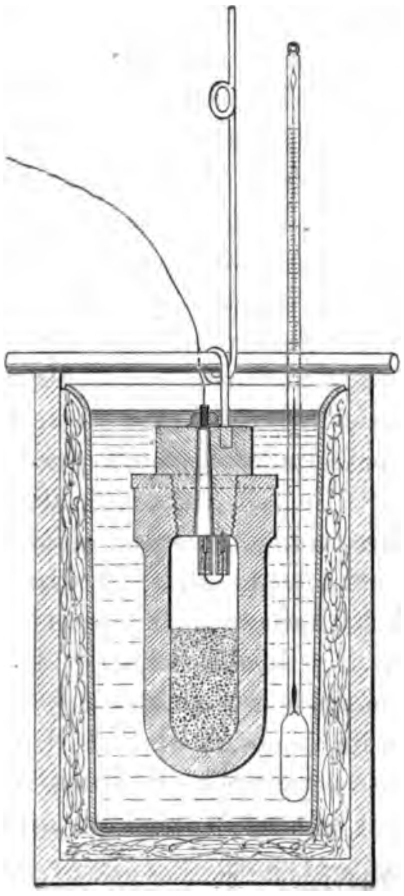
In fact, one of the principal objects of our experiments was to determine this very point, and we therefore considered it necessary, at all events in the first instance, to determine the heat generated when gunpowder is fired in considerable quantities and under high tensions.

To do this, vessels of great strength, consequently of great weight, and therefore not well suited for calorimetric observations, were necessary.

It will presently be seen, however, that the difference between the determinations made by us and those of the other experimenters alluded to by General MORIN and M. BERTHELOT are due, not to error in our determinations, but to essential and striking differences in the decomposition of different descriptions of powder.

The conclusion of the whole of our analyses (of which a complete table is given at p. 207 of this memoir) has shown that, with certain slight exceptions, to which we have elsewhere adverted, the products of combustion are not seriously affected either by differences in the quantity or the gravimetric density of the charge exploded. It would appear indeed as if there were occasionally quite as great differences in the transformation which takes place between charges exploded as nearly as possible under the same circumstances and others exploded under widely different conditions.

We were therefore enabled in the experiments which we are about to describe to make use of the following apparatus.



Two explosion-vessels, both of the general form shown in the annexed figure, but weighing, one, 21,311·6 grs. (1381 grms.), with a capacity of 501·54 grs. (32·50 grms.), the other weighing 52,931·6 grs. (3430 grms.), with a capacity of 1833·75 grs. (118·83 grms.), were prepared. The specific heats of both these vessels were carefully determined, and the amount of heat absorbed by the calorimeter for various changes of temperature was also carefully determined, and corresponding tables for convenient use formed. Thermometers specially made for the purpose and capable of being read to 0°·01 F. (0°.0055 C.) were used in these experiments.

Full details of the determination of the specific heat of the vessels and of the absorption of the heat by the calorimeter are given in the appendix, pp. 264 to 267.

To determine the heat generated, a charge of from 150 to 200 grs. (9·72 to 12·96 grms.) in the smaller cylinder, of 400 grs. (25·92 grms.) in the larger cylinder, was carefully weighed and placed in the explosion-vessel. The explosion-vessel was then immersed in the water of the calorimeter and the charge fired in the usual way, the attached thermometer being read before the explosion, and afterwards continuously until the maximum temperature (which was usually reached in about 2 minutes) was attained.*

In order to make our new calorimetric determinations as complete as possible, and with the view of exhibiting the differences in the heat evolved due to changes in the composition of the powder, we have not only found separately the heat given off by the three principal powders described in our first memoir, but have added to these three other powders differing widely in their composition, viz.: ordinary English mining, CURTIS and HARVEY's well-known powder No. 6, and Spanish spherical. The composition of all these powders has been already given either in the present or in our former memoir. But for our present purpose it is convenient to place in juxtaposition the composition of these six powders.

TABLE IV.—Exhibiting the differences in the composition of the powders experimented with which are described in this memoir.

Nature of powder.	Saltpetre.	Sulphate.	Potassium Chloride.	Sulphur.	Carbon.	Hydrogen.	Oxygen.	Ash.	Water.
Pebble powder	·7467	·0009	..	·1007	·1212	·0042	·0145	·0023	·0095
R. L. G., W. A.	·7443	·0013	..	·1009	·1240	·0040	·0127	·0022	·0106
F. G., W. A.	·7355	·0036	..	·1002	·1136	·0049	·0257	·0017	·0148
CURTIS and HARVEY's No. 6 .	·7440	·0029	Trace	·1037	·1066	·0052	·0229	·0031	·0117
Mining powder	·6166	·0012	·0014	·1506	·1793	·0066	·0223	·0059	·0161
Spanish spherical	·7530	·0027	·0002	·1242	·0865	·0038	·0168	·0063	·0065

The decomposition they experienced is exhibited in Tables I.† and II. of the present memoir.

---

* It is scarcely necessary to state that the correction for the absorption of heat by the calorimeter and the effect of cooling was in all cases made.

† See also Tables XII. and XIII.

With the small explosion-vessel the results obtained (experiments 146 to 166) were as follows, the numbers given below indicating the gramme units of heat evolved by the combustion of 1 gramme of each description of powder employed:—

Nature of powder.	Gramme-units of heat evolved.				
	I.	II.	III.	IV.	Means.
Pebble	711·9	734·1	694·4	710·0	712·6
R. L. G.	725·1	718·4	707·5	. .	717·0
F. G.	706·5	738·9	731·7	. .	725·7
Curtis and Harvey's No. 6 .	784·0	755·7	744·9	732·9	754·3
Mining	512·7	505·5	507·9	. .	508·7
Spanish	762·5	771·4	753·4	. .	762·4

With the larger explosion-vessel the results of experiments 171 to 179 and 181 to 192 gave

Nature of powder.	Gramme-units of heat evolved.			
	I.	II.	III.	Means.
Pebble	728·5	714·7	703·4	715·5
R. L. G.	713·4	724·7	717·7	718·6
F. G.	731·1	722·1	730·7	728·0
Curtis and Harvey's No. 6 .	751·3	765·3	750·6	756·1
Mining	520·0	507·0	499·6	508·9
Spanish.	771·3	761·8	754·0	762·3

From the whole of these experiments, and giving to the second series, as probably the more accurate, twice the weight of the first series, we arrive at the conclusion that the heat generated by the combustion of the powders *as actually used* is as follows:—

1 gramme of pebble powder generates 714·5 gramme-units
1 ,, ,, R. L. G. ,, ,, 718·1 ,, ,,
1 ,, ,, F. G. ,, ,, 727·2 ,, ,,
1 ,, ,, C. and H. No. 6 ,, 755·5 ,, ,,
1 ,, ,, mining ,, ,, 508·8 ,, ,,
1 ,, ,, Spanish ,, ,, 762·3 ,, ,,

From an examination of the whole of the above results it is obvious: Firstly, that the heat generated by the combustion of gunpowder is subject to very wide variations, dependent upon the particular nature* of the powder employed (the Spanish powder, for example, generates just 50 per cent. more heat than the mining powder); and, secondly, that the heat evolved by the same description of powder varies in different experiments to a greater extent than is to be accounted for by errors of observation. And this was, indeed, to be expected, since the very considerable variations in the products of combustion, under the same circumstances, as disclosed by our analyses, could hardly be supposed to exist without some corresponding variation in the heat evolved.

---

* Both physical and chemical.

Our views on this head are confirmed by the calorimetric determinations we have made in our researches on gun-cotton. In these determinations, which have been carried on with precisely the same apparatus, we have found no appreciable difference in the heat evolved in the various experiments. The deduction is that the discrepancies between the several observations in the case of gunpowder are due to differences in the metamorphoses, not to defects in the observations.

The units of heat liberated given above are those furnished by the powders as actually used; but as these powders had different amounts of moisture in their composition, and as, in use, these amounts of moisture are found to vary considerably, giving rise, especially when the powders are used in guns, to very different pressures, and generating very different energies, we have considered it desirable to correct the above figures, and we place below those that would have had place had the powders, when fired, been perfectly free from moisture.

TABLE V.—Showing the heat, in gramme-units, generated by the combustion of 1 gramme of the undermentioned perfectly dry powders.

1	gramme	pebble powder . . .	generates	721·4	gramme-units.
1	,,	W. A. R. L. G. powder	,,	725·7	,,
1	,,	W. A. F. G. powder .	,,	738·3	,,
1	,,	C. and H. No. 6 powder	,,	764·4	,,
1	,,	mining powder . . .	,,	516·8	,,
1	,,	Spanish pellet powder .	,,	767·3	,,

The data for computing the mean specific heats of the products of explosion of the six powders, as well as the results of the necessary calculations, are given in the annexed Table, No. VI.

TABLE VI.—Giving data and results of calculation of the mean specific heats (at natural temperatures) of the products of explosion of the six powders experimented with.

Products of combustion.	A. Specific heat.	Pebble, W. A.		R. L. G., W. A.		F. G., W. A.		CURTIS and HARVEY'S No. 6.		Mining.		Spanish spherical.	
		1. Weight in a gramme.	Product of columns A. and 1.	2. Weight in a gramme	Product of columns A. and 2.	3. Weight in a gramme.	Product of columns A. and 3.	4. Weight in a gramme.	Product of columns A. and 4.	5. Weight in a gramme.	Product of columns A. and 5.	6. Weight in a gramme.	Product of columns A. and 6.
Carbonic anhydride . . .	·172	·2651	·04560	·2608	·04486	·2634	·04530	·2606	·04482	·2290	·03939	·2438	·04193
Carbonic oxide	·174	·0473	·00823	·0418	·00727	·0348	·00605	·0248	·00431	·1531	·02664	·0140	·00244
Nitrogen	·173	·1110	·01920	·1106	·01913	·1100	·01903	·1137	·01977	·0861	·01489	·1097	·01898
Sulphydric acid	·184	·0110	·00202	·0107	·00197	·0100	·00184	·0083	·00153	·0391	·00719	·0097	·00178
Marsh gas	·468	·0006	·00028	·0008	·00037	·0003	·00014	·0046	·00215	·0071	·00332	..	..
Hydrogen	2·411	·0006	·00145	·0005	·00121	·0007	·00169	·0008	·00193	·0017	·00410	·0003	·00072
Oxygen	·155	..	..	·0002	·00003	·0003	·00005	..	..	..	..	·0007	·00011
Potassium carbonate . . .	·206	·3226	·06646	·3392	·06987	·2804	·05776	·3436	·07078	·1970	·04058	·2173	·04476
„ sulphate . . .	·196	·0703	·01378	·0838	·01642	·1225	·02401	·1258	·02466	·0028	·00055	·2962	·05805
„ hyposulphite . .	·197	·0789	·01554	·0760	·01497	·1440	·02837	·0231	·00455	·0281	·00554	·0473	·00932
„ monosulphide .	·108	·0574	·00620	·0360	·00387	·0140	·00151	·0589	·00636	·1612	·01741	·0197	·00213
„ sulphocyanate .	·200	·0016	·00032	·0013	·00026	·0007	·00014	..	..	·0140	·00280	·0003	·00006
„ nitrate	·239	·0013	·00031	·0015	·00036	·0009	·00021	·0017	·00041	·0004	·00010	·0058	·00139
„ oxide	·200	·0009	·00018	..	..	·0075	·00150	..	..	..	..	..	..
„ sesquicarbonate .	·350	·0005	·00017	·0004	·00014	·0003	·00010	·0005	·00018	·0085	·00298	·0002	·00007
Sulphur	·171	·0309	·00528	·0357	·00610	·0102	·00174	·0336	·00575	·0623	·01065	·0350	·00598
Charcoal	·242	..	..	·0007	·00017	..	..	..	..	·0096	·00232	..	..
Mean specific heats . . .	..	..	·18503	..	·18704	..	·18946	..	·18720	..	·17846	..	·18773

There is no material difference in the specific heats if the hyposulphite be supposed to be an accidental product formed by the oxidation of the sulphide.

We have pointed out in our first memoir our reasons for considering fallacious a temperature of explosion deduced (as has been done by some authors) by dividing the number of gramme units of heat by the mean specific heat of the exploded powder at 0° C.; but for purposes of comparison, to exhibit the striking differences between the powders, as well as because we have not data at our disposal to enable us to deduce the actual temperature of explosion in the case of each of the six powders, we give below the temperature of each powder calculated upon the above hypothesis.

They are as follow:—

Temperature of explosion on above hypothesis of

W. A. pebble powder		3899° C.
W. A. R. L. G.	„	3880 „
W. A. F. G.	„	3897 „
C. and H. No. 6	„	4083 „
Mining	„	2896 „
Spanish pellet	„	4087 „

The volumes of the permanent gases generated by the explosion of each of the six powders are as follow:—

1 gramme	of	W. A. pebble	powder	generated	275·7	cubic	centimetres
1	„	„ W. A. R. L. G.	„	„	271·3	„	„
1	„	„ W. A. F. G.	„	„	259·2	„	„
1	„	„ C. and H. No. 6	„	„	238·2	„	„
1	„	„ mining	„	„	354·6	„	„
1	„	„ Spanish pellet	„	„	232·7	„	„

The above volumes are all calculated for a temperature of 0° C. and a barometric pressure of 760 millims. of mercury.

If we correct, as before, the quantities of permanent gases formed for the amounts of moisture contained in the powders when experimented with, we shall have the values given in Table VII.

Table VII.—Giving the volumes of permanent gases generated by the explosion of 1 gramme of the undermentioned perfectly dry powders.

1 gramme	W. A. pebble	powder	generates	278·3	cubic	centimetres.*
1 „	W. A. R. L. G.	„	„	274·2	„	„
1 „	W. A. F. G.	„	„	263·1	„	„
1 „	C. and H. No. 6	„	„	241·0	„	„
1 „	mining	„	„	360·3	„	„
1 „	Spanish pellet	„	„	234·2	„	„

---

* It is perhaps necessary to remind readers not familiar with the French metrical system that the assertion that a gramme of powder generates 278·3 cubic centimetres of permanent gases at temperature

It is of high importance to observe that the volume of the permanent gases generated is in every case in inverse ratio to the units of heat evolved. Thus, if Tables V. and VII. be compared, and if from Table V. we arrange the powders in descending order of units of heat, we have the order exhibited in Table VIII.; and if from Table VII. we place the powders in ascending order of volumes of gas produced, we find that we have precisely the same arrangement.

Table VIII.—Showing the arrangement of the six powders when placed either according to the amount of heat generated in a descending, or according to the quantity of gas evolved in an ascending, scale.

Nature of powder.	Units of heat per gramme exploded.	Cubic cents. of gas per gramme exploded.
Spanish pellet powder	767·3	234·2
Curtis and Harvey's No. 6 powder ..	764·4	241·0
W. A. F. G. powder	738·3	263·1
W. A. R. L. G. powder	725·7	274·2
W. A. pebble powder	721·4	278·3
Mining powder	516·8	360·3

The results given in this table are very striking. If we take the two natures which commence and close the list, it will be observed that on the one hand the heat generated by the Spanish powder is about 50 per cent. higher than that generated by the mining powder, and that on the other hand the quantity of permanent gases evolved by the mining powder is about 50 per cent. greater than that given off by the Spanish.

Thus it appears that the great inferiority of heat developed by the mining as compared with the Spanish powder is compensated, or at least approximately so, by the great superiority in volume of permanent gases produced. A similar relation is observed in respect to the other powders, and it may indeed be noted that the products of the figures given in columns 2 and 3 in Table VIII. do not differ greatly from a constant value; thus pointing towards the conclusion that the pressures at any given density and the capacity for performing work of the various powders are not very materially different.

This fact has been entirely verified for the whole of the Waltham Abbey powders, and in a less degree for the three other powders also.

Thus at the points where the Spanish, mining, and Curtis and Harvey's No. 6 powders have been compared with the standard pressure curves determined from

---

and pressure specified, is equivalent to the assertion that the permanent gases occupy 278·3 times the space which the powder occupied in its unexploded state, the gravimetric density of the powder being assumed to be unity.

Waltham Abbey powder, the agreement is very close ; the departure from the normal curve not in any case exceeding that obtained in particular experiments with the Waltham Abbey powders themselves.

With respect to the pressure given by mining powder, the peculiarities shown by this powder were so interesting that it appeared to us important to determine its tension when fired under a high gravimetric density. We accordingly fired (experiment 230) 11,560 grs. (749 grms.) of this powder under a gravimetric density of unity.

The pressures developed by two very accordant observations was, when corrected, 44 tons on the square inch (6706 atmospheres). The pressure obtained under similar circumstances from Waltham Abbey powder was 43 tons on the square inch (6554 atmospheres).

It will afterwards be seen that the capacity for performing work of the various descriptions of powder is also not very different, and this similarity of result is the more remarkable when it is remembered that with, at all events, three of the powders, there were striking differences both in their composition and in the decomposition that they experienced, and when, in consequence, material variations both in pressures at different densities and in potential energy might have been expected.

But returning to the great difference in heat evolved, for example, by the Spanish and mining powders, we think it is difficult to resist the conclusion that the small number of units of heat evolved by the latter is in great measure due to the quantity of heat that has been absorbed in placing the very much larger proportion of the products of combustion in the form of permanent gases. This suggestion would, we think, also fully explain the fact alluded to in our first memoir,* and to which we had been led purely by experiment, namely, "that the variations observed by us in the decomposition of gunpowder do not, even when very considerable, materially affect either its tension or capacity for performing work."

The above facts and remarks would also show that a comparison between different gunpowders, or a comparison between gunpowder and other explosive agents cannot, as has been proposed,† be determined by a simple measurement of the corresponding units of heat they evolve.

Did such a law hold, the Spanish powder should have more than 50 per cent. advantage over the mining powder, but as a matter of fact, although not very widely different, the mining powder had the advantage both in respect to the tension observed in a close vessel and to the energy developed in the bore of a gun.

As regards the *actual* temperature of explosion, we have little doubt, from the results of the further experiments detailed in this paper, that the temperature named in our first memoir, viz., 2200° C., is not far removed from the truth for the principal powders with which we were then experimenting.

---

* *Loc. cit.*, p. 88.

† De Tromenec, 'Comptes Rendus,' tom. lxxvii., p. 128.

That temperature may not improbably be taken at the high limit of the temperature of explosion for the Spanish pellet, which, as conjectured by us, has been proved to have developed a higher temperature than any other powder with which we have experimented.

The complete fusion of the platinum with this powder, and with this alone, is thus shown not to be an isolated or accidental occurrence, but to depend on a real difference of temperature, and we are thus by two converging lines of reasoning brought to the conclusion that for pebble or R. L. G. powder the temperature of explosion may be taken as a little above the melting point of platinum, say about 2100° C., while the temperature of explosion of a powder like the Spanish pellet may be taken as occasionally ranging up to 2200° C.

The data at our disposal are not sufficient to enable us to determine the temperature of explosion of the English mining powder with the same accuracy, but it is probable that 2000° C. and 1800° C. may be assigned as the limits between which the true temperature may be placed.

After our remarks on the slight differences or accidents which appear to give rise to not inconsiderable variations in the products of decomposition of gunpowder, it is hardly necessary to point out that such differences in decomposition are nearly sure to give rise to corresponding variations in the temperature of explosion, and that therefore this temperature, even in one and the same powder, cannot be supposed to be always identical.*

The relation between the tension of the gases developed by the explosion of gunpowder (when it is expanded in the bore of a gun with production of work) and the volume which these gases occupy is in our first memoir† expressed by the relation

$$\frac{p}{p_0}=\left\{\frac{v_0(1-\alpha)}{v-\alpha v_0}\right\}^{\frac{C_p+\beta\lambda}{C_v+\beta\lambda}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

where p is the tension of the permanent gases corresponding to volume v, C_p the specific heat of the permanent gases at constant pressure, C_v the specific heat at constant volume, α the ratio which the volume of the non-gaseous products of explosion bears to the volume of the unexploded powder, β the ratio between the weights of the non-gaseous and gaseous portions of the products of explosion, λ the specific heat of the non-gaseous products; but in that memoir the values of the constants C_p, C_v, and β were calculated from a few of the analyses that were first made. The completion of the whole of our analyses has enabled us to recalculate these three constants, and their values, together with those of the other constants used in equation (30), are given in Table IX.

---

* It is by no means improbable that owing to the larger proportion of carbon which assumes the higher state of oxidation as the pressure under which the explosion takes place is increased, the temperatures at high tensions may be somewhat greater than those which occur when the powder is fired under low tensions.

† Phil. Trans., 1875, Part I., p. 129.

With regard to the value of λ it is necessary to say a few words. If the tensions given in Table XVIII.[*] of our first memoir calculated from (30) be compared with the tensions actually observed in the bores of guns given in the same table, or if a comparison be made between curves B and C in Plate 24 of the same memoir, it will be observed that for densities of the products of combustion of ·4 and below, or, in other words, for 2½ expansions and higher, the tensions actually observed with R. L. G. powder are in all cases higher than those calculated from equation (30). This, of course, should not be, as those last tensions should in actual practice only be reached if there were no loss from windage, vent, or other causes, if there were perfect combustion of the charge, and if the charge were expanded in a gun perfectly impervious to heat.

We surmised[†] as one of the causes of this difference that in our calculations we had taken λ at its mean value, whereas we pointed out that as the specific heat of the non-gaseous products must, according to our hypothesis, increase rapidly with the temperature, λ should for our purposes be taken at a considerably higher value.

As however the agreement between the observed and calculated tensions was exceedingly close, as when the calculations were applied to guns, the "factor of effect" alone would practically be altered, and as, finally, we had not sufficient data to enable us to correct this constant with any certainty, we did not feel justified in attempting hypothetical corrections.

But since the date of the submission of our first memoir to the Royal Society, our knowledge of the action of large charges in the bores of guns has been greatly increased; not only have charges seven or eight times as great[‡] as the largest of those then discussed been fired, but we have submitted to careful calculation, with a full knowledge of all the necessary data, the results of a very large number of rounds fired from guns of all sizes, from the 100-ton gun down to the smallest gun in H.B.M.'s service. In some of these guns also the charges were so arranged as to suffer a high degree of expansion, and we have thus accumulated data which have enabled us to deduce a corrected value of λ, and which gives pressures and energies more closely in accordance with the whole of the experiments we have discussed.

---

[*] Phil. Trans., 1875, Part I., p. 129.

[†] Phil. Trans., *loc. cit.*, p. 131.

[‡] The highest charge fired by the distinguished Italian artillerists who conducted the recent experiments with the 100-ton guns reached the great weight of 260 kilos. (573 lb.) of FOSSANO powder—a powder singularly well adapted for use in large guns. The velocity given to a projectile of 908 kilos. (2001·5 lb.) was (525·9 metres) 1725·3 feet per second, and the energy of the shot at the muzzle reached the enormous amount of (12,800 metre tonnes) 41,333 foot-tons. The highest charge fired from the 80-ton gun has been (208·7 kilos.) 460 lb. prismatic powder. This charge gave to a projectile of 1705 lb. a muzzle velocity of (495·6 metres) 1626 feet per second and an energy of (9680 metre tonnes) 31,257 foot-tons.

The value of p_0, that is, the pressure corresponding to a gravimetric density of unity, is taken at 6554 atmospheres =43 tons per square inch, this value being the result of our further experiments corrected for perfectly dry powder.

TABLE IX.—Showing the value of the constants in equation (30).

p_o =43 tons per square inch =6554 atmospheres.
$\alpha =$ ·57
$\beta =$ 1·2957
$C_p =$ ·2324
$C_v =$ ·1762
$\lambda =$ ·45

From equation (30), with the values of the constants given in Table IX., we have calculated the tensions for various gravimetric densities ranging from 1 to ·05, and have expressed these tensions in kilogrammes per square centimetre, in tons per square inch, and in atmospheres.

We have placed in the same table, for purposes of comparison, the pressures which would rule were the gases permitted to expand without production of work in a vessel impervious to heat, or, what amounts to the same thing, the pressures that would have place were the charges exploded in a perfectly closed cylinder with the corresponding gravimetric densities.

TABLE X.—Giving in terms of the density the tensions existing in the bores of guns calculated from equation (30); giving also the tensions if the gases are suffered to expand without production of work. In both cases the powder is supposed to be perfectly dry.

Mean density of products of combustion.	Corresponding expansions.	Tensions calculated from equation (30).						Tensions in close cylinders or where gases expand without cooling or production of work.					
		Kilos. per square cm.	Differences.	Tons per square inch.	Differences.	Atmospheres.*	Differences.	Kilos. per square c m.	Differences.	Tons per square inch.	Differences.	Atmospheres.	Differences.
1·00	1·000	6772·2		43·00		6554·0		6772·2		43·00		6554·0	
			789·0		5·01		763·6		738·6		4·69		715·9
·95	1·053	5983·2		37·99		5790·4		6033·6		38·31		5839·1	
			693·0		4·40		670·7		652·1		4·14		631·0
·90	1·111	5290·2		33·59		5119·7		5381·5		34·17		5208·1	
			609·5		3·88		589·8		579·5		3·68		560·9
·85	1·176	4680·7		29·72		4529·9		4802·0		30·49		4647·2	
			541·8		3·44		524·4		519·8		3·30		503·0
·80	1·250	4138·9		26·28		4005·5		4282·2		27·19		4144·2	
			481·9		3·07		466·4		467·7		2·97		452·6
·75	1·333	3657·0		23·22		3539·1		3814·5		24·22		3691·6	
			434·7		2·76		420·6		422·1		2·68		408·5
·70	1·429	3222·3		20·46		3118·5		3392·4		21·54		3283·1	
			390·6		2·48		378·0		385·9		2·45		373·4
·65	1·539	2831·7		17·98		2740·5		3006·5		19·09		2909·7	
			354·3		2·25		343·0		351·2		2·23		339·9
·60	1·667	2477·4		15·73		2397·5		2655·3		16·86		2569·8	
			321·3		2·04		310·9		322·8		2·05		312·5
·55	1·818	2156·1		13·69		2086·6		2332·5		14·81		2257·3	
			292·9		1·86		283·5		296·1		1·88		286·5
·50	2·000	1863·2		11·83		1803·1		2036·4		12·93		1970·8	
			267·8		1·70		259·1		274·1		1·74		265·2
·45	2·222	1595·4		10·13		1544·0		1762·3		11·19		1705·6	
			245·7		1·56		237·8		253·5		1·61		245·4
·40	2·500	1349·7		8·57		1306·2		1508·8		9·58		1460·2	
			225·2		1·43		217·9		236·3		1·50		228·7
·35	2·857	1124·5		7·14		1088·3		1272·5		8·08		1231·5	
			206·3		1·31		199·7		218·9		1·39		211·8
·30	3·333	918·2		5·83		888·6		1053·6		6·69		1019·7	
			190·6		1·21		184·4		204·7		1·30		198·2
·25	4·000	727·6		4·62		704·2		848·9		5·39		821·5	
			174·8		1·11		169·2		192·2		1·22		185·9
·20	5·000	552·8		3·51		535·0		656·7		4·17		635·6	
			160·6		1·02		155·5		179·5		1·14		173·8
·15	6·667	392·2		2·49		379·5		477·2		3·03		461·8	
			146·5		·93		141·7		168·5		1·07		163·1
·10	10·000	245·7		1·56		237·8		308·7		1·96		298·7	
			132·3		·84		128·1		159·1		1·01		153·9
·05	20·000	113·4		·72		109·7		149·6		·95		144·8	

We have shown[†] that the theoretic work which a charge of gunpowder is capable of effecting in expanding to any volume v is expressed by the equation

$$W=\frac{p_0v_0(1-\alpha)(C_v+\beta\lambda)}{C_p-C_v}\left\{1-\left(\frac{v_0(1-\alpha)}{v-\alpha v_0}\right)^{\frac{C_p-C_v}{C_v+\beta\lambda}}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

---

[*] We are indebted to Colonel ALONCLE and the Commandant HEDON, of the Marine Artillery of France, translators of our memoir, for pointing out that although we have adopted as our standard atmosphere that used in France, viz.: a barometric pressure of 760 m.m. mercury and a temperature of 0° C., the coefficient we have used in converting tons per square inch into atmospheres is not exactly in accordance with our supposition. In the present memoir this discrepancy is corrected.

[†] Phil. Trans., 1875, Part I., p. 132.

And on account of its very great utility we give in Table XI. the results of the calculations of W for various values of v up to and inclusive of $v=50$, the powder used being supposed to be of the normal type and free from moisture, and the constants having the values given in Table IX.

The work is expressed in the table both in metre tonnes per kilogramme and foot-tons per pound of powder burned, and by use of a proper factor of effect is applicable also to powders differing very materially from those of Waltham Abbey manufacture—for example, to mining or the Spanish powders.

But before entering upon these special cases we shall, as the principles embodied in this table have led to the greatly increased charges of recent guns and the consequent high velocities attained, give one or two illustrations of its application.

TABLE XI.—Giving for value of v up to $v=50$, the total work that dry gunpowder of the Waltham Abbey standard is capable of performing in the bore of a gun in metre tonnes per kilogramme, and foot-tons per lb. of powder burned.

Number of volumes of expansion.	Corresponding density of products of combustion.	Total work that gunpowder is capable of performing.			
		Per kilogramme burned in metre tonnes.	Difference.	Per lb. burned in foot-tons.	Difference.
1·00	1·000	..	..	..	..
1·01	·990	0·669	·669	·980	·980
1·02	·980	1·322	·653	1·936	·956
1·03	·971	1·960	·638	2·870	·934
1·04	·962	2·583	·623	3·782	·912
1·05	·952	3·192	·609	4·674	·892
1·06	·943	3·788	·596	5·547	·873
1·07	·935	4·370	·582	6·399	·852
1·08	·926	4·940	·570	7·234	·835
1·09	·917	5·498	·558	8·051	·817
1·10	·909	6·045	·547	8·852	·810
1·11	·901	6·581	·536	9·637	·785
1·12	·893	7·106	·525	10·406	·769
1·13	·885	7·621	·515	11·160	·754
1·14	·877	8·126	·505	11·899	·739
1·15	·870	8·622	·496	12·625	·726
1·16	·862	9·109	·487	13·338	·713
1·17	·855	9·587	·478	14·038	·700
1·18	·847	10·056	·469	14·725	·687
1·19	·840	10·517	·461	15·400	·675
1·20	·833	10·970	·453	16·063	·663
1·21	·826	11·416	·446	16·716	·653
1·22	·820	11·855	·439	17·359	·643
1·23	·813	12·287	·432	17·992	·633
1·24	·806	12·712	·425	18·614	·622
1·25	·800	13·130	·418	19·226	·612
1·26	·794	13·541	·411	19·828	·602
1·27	·787	13·945	·404	20·420	·592
1·28	·781	14·342	·397	21·001	·581
1·29	·775	14·732	·390	21·572	·571
1·30	·769	15·115	·383	22·133	·561

TABLE XI.—continued.

Number of volumes of expansion.	Corresponding density of products of combustion.	Total work that gunpowder is capable of performing.			
		Per kilogramme burned in metre tonnes.	Difference.	Per lb. burned in foot-tons.	Difference.
1·32	·758	15·875	·760	23·246	1·113
1·34	·746	16·611	·736	24·324	1·078
1·36	·735	17·326	·715	25·371	1·047
1·38	·725	18·021	·695	26·389	1·018
1·40	·714	18·698	·677	27·380	·991
1·42	·704	19·358	·660	28·348	·968
1·44	·694	20·002	·644	29·291	·943
1·46	·685	20·630	·628	30·211	·920
1·48	·676	21·243	·613	31·109	·898
1·50	·667	21·842	·599	31·986	·877
1·52	·658	22·427	·585	32·843	·857
1·54	·649	22·999	·572	33·681	·838
1·56	·641	23·558	·559	34·500	·819
1·58	·633	24·105	·547	35·301	·801
1·60	·625	24·641	·536	36·086	·785
1·62	·617	25·166	·525	36·855	·769
1·64	·610	25·680	·514	37·608	·753
1·66	·602	26·184	·504	38·346	·738
1·68	·595	26·678	·494	39·069	·723
1·70	·588	27·162	·484	39·778	·709
1·72	·581	27·637	·475	40·474	·696
1·74	·575	28·103	·466	41·156	·682
1·76	·568	28·561	·458	41·827	·671
1·78	·562	29·011	·450	42·486	·659
1·80	·555	29·453	·442	43·133	·647
1·82	·549	29·887	·434	43·769	·636
1·84	·543	30·314	·427	44·394	·625
1·86	·537	30·734	·420	45·009	·615
1·88	·532	31·147	·413	45·614	·605
1·90	·526	31·553	·406	46·209	·595
1·92	·521	31·953	·400	46·795	·586
1·94	·515	32·347	·394	47·372	·577
1·96	·510	32·735	·388	47·940	·568
1·98	·505	33·117	·382	48·499	·559
2·00	·500	33·493	·376	49·050	·551
2·05	·488	34·403	·910	50·383	1·333
2·10	·476	35·284	·881	51·673	1·290
2·15	·465	36·137	·853	52·922	1·249
2·20	·454	36·963	·826	54·132	1·210
2·25	·444	37·763	·800	55·304	1·172
2·30	·435	38·538	·775	56·439	1·135
2·35	·425	39·289	·751	57·539	1·100
2·40	·417	40·017	·728	58·605	1·066
2·45	·408	40·723	·706	59·639	1·034
2·50	·400	41·408	·685	60·642	1·003
2·55	·392	42·073	·665	61·616	·974
2·60	·384	42·720	·647	62·563	·947
2·65	·377	43·350	·630	63·486	·923
2·70	·370	43·964	·614	64·385	·899
2·75	·363	44·563	·599	65·262	·877
2·80	·357	45·148	·585	66·119	·857
2·85	·351	45·719	·571	66·955	·836
2·90	·345	46·276	·557	67·771	·816

TABLE XI.—continued.

Number of volumes of expansion.	Corresponding density of products of combustion.	Total work that gunpowder is capable of performing.			
		Per kilogramme burned in metre tonnes.	Difference.	Per lb. burned in foot-tons.	Difference.
2·95	·339	46·820	·544	68·568	·797
3·00	·333	47·352	·532	69·347	·779
3·05	·328	47·872	·520	70·109	·762
3·10	·322	48·381	·509	70·854	·745
3·15	·317	48·880	·499	71·585	·731
3·20	·312	49·369	·489	72·301	·716
3·25	·308	49·848	·479	73·002	·701
3·30	·303	50·318	·470	73·690	·688
3·35	·298	50·779	·461	74·365	·675
3·40	·294	51·231	·452	75·027	·662
3·45	·290	51·675	·444	75·677	·650
3·50	·286	52·111	·436	76·315	·638
3·55	·282	52·539	·428	76·940	·625
3·60	·278	52·959	·420	77·553	·613
3·65	·274	53·371	·412	78·156	·603
3·70	·270	53·776	·405	78·749	·593
3·75	·266	54·174	·398	79·332	·583
3·80	·263	54·565	·391	79·905	·573
3·85	·260	54·950	·385	80·469	·564
3·90	·256	55·329	·379	81·024	·555
3·95	·253	55·702	·373	81·570	·546
4·00	·250	56·069	·367	82·107	·537
4·10	·244	56·786	·717	83·157	1·050
4·20	·238	57·482	·696	84·176	1·019
4·30	·232	58·158	·676	85·166	·990
4·40	·227	58·815	·657	86·128	·962
4·50	·222	59·454	·639	87·064	·936
4·60	·217	60·076	·622	87·975	·911
4·70	·213	60·671	·605	88·861	·886
4·80	·208	61·260	·589	89·724	·863
4·90	·204	61·834	·574	90·565	·841
5·00	·200	62·394	·560	91·385	·820
5·10	·196	62·941	·547	92·186	·801
5·20	·192	63·475	·534	92·968	·782
5·30	·188	63·997	·522	93·732	·764
5·40	·185	64·507	·510	94·479	·747
5·50	·182	65·006	·499	95·210	·731
5·60	·178	65·494	·488	95·925	·715
5·70	·175	65·972	·478	96·625	·700
5·80	·172	66·440	·468	97·310	·685
5·90	·169	66·898	·458	97·981	·671
6·00	·166	67·347	·449	98·638	·657
6·10	·164	67·787	·440	99·282	·644
6·20	·161	68·219	·432	99·915	·633
6·30	·159	68·643	·424	100·536	·621
6·40	·156	69·059	·416	101·145	·609
6·50	·154	69·468	·409	101·744	·599
6·60	·151	69·870	·402	102·333	·589
6·70	·149	70·265	·395	102·912	·579
6·80	·147	70·653	·388	103·480	·568
6·90	·145	71·034	·381	104·038	·558
7·00	·143	71·411	·374	104·586	·548
7·10	·141	71·779	·368	105·125	·539

TABLE XI.—concluded.

Number of volumes of expansion.	Corresponding density of products of combustion.	Total work that gunpowder is capable of performing.			
		Per kilogramme burned in metre tonnes.	Difference.	Per lb. burned in foot-tons.	Difference.
7·20	·139	72·141	·362	105·655	·530
7·30	·137	72·497	·356	106·176	·521
7·40	·135	72·847	·350	106·688	·512
7·50	·133	73·191	·344	107·192	·504
7·60	·131	73·530	·339	107·688	·496
7·70	·130	73·864	·334	108·177	·489
7·80	·128	74·193	·329	108·659	·482
7·90	·126	74·517	·324	109·133	·474
8·00	·125	74·836	·319	109·600	·467
8·10	·123	75·150	·314	110·060	·460
8·20	·122	75·460	·310	110·514	·454
8·30	·120	75·766	·306	110·962	·448
8·40	·119	76·068	·302	111·404	·442
8·50	·117	76·366	·298	111·840	·436
8·60	·116	76·660	·294	112·270	·430
8·70	·115	76·950	·290	112·695	·425
8·80	·114	77·236	·286	113·114	·419
8·90	·112	77·519	·283	113·528	·414
9·00	·111	77·798	·279	113·937	·409
9·10	·110	78·074	·276	114·341	·404
9·20	·109	78·346	·272	114·739	·398
9·30	·108	78·615	·269	115·133	·394
9·40	·106	78·880	·265	115·521	·388
9·50	·105	79·142	·262	115·905	·384
9·60	·104	79·401	·259	116·284	·379
9·70	·103	79·657	·256	116·659	·375
9·80	·102	79·910	·253	117·029	·370
9·90	·101	80·160	·250	117·395	·366
10	·100	80·407	·247	117·757	·362
11	·091	82·734	2·327	121·165	3·408
12	·083	84·833	2·099	124·239	3·074
13	·077	86·743	1·910	127·036	2·797
14	·071	88·945	1·752	129·602	2·566
15	·066	90·112	1·617	131·970	2·368
16	·062	91·613	1·501	134·168	2·198
17	·059	93·013	1·400	136·218	2·050
18	·055	94·324	1·311	138·138	1·920
19	·052	95·557	1·233	139·944	1·806
20	·050	96·720	1·163	141·647	1·703
21	·047	97·820	1·100	143·258	1·611
22	·045	98·865	1·045	144·788	1·530
23	·043	99·858	·993	146·242	1·454
24	·042	100·805	·947	147·629	1·387
25	·040	101·709	·904	148·953	1·324
30	·033	105·701	3·992	154·800	5·847
35	·028	109·024	3·323	159·667	4·867
40	·025	111·865	2·841	163·828	4·161
45	·022	114·342	2·477	167·456	3·628
50	·020	116·537	2·195	170·671	3·215

Thus if we wished to know the maximum work of a given charge fired in a gun with such capacity of bore that the charge suffered five expansions during the motion of the projectile in the gun, the gravimetric density of the charge being unity, the table shows us that for every pound or kilogramme in the charge, an energy of 91·4 foot-tons or 62,394 kilogrammetres will as a maximum be generated.

If the factor of effect for the powder be known, the above values multiplied by that factor will give the energy per pound or kilogramme that may be expected to be raised in the projectile.

But it rarely happens, especially with the very large charges used in the most recent guns, that gravimetric densities so high as unity are employed, and in such cases, from the total realisable energy must be deducted the energy which the powder would have generated had it expanded from a density of unity to that actually occupied by the charge.

Thus in the instance above given, if we suppose the charge instead of a gravimetric density of unity to have a gravimetric density of ·8, which corresponds to a volume of expansion of 1·25, we see from Table XI. that from the 91·4 foot-tons or 62,394 kilogrammetres above given there must be subtracted* 19·23 foot-tons, or 13,127·3 kilogrammetres, leaving 72·17 foot-tons or 49,272·8 kilogrammetres as the maximum energy realisable under the given conditions per pound or per kilogramme of the charge.

As before, these values must be multiplied by the factor of effect to obtain the energy realisable in the projectile.

But to apply these principles to an actual case. The factor of effect of a certain brand of pebble powder having been found in a powder-proof gun to be with that gun between ·82 and ·84, let us examine what are the energies likely to be realised with charges of 70·90 and 100 lb. in an 8-inch gun, of 130 and 140 lb. in a 10-inch gun, and of 235 lb. in an 11-inch gun.

We have selected these instances both because the same powder was used in the experiments, and because they offer considerable variety with respect to the number of expansions and the gravimetric densities of the charges.

Taking first the 8-inch gun. The number of expansions that the charges experienced in the bore of the gun and the original gravimetric densities of the charges were as follow :—

For a charge of 70 lb., number of expansions 6·12, gravimetric density ·605
" " 90 " " " 4·76 " " ·780
" " 100 " " " 4·29 " " ·865

Hence from Table XI. the maximum energy realisable is

For the 70 lb. charge, 99·4 foot-tons −37·60 foot-tons =61·80 foot-tons per lb.
" 90 " " 89·3 " −20·86 " =68·44 " "
" 100 " " 84·9 " −13·66 " =71·24 " "

---

* This correction, as has been elsewhere pointed out by one of us, is only approximate.

Multiplying these energies by the factors of effect obtained from the proof of the powder and by the number of pounds in the charge, we should expect the energy realised from the

70 lb. charge to lie between 3547·3 and 3633·8 foot-tons
90 „ „ „ „ 5050·9 „ 5174·0 „
100 „ „ „ „ 5841·7 „ 5984·2 „

Compare now these results with the actual experiments. On firing the above charges it was found that the 70 lb. charge gave to an 180 lb. shot a velocity of 1694 feet per second, corresponding to a total energy of 3637 foot-tons, the 90 lb. charge a velocity of 2027 feet per second, or a total energy of 5133 foot-tons, and the 100 lb. charge a velocity of 2182 feet per second, or an energy of 5940 foot-tons.

In the 10-inch gun the number of expansions and the gravimetric densities were

For the charge of 130 lb., number of expansions 4·294, gravimetric density 0·792
„ „ 140 „ „ „ 4·050 „ „ 0·840

Hence, as before, the maximum energies realisable are found to be

For the 130 lb., 65·14 foot-tons per lb. (84·94—19·80)
„ 140 „ 66·84 „ „ „ (82·50—15·66)

and multiplying by the same factors of effect, the total energy realised would lie between 6943·9 and 7113·3 foot-tons for the former, and between 7673·2 and 7860·4 foot-tons for the latter charge.

The actual results obtained with the 10-inch gun were for the 130 lb. charge a velocity of 1605 feet per second and an energy of 7158 foot-tons; for the 140 lb. charge a velocity of 1706 feet per second and an energy of 8092 foot-tons. With the 235 lb. charge fired from the 11-inch gun the number of expansions was 4·214, the gravimetric density of the charge ·770, while the energy realised was found to be 13,066 foot-tons, or 55·6 foot-tons per pound of powder used.

It will be noted that with the 8-inch gun (it happens that this calibre is the same as that of the gun used for powder proof, although the charges employed with this last are from one-half to one-third of those we are now discussing) the results realised are in each case very close to those predicted; ·84 as a factor of effect gives calculated results all but coincident with the high limit above given, but it must not be expected that results so closely accordant can always be obtained. Even when the same charges of the same powder and under precisely the same conditions are fired, considerable variations in energy sometimes have place. In the 10-inch gun, and with the larger charges of 130 and 140 lb., it will be observed that the realised energies are in both instances higher than the highest expected energy above given. In other words, for this gun and these charges a factor of from ·85 to ·86 should be used instead of the factor ·84. Again, with the 11-inch gun, when the still larger charge of 235 lb. was employed, it will be found from the figures above given that the factor of effect for this gun, powder, and charge is about ·89.

Hence the factor of effect with the same powder has gradually increased from about ·83 in the powder-proof gun, to ·84 in the 8-inch gun, to ·86 in the 10-inch, and to ·89 in the 11-inch gun. And generally we must point out that not only may the factors of effect differ very much with the powders employed, being in this respect dependent upon circumstances, such as the density of the powder, its size of grain, amount of moisture, chemical composition, nature of charcoal used, &c., but they may also vary considerably even with the same powder if the charges be not fired under precisely the same circumstances. For example, especially with slow-burning powders, the weight of the shot fired exerts a very material influence upon the factor of effect, and the reason is obvious: the slower the shot moves at first the earlier in its passage up the bore is the charge entirely consumed and the higher is the energy realised. The same effect, unless modified by other circumstances, is produced when the charge is increased with the same weight of projectile. In this case the projectile has to traverse a greater length of bore before the same relief due to expansion is attained. The higher pressures which consequently rule react upon the rate of combustion of the powder, and again a somewhat higher energy is obtained.

But these increased effects, of course, correspond to an increased initial tension of the powder gases; but, especially with the smaller guns, a very great difference in the realised energy may arise from other causes. Thus, it having been found that with certain breechloading guns a superior effect was attained by substituting copper rings for lead coating, it was assumed that the cause of this superiority was due to the less friction of the copper rings in the passage of the shot up the bore. But it occurred to one of us that the superior effect was in all probability not due to less friction, and the following experiments were made.

Three rounds were fired from a 12-centimetre B. L. gun with 7 lb. R. L. G. powder and the ordinary service lead-coated shot. The energy realised per lb. of powder was 80·65 foot-tons.

Three more rounds were fired with the lead considerably reduced, and so as barely to fill the grooves. The mean energy realised per lb. was 78·68 foot-tons, thus showing that no superior effect was thereby obtained.

Two rounds were then fired with shell fitted with copper rings. The energy obtained was 82 foot-tons per lb. and the gain was real, but the chamber pressure ran up from a mean of 16·8 tons on the square inch to a mean of 18·6 tons, thus showing that, at all events in great measure, the increased effect is caused not by the copper bands giving rise to less loss by friction, but to the fact that the increased difficulty of forcing the copper bands into the grooves permits the powder to become fully burned at an earlier point in the bore, and thus an increased effect from the powder is realised.

But to show the effect of a greater or less degree of retention of the shot in its chamber in as clear a light as possible, the following experiments were made. Four projectiles for a 12-centimetre B. L. gun were manufactured of precisely the same weight, and which differed from one another in the following respect only: that two of these were fitted with a rotating gas-check of such a form that a high pressure would be necessary to force the projectile into the bore; the two others being fitted with gas-checks of a form such that a comparatively feeble pressure only would be requisite. The copper surfaces in contact with the bore were the same in each case.

Two rounds, one with each form of gas-check, were then fired with a charge of 7 lb. of R. L. G. powder, every condition, except as noted, being precisely the same; the velocities with the two forms were respectively 1609 feet per second and 1512 feet per second, giving rise to 82·04 and 72·44 foot-tons per lb. of powder. The chamber pressures were respectively 15·2 and 12·0* tons per square inch. Two further rounds were then fired with charges of 7½ lb. R. L. G., when velocities of 1644 and 1544 feet per second, or energies per lb. of 79·94 and 70·51 foot-tons were respectively obtained, the chamber pressures in this case being 16·4 and 14·1 tons per square inch.

These experiments prove in the most complete manner that although there may be, and doubtless is, some difference in the amount of friction due to the employment of lead or copper as the driving or rotating material, that difference is perfectly insignificant when compared to the alteration in energy due to the projectile being more or less retained in its initial position, and thus permitting the powder to be consumed earlier and in a more complete manner.

In cases where the projectile has been removed for a considerable distance from the charge, that is, when there is a considerable air space between the charge and the projectile, it has been found that the energy developed in the projectile is materially higher than that due to the expansion of the powder gases through the space traversed by the projectile, and the cause of this appears to us clear. When the charge is ignited at one end of the bore and the ignited products have to travel a considerable distance before striking the projectile, these ignited products possess considerable energy, and a portion of this energy will be communicated to the projectile by direct impact.

With the great lengths of charges used in the larger guns of the present day, some action of this sort doubtless, under ordinary circumstances, frequently happens, thus giving rise to somewhat more energy in the projectile than that due to the expansion of the gases from their initial density in the powder chamber to their final density when the projectile reaches the muzzle of the gun.†

---

* The figures given denote the pressures on the bottom of the chamber and the base of the projectile respectively. The pressures are given as observed, but that on the base of the projectile requires an addition not generally made in actual practice.

† As bearing upon the energy which is usually assigned to a projectile, we may remark that it is customary in correcting the measured to the muzzle velocity to assume that the loss due to the resistance of the air has accrued from the instant the shot quitted the muzzle. But, especially with the large charges and high-muzzle pressures now employed, we believe this rule should be greatly modified.

For a considerable distance from the muzzle of the gun the projectile will be moving in an atmosphere

A considerable number of rounds have been fired from small guns with mining powder. The particular powder of which the analysis is given in this memoir generated an energy differing but little from those obtained with R. L. G. under like conditions. Another sample of mining powder, however, differing from the first sample in containing a higher proportion of saltpetre, generated an energy higher than did any of the R. L. G. powders with which it was compared. This last powder, we may remark, was that which gave the pressure of 44 tons on the square inch when fired in an absolutely close vessel with a gravimetric density of unity.

The Spanish spherical pellet powder generated rather less energy than did the mining powder, but in neither powder did the realised effect vary more from that generated by the normal service Waltham Abbey powders than do occasional samples of these last—a sufficiently curious result, as we have already remarked, when the differences in the composition and the great differences in the decomposition of the various powders are taken into account.

The same remarks as to energy apply generally to the small-grained powders.

These, indeed, cannot be fired satisfactorily without special arrangements in very large charges, chiefly, among other reasons, from their tendency to cake under the pressure of the first ignited portion; but for smaller charges the tables in this memoir alike apply.

Of course, were similar weights of pebble, R. L. G., and F. G. fired in the same gun, the gun being supposed to be of small calibre, the energy realised by the F. G. would be greater than that realised by R. L. G., and still greater than that realised by pebble, on account of its much more rapid combustion. The maximum pressures developed in the bore would correspond with the energies realised.

When the maximum chamber pressure as well as the energy developed by a given charge in a given gun are known, we are able from Table X. to fix very approximately the position of the shot in the bore when the combustion of the charge may practically be considered to be effected. Thus, if with a given energy in the projectile it is found that the maximum chamber pressure is 3118 atmospheres (20·46 tons on the square inch), we learn from Table X. that this tension corresponds to a density of the products of combustion of ·70 ; and hence the charge may be supposed to be practically consumed when the projectile is in such a position in the bore that the products have this mean density. Again, if the observed pressure was 2400 atmospheres (15·73 tons per square inch), the same table shows us that, when this pressure is reached, the position of the projectile in the bore corresponds to a density of charge of ·60.

with a velocity higher than its own, and for some short distance it appears to us probable that its velocity may be receiving an appreciable increase. As corroborative of our views, we may note the great indications of pressure upon gas-checks on the projectiles after these last are released from the support of the bore. Also the fact that when muzzle velocities calculated from data measured inside the bore were compared with those calculated from data measured outside, the latter were in all cases somewhat higher.

It will be gathered from what has just been said that, with a little experience, if the factor of effect and maximum chamber pressure in any gun be known, the behaviour of the same powder in other guns, or in the same gun with other charges or weights of shot, can be anticipated. It was the consideration of the results embodied in Tables X. and XI. that allowed the high energies (more than twice as great as those obtained from the same calibres a few years back) to be predicted and realised; to be realised, also, with less strain to the gun than when much smaller charges were fired, without attention being paid to the all-important point of the density of the products of explosion at the moment when such explosion may be considered to be completed.

It will readily be understood from our remarks upon Table XI. that, to the artillerist, two descriptions of factors of effect are useful. One of these factors is employed to give the ratio between the work actually realised in a given gun, and the maximum work attainable by the charge (its gravimetric density being supposed to be unity). The value of this factor shows whether or not the charge is economically employed. The other factor is employed to denote the ratio between the work actually realised and the maximum work realisable by the charge, in expanding from the gravimetric density of the powder chamber to the mean density of the products of explosion, at the moment when the projectile leaves the muzzle of the gun.

With respect to the first class of factors, it would obviously be difficult to lay down general rules. The value of the factor depends mainly upon the gravimetric density of the charge, but we may remark that, in the most modern guns, even with the advantage they possess of great absolute length, the powder is very uneconomically burned. With the very high charges and consequent small number of expansions, with the low gravimetric density also of the charge, the realised energy per pound of powder is necessarily much lower than was the case with the older guns.

The same difficulty does not exist with regard to the second class of factors of effect. With respect to these, it may be enough to state that, in the smaller guns with R. L. G. powder, the factors of effect vary with a mild brand of powder from ·71 to ·76; with a specially violent brand, from ·82 to ·89, the variations being chiefly dependent upon the principles we have already explained.

In 6-inch guns, firing pebble powder of the normal quality, the factors of effect vary from ·75 to ·82; and, as the calibres of the guns are increased, the factors of effect likewise gradually increase, until, in the 80 and 100-ton guns, factors of from ·89 to ·96 have usually been reached.

In concluding this memoir we desire to remark that, although the agreement between the results of the long and laborious series of experiments and calculations which we now bring to a close have far exceeded the expectations we had formed when we commenced our task, it would yet be idle to suppose that many of our deductions, referring as they do to temperatures and tensions far above the range of ordinary research, will not require some subsequent correction.

But although certain minor points may, as we have said, require considerable correction, we have little doubt that the main theories upon which we insist—confirmed, as they are, by experiments made or facts obtained under an almost infinite variety of circumstances—must be accepted as very approximately correct. It is satisfactory to find that the laws which rule the tensions and temperatures of gases under ordinary circumstances do not lose their physical significance, but are still approximately applicable at the high temperatures and pressures we have been considering.

At all events, whether we are right or wrong in taking this view, it appears to us certain that the rules and tables we have laid down, based on our analyses, experiments, and calculations, may for all practical purposes be accepted as correct, and may, bearing in mind the restrictions to which we have referred in this memoir, be applied to nearly every question of internal ballistics.

Memorandum showing the elementary substances found in the products of explosion, and existing in the powder before combustion.

EXPERIMENT 8.—102·77 grms. pebble, density =·1.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	56·950	45·820
„ „ gas	58·411	44·359
„ „ mean . . .	57·680	45·090

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	. .	9·518	1·294	·153	22·298	11·827	
Solid state	30·039	2·803	7·227	·008	17·517	·086	
Total found	30·039	12·321	8·521	·161	39·815	11·913	
Originally in powder .	29·660	12·456	10·349	·534	38·720	12·160	
Difference	+·379	−·135	−1·828	−·373	+1·092	−·247	

Oxygen in hyposulphite, 3·013 grms.

EXPERIMENT 7.—204·117 grms. pebble, density = ·2.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	111·271	92·846
,, ,, gas. . . .	117·195	86·922
,, ,, mean . . .	114·233	89·884

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	19·06	3·53	·41	44·04	22·84	
Solid state	60·47	5·73	18·19	·01	29·74	·05	
Total found	60·47	24·79	21·72	·42	73·78	22·89	
Originally in powder .	58·91	24·74	20·56	1·06	76·58	23·01	
Difference	+1·56	+0·05	+1·16	−·64	−2·80	−0·12	

Oxygen in hyposulphite, 1·070 grms.

EXPERIMENT 9.—306·175 grms. pebble, density = ·3.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	167·338	138·837
,, ,, gas	174·011	132·164
,, ,, mean . . .	170·674	135·501

Elements in —	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	29·119	3·596	·442	67·927	34·417	
Solid state	90·126	8·904	26·166	·024	45·246	·208	
Total found	90·126	38·023	29·762	·466	113·173	34·625	
Originally in powder .	88·362	37·108	30·832	1·592	114·771	34·006	
Difference	−1·764	+0·915	−1·070	−1·126	−1·598	+0·619	

Oxygen in hyposulphite, 1·858 grms.

EXPERIMENT 12.—411·085 grms. pebble, density =·4.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids	237·717	173·368
,, ,, gas	230·767	180·318
,, ,, mean . . .	234·242	176·843

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	38·314	2·712	·466	90·344	45·007	
Solid state	116·983	11·342	39·047	·013	64·703	·262	
Total found	116·983	49·656	41·759	·479	155·047	45·269	
Originally in powder .	118·639	49·824	41·396	2·138	154·058	45·235	
Difference	−1·656	−0·168	+·363	−1·659	+·989	+·034	

Oxygen in hyposulphite, 8·311 grms.

EXPERIMENT 14.—513·856 grms. pebble, density =·5.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	278·232	235·624
,, ,, gas. . . .	288·739	225·117
,, ,, mean . . .	283·485	230·370

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	49·715	4·073	·676	117·361	58·520	
Solid state	151·125	13·957	45·428	·018	72·595	·361	
Total found	151·125	63·672	49·501	·694	189·956	58·881	
Originally in powder .	148·299	62·279	51·745	2·672	192·505	56·244	
Difference	+2·826	+1·393	−2·244	−1·978	−2·549	+2·637	

Oxygen in hyposulphite, 4·372 grms.

Experiment 37.—295·488 grms. pebble, density =·6.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	167·161	128·327
„ „ gas	166·076	129·412
„ „ mean . . .	166·618	128·870

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	27·874	2·608	·398	65·580	32·410	
Solid state	85·001	8·424	26·806	·008	45·911	·468	
Total found	85·001	36·298	29·414	·406	111·491	32·878	
Originally in powder .	85·277	35·813	29·756	1·536	110·672	32·254	
Difference	−0·276	+0·485	−0·342	−1·130	+0·819	+0·624	

Oxygen in hyposulphite, 4·178 grms.

Experiment 38.—344·736 grms. pebble, density =·70.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	202·460	142·276
„ „ gas	192·885	151·851
„ „ mean . . .	197·673	147·063

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	31·133	4·193	·484	74·178	37·075	
Solid state	97·174	8·742	31·291	·009	60·143	·314	
Total found	97·174	39·875	35·484	·493	134·321	37·389	
Originally in powder .	99·491	41·782	34·715	1·793	129·060	37·413	
Difference	−·317	−1·907	+·709	−1·300	+5·261	−·024	

Oxygen in hyposulphite, 16·030 grms.

EXPERIMENT 43 (76).—393·984 grms. pebble, density =·8.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	229·932	164·052
" " gas	219·181	174·803
" " mean . . .	224·557	169·427

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	36·157	2·965	·402	86·801	43·102	
Solid state	111·046	11·647	34·704	·015	66·653	·490	
Total found	111·046	47·804	37·669	·417	153·454	43·592	
Originally in powder .	113·704	47·751	39·674	2·049	147·519	42·801	
Difference	−2·658	+·053	−2·005	−1·632	+5·935	+·791	

Oxygen in hyposulphite, 11·708 grms.

EXPERIMENT 77.—417·31 grms. pebble, density =·9.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	246·150	171·160
" " gas	232·380	184·930
" " mean . . .	239·260	178·050

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	37·765	3·385	·547	90·866	45·492	
Solid state	117·064	13·570	37·799	·023	70·127	·676	
Total found	117·064	51·335	41·184	·570	160·993	46·168	
Originally in powder .	120·436	50·578	42·023	2·170	156·237	45·306	
Difference	−3·372	+·757	−·839	−1·600	+4·756	+·862	

Oxygen in hyposulphite, 8·001 grms.

EXPERIMENT 1.—90·719 grms. R. L. G., density =·1.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	52·256	38·463
„ „ gas	51·565	39·154
„ „ mean . . .	51·910	38·809

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	7·648	1·421	·149	18·698	10·894	
Solid state	26·113	2·747	6·774	·003	16·222	·051	
Total found	26·113	10·395	8·195	·152	34·920	10·945	1st analysis of powder top of barrel.
Originally in powder .	26·299	9·852	9·344	·490	34·698	11·149	
Difference	−·186	+·543	−1·149	−·338	+·222	−·204	

Oxygen in hyposulphite, 2·665 grms.

EXPERIMENT 3.—190·538 grms. R. L. G., density =·2.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	108·556	81·982
„ „ gas	109·503	81·035
„ „ mean . . .	109·030	81·508

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	16·256	2·669	·289	39·397	22·896	
Solid state	55·479	5·222	16·191	·005	32·037	·100	1st analysis.
Total found	55·479	21·478	18·860	·294	71·434	22·996	
Originally in powder .	55·237	20·692	19·625	1·029	72·559	22·169	
Difference	+·242	+·786	−·765	−·735	−1·125	+·827	

Oxygen in hyposulphite, 1·580 grms.

EXPERIMENT 4.—285·833 grms. R. L. G., density =·3.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	165·992	119·841
„ „ gas	163·174	122·659
„ „ mean . . .	164·583	121·250

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	25·142	3·423	·421	60·582	31·683	
Solid state	82·167	7·521	24·770	·005	50·068	·053	1st analysis.
Total found	82·167	32·663	28·193	·426	110·650	31·736	
Originally in powder .	82·863	31·041	29·441	1·543	108·674	32·756	
Difference	—·696	+1·622	—1·248	—1·117	+1·976	—1·020	

Oxygen in hyposulphite, 5·330 grms.

EXPERIMENT 11.—381·091 grms. R. L. G., density =·4.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	224·951	156·140
„ „ gas	216·358	164·753
„ „ mean . . .	220·655	160·436

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	33·338	2·793	·451	80·520	43·334	
Solid state	108·521	9·400	33·047	·007	69·500	·180	1st analysis.
Total found	108·521	42·738	35·840	·458	150·020	43·514	
Originally in powder .	110·478	41·386	39·252	2·058	144·859	43·278	
Difference	—1·957	+1·352	—3·412	—1·600	+5·161	+·236	

Oxygen in hyposulphite, 13·379 grms.

EXPERIMENT 70.—246·286 grms. R. L. G., density =·5.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	143·997	102·289
„ „ gas	141·824	104·462
„ „ mean . . .	142·910	103·376

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	23·215	1·537	·373	53·089	25·162	
Solid state	70·862	7·610	22·013	·011	42·133	·281	
Total found	70·862	30·825	23·550	·384	95·222	25·443	
Originally in powder .	70·906	30·535	24·917	1·278	90·768	27·873	
Difference	—·044	+·290	—1·367	—·894	+4·454	—2·430	

Oxygen in hyposulphite, 9·122 grms.

EXPERIMENT 39.—295·483 grms. R. L. G., density =·6.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	167·871	127·612
„ „ gas	168·697	126·786
„ „ mean . . .	168·284	127·199

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	27·734	1·856	·020	65·477	31·812	N.B.—2nd analysis taken.
Solid state	85·451	9·421	26·182	·015	47·016	·199	
Total found	85·451	37·155	28·038	·335	112·493	32·011	
Originally in powder .	85·070	36·634	29·894	1·534	108·131	32·766	
Difference	+·381	+·521	—1·856	—1·199	+4·362	—·755	

Oxygen in hyposulphite, 2·748 grms.

Experiment 96.—295·488 grms. R. L. G., density =·6.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	169·403	126·085
„ „ gas	169·256	126·232
„ „ mean . . .	169·330	126·158

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	. .	26·454	4·938	·511	61·681	32·574	
Solid state	85·031	9·694	25·771	·012	48·514	·308	2nd analysis.
Total found	85·031	36·148	30·709	·523	110·195	32·882	
Originally in powder .	85·071	36·635	29·895	1·534	108·870	33·250	
Difference	−·040	−·487	+·814	−1·011	+1·325	−·368	

Oxygen in hyposulphite, 2·940 grms.

Experiment 41.—344·736 grms. R. L. G., density =·7.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	201·915	142·821
„ „ gas	194·931	149·805
„ „ mean . . .	198·423	146·313

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	. .	31·195	3·745	·642	74·051	36·681	
Solid state	98·211	10·545	30·085	·013	59·083	·486	2nd analysis.
Total found	98·211	41·740	33·830	·655	133·134	37·167	
Originally in powder .	99·249	42·740	34·877	1·787	126·987	38·803	
Difference	−1·038	−1·000	−1·047	−1·132	+6·147	−1·636	

Oxygen in hyposulphite, 9·300 grms.

Experiment 44.—393·978 grms. R. L. G., density =·8.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	221·009	172·969
" " gas	231·376	162·602
" " mean . . .	226·192	167·785

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	36·327	2·338	·452	86·473	42·195	
Solid state	111·655	13·185	39·264	·021	61·613	·452	2nd analysis.
Total found	111·655	49·512	41·602	·473	148·086	42·647	
Originally in powder .	113·426	48·845	39·859	2·045	145·103	42·987	
Difference	−1·771	+·667	+1·743	−1·572	+2·983	−·340	

Oxygen in hyposulphite, 1·761 grms.

Experiment 68.—443·23 grms. R. L. G., density =·9.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	257·858	185·372
" " gas	248·696	194·534
" " mean . . .	253·277	189·953

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	40·551	3·199	·500	97·626	48·077	
Solid state	126·402	14·635	40·142	·031	71·412	·643	2nd analysis.
Total found	126·402	55·186	43·341	·531	169·038	48·720	
Originally in powder .	127·606	54·952	44·842	2·300	163·233	49·735	
Difference	−1·204	+·234	−1·501	−1·769	+5·805	−1·015	

Oxygen in hyposulphite, 5·490 grms.

Experiment 16.—102·771 grms. F. G., density =·10.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	55·762	47·009
„ „ gas	61·736	41·038
„ „ mean . . .	58·749	44·022

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	9·436	0·928	·164	21·784	11·710	
Solid state	30·905	2·524	7·721	·003	18·028	·068	
Total found	30·905	11·960	8·149	·167	39·812	11·778	
Originally in powder .	29·331	11·675	10·380	·668	39·795	11·927	
Difference	+1·574	+·285	−2·231	−·501	+·017	−·149	

Oxygen in hyposulphite, 3·518 grms.

Experiment 17.—205·542 grms. F. G., density =·2.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	116·719	88·823
„ „ gas	122·390	83·152
„ „ mean . . .	119·554	85·987

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	17·770	2·977	·393	42·430	22·417	
Solid state	60·080	6·202	16·091	·015	37·099	·068	
Total found	60·080	23·972	19·068	·408	79·527	22·485	
Originally in powder .	58·662	23·349	20·760	1·336	79·281	23·055	
Difference	+1·413	+·623	−1·692	−·928	+·246	−·570	

Oxygen in hyposulphite, 1·597 grms.

Experiment 18.—308·32 grms. F. G., density =·3.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	177·44	130·88
„ „ gas	181·80	126·52
„ „ mean . . .	179·62	128·70

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	27·055	2·579	·419	65·764	32·883	
Solid state	89·076	7·219	26·863	·002	56·388	·072	
Total found	89·076	34·274	29·442	·421	122·152	32·955	
Originally in powder .	89·994	35·025	31·140	2·004	118·750	33·088	
Difference	−·918	−·751	−1·698	−1·583	+3·402	−·133	

Oxygen in hyposulphite, 12·484 grms.

Experiment 19.—411·085 grms. F. G., density =·4.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	237·880	173·210
„ „ gas	240·970	170·116
„ „ mean . . .	239·430	171·660

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	36·039	3·086	·519	88·006	44·015	
Solid state	118·118	9·382	37·849	·008	73·990	·086	
Total found	118·118	45·421	40·935	·527	161·996	44·101	
Originally in powder .	117·325	46·700	41·520	2·672	158·294	45·030	
Difference	+·793	−1·279	−·585	−2·145	+3·702	−·929	

Oxygen in hyposulphite, 17·262 grms.

EXPERIMENT 75.—246·286 grms. F. G., density =·5.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	141·760	104·526
" " gas	144·474	101·812
" " mean . . .	143·117	103·169

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	21·526	2·049	·334	52·261	26·999	
Solid state	70·963	7·000	21·291	·009	43·768	·085	
Total found	70·963	28·526	23·340	·343	96·029	27·084	
Originally in powder .	70·290	27·978	24·875	1·601	94·805	26·849	
Difference	+·673	+·548	−1·535	−1·258	+1·224	+·235	

Oxygen in hyposulphite, 4·840 grms.

EXPERIMENT 40.—295·488 grms. F. G., density =·6.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	170·268	125·220
" " gas	172·509	122·979
" " mean . . .	171·388	124·099

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	25·770	2·511	·368	63·097	32·353	
Solid state	84·762	6·281	26·233	·002	53·973	·137	
Total found	84·762	32·051	28·744	·370	117·070	32·490	
Originally in powder .	84·332	33·567	29·844	1·921	113·765	32·106	
Difference	+·430	−1·516	−1·100	−1·551	+3·305	+·384	

Oxygen in hyposulphite, 13·786 grms.

EXPERIMENT 42.—344·738 grms. F. G., density =·7.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	200·191	144·547
„ „ gas	200·220	144·520
„ „ mean . . .	200·210	144·533

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	29·628	2·614	·413	73·372	38·503	
Solid state	98·540	7·474	30·531	·112	63·417	·135	
Total found	98·540	37·102	33·145	·525	136·789	38·638	
Originally in powder .	98·388	39·162	34·819	2·241	132·644	37·357	
Difference	+·152	−2·060	−1·674	−1·716	+4·145	+1·281	

Oxygen in hyposulphite, 16·182 grms.

EXPERIMENT 47.—393·987 grms. F. G., density =·8.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	231·652	162·335
„ „ gas	229·392	164·595
„ „ mean . . .	230·522	163·465

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	33·138	3·800	·492	81·851	44·184	
Solid state	111·893	8·830	36·136	·008	73·363	·292	
Total found	111·893	41·968	39·936	·500	155·214	44·476	
Originally in powder .	112·444	44·757	39·793	2·561	151·572	42·614	
Difference	—·551	−2·789	+·143	−2·061	+3·642	+1·862	

Oxygen in hyposulphite, 20·106 grms.

Experiment 69.—443·230 grms. F. G., density =·90.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	255·256	187·974
„ „ gas	257·131	186·099
„ „ mean . . .	256·193	187·037

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	. .	38·027	4·875	·545	94·624	48·966	
Solid state	127·001	11·372	38·012	·006	79·476	·327	
Total found	127·001	49·399	42·887	·551	174·100	49·293	
Originally in powder .	126·495	50·350	44·765	2·881	170·498	47·878	
Difference	+·506	−·951	−1·878	−2·330	+3·602	+1·415	

Oxygen in hyposulphite, 16·702 grms.

Experiment 78.—344·738 grms. R. F. G., density =·7.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	202·222	142·516
„ „ gas	200·960	143·780
„ „ mean . . .	201·591	143·148

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	. .	29·469	2·048	·346	72·664	38·621	
Solid state	99·765	10·360	29·551	·008	61·834	·073	
Total found	99·765	39·829	31·599	·354	134·498	38·694	
Originally in powder .	100·077	36·784	34·336	2·103	134·747	36·323	
Difference	−·312	+3·045	−2·737	−1·749	−·250	+2·371	

Oxygen in hyposulphite, 4·140 grms,

EXPERIMENT 79.—344·738 grms. Spanish, density =·7.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	214·917	129·821
„ „ gas	213·875	130·863
„ „ mean . . .	214·396	130·342

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	24·953	3·140	·288	64·032	37·930	
Solid state	100·418	6·545	38·338	·007	68·758	·330	
Total found	100·418	31·498	41·478	·295	132·790	38·260	
Originally in powder .	100·663	29·820	42·989	1·551	131·506	36·461	
Difference	—·245	+1·678	—1·511	—1·256	+1·284	+1·799	

Oxygen in hyposulphite, 4·106 grms.

EXPERIMENT 196.—301·315 grms. CURTIS and HARVEY's No. 6, density =·3.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	168·827	128·963
„ „ gas	180·818	116·972
„ „ mean . . .	174·823	122·967

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	25·389	2·337	·725	60·639	33·877	
Solid state	90·017	8·918	24·324	·013	51·435	·116	
Total found	90·017	34·307	26·661	·738	112·074	33·993	
Originally in powder .	86·929	32·120	31·427	1·567	113·687	31·156	
Difference	+3·088	+2·187	—4·766	—·829	—1·613	+2·837	

Oxygen in hyposulphite, 1·731 grms.

EXPERIMENT 194.—301·315 grms. mining powder, density =·3.

	Calculated solid products. grms.	Calculated gaseous products. grms.
From analysis of solids . . .	142·743	153·921
" " gas	143·895	152·569
" " mean . . .	143·319	153·145

Elements in—	K.	C.	S.	H.	O.	N.	Remarks.
Gaseous state	..	39·601	10·909	1·714	75·330	25·591	
Solid state	70·596	11·801	35·901	·205	23·488	1·328	
Total found	70·596	51·402	46·810	1·919	98·818	26·919	
Originally in powder .	72·014	54·026	45·378	1·989	95·336	25·943	
Difference	−1·418	−2·624	+1·432	−·070	+3·482	+·976	

Oxygen in hyposulphite, 2·062 grms.

ABSTRACT OF EXPERIMENTS.

In this abstract the following abbreviations are used :—

δ, to represent the mean density of the products of explosion; A, the area of the piston of the crusher-gauge; *a*, the sectional area of the copper cylinder.

Experiment 96.—4560 grs. (295·49 grms.) R. L. G.; this experiment is No. 39 repeated. On opening cylinder appearances as usual—colour of fracture, a dark bluish grey; surface not smooth but wavy; tears remained adhering to the side, and the usual sooty deposit observed.

δ.	A.	*a*.	Crush.	Pressure.
·57	·0833	·0417	·134	14·32 tons per square inch.

Experiment 121.—Fired 5960 grs. (386·21 grms.) R. L. G. in large cylinder. Density ·4.

On opening, appearances much as usual. Fracture, slaty grey with yellowish portions. Divided deposit into two portions, top and bottom. Bottom decidedly more yellow than the top. Divided again top and bottom portions into two parts, one bottled and sealed with as little exposure to the air as possible, the other ground and freely exposed to the air for 48 hours.

The ground portion heated but slightly, the bottom portion showing this tendency in the highest degree, but the heating was on the whole very abnormally low.

During exposure the colour of the ground deposit became considerably lighter.

Experiment 122.—Fired 5960 grs. (386·21 grms.) pebble in large cylinder. Density ·4.

On opening the cylinder observed that the deposit was lighter in colour than in the case of the R. L. G. The fracture was also different, being lighter and having several isolated portions yellow or greenish yellow.

Divided, as in last experiment, the deposit into top and bottom, a portion of each being bottled with as little exposure as possible, and a portion of each being finely ground and exposed to the atmosphere for about 48 hours.

The bottom part of the deposit was lighter and yellower than the top portion.

The top ground deposit began to heat when placed on the paper, the deposit on the apex and in the interior, where the greatest heat prevailed, changing rapidly to a light sulphury yellow with a tinge of green.

It attained its greatest heat in about 10 minutes, and in about 15 minutes later was not *hot* to the hand.

The bottom ground portion exhibited the tendency to heat in a much higher degree than the top portion, commencing to heat immediately; the colour of the residue darkening while an orange coloured deposit formed on the surface.

During the exhibition of heat the ground residue smoked considerably, the orange colour on the surface was doubtless due to this vapour.

The smell was very peculiar, SH_2 was quite perceptible, but was not the dominant odour.

The maximum temperature occurred at about 20 minutes after exposure, and a thermometer placed in the centre showed a temperature of over 600° F. (310°·6 C.). The temperature might have been somewhat higher, as the thermometer had to be withdrawn for fear of fracture.

The paper was burned through on which the deposit was placed. After half an hour the deposit began to cool rapidly.

It is to be noticed that the heat appears to play an important part in the changes which take place, as it was observed that the residue at the base of the cone remained unchanged in appearance although more exposed to the action of the air than other portions.

It may also be noted that after the residue has gone through this heating process the physical characteristics are considerably changed.

When taken out of the exploding vessel the residue is always difficult to pound in the mortar, being somewhat unctuous or greasy to the touch, but after the development of the heating phase it becomes crisp and powdery.

Determinations of heat absorbed by calorimeter. Temperature of room 62°.

Experiment 129.

Temperature of calorimeter	58°·95	F.
„ 30,000 grs. water	79·0	„
Water poured into calorimeter.		
After 1 minute, temperature	78·24	„
„ 2 „ „	78·22	„
„ 4 „ „	78·10	„
„ 6 „ „	78·00	„

Hence loss of heat 0°·76 F. in one minute.

Experiment 130.

Temperature of calorimeter	60°·2	F.
„ water	77·6	„
After 1 minute, temperature	76·8	„
„ 2 „ „	76·7	„
„ 3 „ „	76·62	„
„ 4 „ „	76·6	„
„ 6 „ „	76·5	„

Hence loss of heat 0°·8 F. in one minute.

Experiment 131.

Temperature of calorimeter	61°·08	F.
„ water	71·4	„
After 1 minute, temperature	71·0	„
„ 3 „ „	71·0	„
„ 5 „ „	70·95	„

Loss of heat 0°·4 F. in one minute.

Experiment 132.

Temperature of calorimeter	62°·1	F.
„ water	70·8	„
After 1 minute, temperature	70·4	„
„ 2 „ „	70·4	„

Loss of heat 0°·4 F. in one minute.

Experiment 133.

Temperature of calorimeter	62°·6	F.
„ 30,000 grs. water	70·6	„
Water poured into calorimeter.		
After 1 minute, temperature	70·25	„
„ 2 „ „	70·2	„

Loss of heat 0°·35 F. in one minute.

Experiment 134.

Temperature of calorimeter . . . . 63°·45 F.
„ water . . . . 70·2 „
After 1 minute, temperature . . . . . . 70·0 „
„ 2 „ „ . . . . . . 70·0 „

Loss of heat 0°·2 F. in one minute.

Experiment 135.

Temperature of calorimeter . . . . 64°·05 F.
„ water. . . . . 70·0 „
After 1 minute, temperature . . . . . . 69·82 „
„ 2 „ „ . . . . . . 69·8 „

Hence loss of heat 0°·18 F. in one minute.

From the above and similar experiments the following table of loss of heat (the difference of temperature between water and original state of calorimeter being taken as argument) was arranged.

	Difference of temperature.	Loss of heat.	Loss of heat.	
For 30,000 grs. water	2° F.	0°·08 F.	0°·13 F.	For large explosion-vessel. Calorimeter with 25,000 grs. of water.
	4 „	0·16 „	0·26 „	
	6 „	0·24 „	0·39 „	
	8 „	0·32 „	0·52 „	
	10 „	0·40 „	0·65 „	
	12 „	0·48 „	0·78 „	
	14 „	0·56 „	0·91 „	
	16 „	0·64 „	1·04 „	
	18 „	0·72 „	1·18 „	
	20 „	0·80 „	1·31 „	
	22 „	0·88 „	1·44 „	
	24 „	0·96 „	1·57 „	
	26 „	1·04 „	1·70 „	
	27 „	..	1·77 „	
	28 „	..	1·83 „	
	29 „	..	1·90 „	

Experiments for the determination of the specific heats of the vessels used for determining the heat generated by explosion.

In all cases the vessel was boiled, and then kept for five minutes suspended in the escaping steam; it was then transferred to the calorimeter containing 30,000 grs. of distilled water.

Experiment 141.—Weight of vessel, 21,311·6 grs. (1381·0 grms.).

Temperature of air . . . . 58°·8 F.
„ steam . . . . 211·65 „
„ calorimeter before immersion of vessel . . . . 58·88 „
„ calorimeter after thermometer became stationary . . . . . . 70·08 „
Loss of heat in vessel . . 211°·65−70°·08+0°·2 =141·77 „
Gain of heat in water . . . . . 11°·2 +0°·44= 11·64 „
Hence specific heat of vessel=·1156.

Experiment 142.—The same vessel.

Temperature of air . . . . 59°·0 F.
„ steam . . . . 211·65 „
„ calorimeter before experiment . . 62·21 „
„ „ after „ . . 73·20 „
Loss of heat in vessel . . 211°·65−73°·20+0°·2 =138·65 „
Gain of heat in water . . . . . 10°·99+0°·44= 11·43 „
Hence specific heat of vessel=·1158.

Experiment 143.—The same vessel.

Temperature of air . . . . 60°·20 F.
„ steam . . . . 212·20 „
„ calorimeter before experiment . . 61·11 „
„ „ after „ . . 72·20 „
Loss of heat in vessel . . 212°·20−72°·20+0°·20=140·20 „
Gain of heat in water . . . . . 11°·09+0°·44= 11·53 „
Hence specific heat of vessel=·1155.

Experiment 144.—The same vessel.

Temperature of air . . . . 60°·20 F.
„ steam . . . . 212·10 „
„ calorimeter before experiment . . 65·10 „
„ „ after „ . . 75·95 „
Loss of heat in vessel . . 212°·20−75°·95+0°·2 =136·45 „
Gain of heat in water . . . . . 10°·85+0°·44= 11·29 „
Hence specific heat of vessel=·1163.

Hence mean specific heat of vessel from four experiments, ·1158.

Experiment 167.—Weight of vessel, 52,931·6 grs. (3430 grms.).

Temperature of air . . . . 60°·0 F.
„ steam . . . . 211·14 „
„ calorimeter before experiment . . 55·75 „
„ „ after „ . . 84·52 „
Loss of heat in vessel . . 211°·14−84°·52+0°·2 =126·82 „
Gain of heat in water (25,000 grs.) 28°·77+1°·86= 30·63 „

Hence specific heat of vessel=·1140.

Experiment 168.—The same vessel.

Temperature of air . . . . 62°·0 F.
„ steam . . . . 211·14 „
„ calorimeter before experiment . . 55·48 „
„ „ after „ . . 84·10 „
Loss of heat in vessel . . 211°·14−84°·10+0°·2 =127·24 „
Gain of heat in water . . . . . 28°·60+1°·85= 30·40 „

Hence specific heat of vessel=·1132.

Experiment 169.—The same vessel.

Temperature of air . . . . 62°·0 F.
„ steam . . . . 211·14 „
„ calorimeter before experiment . . 55·55 „
„ „ after „ . . 84·30 „
Loss of heat in vessel . . 211°·14−84°·30+0°·2 =127·04 „
Gain of heat in water . . . . . 28°·75+1°·86= 30·61 „

Hence specific heat of vessel=·1138.

Hence mean specific heat of vessel from three experiments=·1137.

Determination of heat evolved by the various powders.

A.—SMALL explosion-vessel.

		Grs.
Weight of water, 30,000 grs.	equivalent in water	30,000·0
„ explosion-vessel, 21,311·6 grs. .	„ „	2,465·8
„ powder products	„ „	28·5
Equivalent in water, of contents of calorimeter		32,494·3
When 200 grs. of powder used, the equivalent in water of the contents of the calorimeter is		32,503·8

Experiment 146.—Exploded 150 grs. Curtis and Harvey's No. 6.

Temperature of calorimeter before explosion . 61°·50 F.
" " after " . 67·78 "
Hence difference . . . . . 6°·28+0°·24= 6·52 "

Hence heat evolved=784·0 gramme-units centigrade.

Experiment 147.—Exploded 150 grs. Spanish.

Temperature of calorimeter before explosion . 65°·10 F.
" " after " . 71·20 "
Hence difference . . . . . 6°·10+0°·24= 6·34 "

Hence heat evolved=762·5 gramme-units centigrade.

Experiment 148.—Exploded 150 grs. R. L. G.

Temperature of calorimeter before explosion . 56°·28 F.
" " after " . 62·07 "
Hence difference . . . . . 5°·79+0°·24= 6·03 "

Hence heat evolved=725·1 gramme-units centigrade.

Experiment 149.—Exploded 150 grs. pebble.

Temperature of calorimeter before explosion . 60°·42 F.
" " after " . 66·10 "
Hence difference . . . . . 5°·68+0°·24= 5·92 "

Hence heat evolved=711·9 gramme-units centigrade.

Experiment 150.—Exploded 150 grs. F. G.

Temperature of calorimeter before explosion . 65°·16 F.
" " after " . 70·80 "
Hence difference . . . . . 5°·64+0°·23= 5·87 "

Hence heat evolved=706·45 gramme-units centigrade.

Experiment 153.—With 200 grs. pebble.

Failure; the plug being spoiled by the explosion.

Experiment 154.—Exploded 150 grs. F. G.

Temperature of calorimeter before explosion . 49°·55 F.
" " after " . 55·45 "
Hence difference . . . . . 59°·0+0°·24= 6·14 "

Hence heat evolved=738·9 gramme-units centigrade.

Experiment 155.—Exploded 150 grs. R. L. G.
Temperature of calorimeter before explosion . 86°·00 F.
,, ,, after ,, . 91·73 ,,
Hence difference . . . . . $5^\circ{\cdot}73+0^\circ{\cdot}24=5{\cdot}97$,,
Hence heat evolved=718·4 gramme-units centigrade.

Experiment 156.—Exploded 150 grs. pebble.
Temperature of calorimeter before explosion . 56°·07 F.
,, ,, after ,, . 61·95 ,,
Hence difference . . . . . . $5^\circ{\cdot}87+0^\circ{\cdot}23=6{\cdot}10$,,
Hence heat evolved=734·1 gramme-units centigrade.

Experiment 157.—Exploded 150 grs. Spanish.
Temperature of calorimeter before explosion . 56°·90 F.
,, ,, after ,, . 63·06 ,,
Hence difference . . . . . $6^\circ{\cdot}16+0^\circ{\cdot}25=6{\cdot}41$,,
Hence heat evolved=771·4 gramme-units centigrade.

Experiment 158.—Exploded 150 CURTIS and HARVEY'S No. 6.
Temperature of calorimeter before explosion . 57°·92 F.
,, ,, after ,, . 63·87 ,,
Hence difference . . . . . $5^\circ{\cdot}95+0^\circ{\cdot}24=6{\cdot}19$,,
Hence heat evolved=744·9 gramme-units centigrade.

Experiment 159.—Exploded 150 grs. mining.
Temperature of calorimeter before explosion . 58°·50 F.
,, ,, after ,, . 62·54 ,,
Hence difference . . . . . $4^\circ{\cdot}04+0^\circ{\cdot}16=4{\cdot}02$,,
Hence heat evolved=505·5 gramme-units centigrade.

Experiment 160.—Exploded 150 grs. R. L. G.
Temperature of calorimeter before explosion . 62°·42 F.
,, ,, after ,, . 68·06 ,,
Hence difference . . . . . $5^\circ{\cdot}64+0^\circ{\cdot}24=5{\cdot}88$,,
Hence heat evolved=707·5 gramme-units centigrade.

Experiment 161.—Exploded 150 grs. pebble.
Temperature of calorimeter before explosion . 49°·05 F.
,, ,, after ,, . 54·59 ,,
Hence difference . . . . . $5^\circ{\cdot}54+0^\circ{\cdot}23=5{\cdot}77$,,
Hence heat evolved=694·4 gramme-units centigrade.

Experiment 162.—Exploded 150 grs. mining.
Temperature of calorimeter before explosion . 56°·80 F.
„ „ after „ . 60·82 „
Hence difference . . . . . 4°·02+0°·20= 4·22 „
Hence heat evolved=507·9 gramme-units centigrade.

Experiment 163.—Exploded 150 grs. Curtis and Harvey's No. 6.
Temperature of calorimeter before explosion . 52°·80 F.
„ „ after „ . 58·65 „
Hence difference . . . . . 5°·85+0°·24= 6·09 „
Hence heat evolved=732·9 gramme-units centigrade.

Experiment 164.—Exploded 150 grs. F. G.
Temperature of calorimeter before explosion . 57°·42 F.
„ „ after „ . 63·26 „
Hence difference . . . . . 5°·84+0°·24= 6·08 „
Hence heat evolved=731·7 gramme-units centigrade.

Experiment 165.—Exploded 150 grs. Spanish.
Temperature of calorimeter before explosion . 55°·70 F.
„ „ after „ . 61·72 „
Hence difference . . . . . 6°·02+0°·24= 6·26 „
Hence heat evolved=753·4 gramme-units centigrade.

Experiment 166.—Exploded 150 grs. pebble.
Temperature of calorimeter before explosion . 61°·12 F.
„ „ after „ . 66·80 „
Hence difference . . . . . 5°·68+0°·22= 5·90 „
Hence heat evolved=710·0 gramme-units centigrade.

Experiment 151.—Exploded 200 grs. mining.
Temperature of calorimeter before explosion . 60°·38 F.
„ „ after „ . 65·87 „
Hence difference . . . . . 5°·49+0°·22= 5·71 „
Hence heat evolved=512·7 gramme-units centigrade.

Experiment 152.—Exploded 200 grs. Curtis and Harvey's No. 6.
Temperature of calorimeter before explosion . 64°·95 F.
„ „ after „ . 73·00 „
Hence difference . . . . . 8°·05+0°·32= 8·37 „
Hence heat evolved=755·7 gramme-units centigrade.

B.—LARGE explosion-vessel.

		Grs.
Weight of water, 25,000 grs.	equivalent in water	25,000·0
„ explosion-vessel, 52,931·6 grs. .	„ „	6,018·3
„ powder products, 400 grs. . .	„ „	76·0
Equivalent in water, of contents of calorimeter		31,094·3

Experiment 171.—Exploded 400 grs. pebble.

Temperature of calorimeter before explosion . 54°·38 F.
„ „ after „ . 69·43 „
Hence difference . . . . . 15°·05+1°·24=16·29 „

Hence heat evolved=703·41 gramme-units centigrade.

Experiment 172.—Exploded 400 grs. R. L. G.

Temperature of calorimeter before explosion . 57°·08 F.
„ „ after „ . 72·44 „
Hence difference . . . . . 15°·36+1°·26=16·62 „

Hence heat evolved=717·7 gramme-units centigrade.

Experiment 173.—Exploded 400 grs. R. L. G.

Temperature of calorimeter before explosion . 57°·22 F.
„ „ after „ . 72·74 „
Hence difference . . . . . 15°·52+1°·26=16·78 „

Hence heat evolved=724·7 gramme-units centigrade.

Experiment 174.—Exploded 400 grs. pebble.

Temperature of calorimeter before explosion . 57°·13 F.
„ „ after „ . 72·42 „
Hence difference . . . . . 15°·29+1°·26=16·55 „

Hence heat evolved=714·7 gramme-units centigrade.

Experiment 175.—Exploded 400 grs. pebble.

Temperature of calorimeter before explosion . 56°·40 F.
„ „ after „ . 72·00 „
Hence difference . . . . . 15°·6+1°·27=16·87 „

Hence heat evolved=728·5 gramme-units centigrade.

Experiment 176.—Exploded 400 grs. R. L. G.
Temperature of calorimeter before explosion . 67°·12 F.
„ „ after „ . 82·38 „
Hence difference . . . . . 15°·26+1°·26=16·52 „
Hence heat evolved=713·4 gramme-units centigrade.

Experiment 177.—Exploded 400 grs. mining.
Temperature of calorimeter before explosion . 53°·27 F.
„ „ after „ . 64·13 „
Hence difference . . . . . 10°·86+0°·71=11·57 „
Hence heat evolved=499·65 gramme-units centigrade.

Experiment 178.—Exploded 400 grs. mining.
Temperature of calorimeter before explosion . 57°·25 F.
„ „ after „ . 68·27 „
Hence difference . . . . . 11°·02+0°·72=11·74 „
Hence heat evolved=507·0 gramme-units centigrade.

Experiment 179.—Exploded 400 grs. mining.
Temperature of calorimeter before explosion . 64°·73 F.
„ „ after „ . 76·03 „
Hence difference . . . . . 11°·30+0°·74=12·04 „
Hence heat evolved=520·0 gramme-units centigrade.

Experiment 181.—Exploded 400 grs. Spanish.
Temperature of calorimeter before explosion . 51°·62 F.
„ „ after „ . 67·76 „
Hence difference . . . . . 16°·14+1°·32=17·46 „
Hence heat evolved=754·0 gramme-units centigrade.

Experiment 182.—Exploded 400 grs. Spanish.
Temperature of calorimeter before explosion . 66°·90 F.
„ „ after „ . 83·21 „
Hence difference . . . . . 16°·31+1°·33=17·64 „
Hence heat evolved=761·8 gramme-units centigrade.

Experiment 183.—Exploded 400 grs. special mining.
Temperature of calorimeter before explosion . 52°·72 F.
„ „ after „ . 63·58 „
Hence difference . . . . . 10°·86+0°·71=11·57 „
Hence heat evolved=499·65 gramme-units centigrade.

Experiment 184.—Exploded 400 grs. Spanish.
Temperature of calorimeter before explosion . 58°·52 F.
„ „ after „ . 75·03 „
Hence difference . . . . . 16°·51+1°·35=17·86 „
Hence heat evolved=771·3 gramme-units centigrade.

Experiment 185.—Exploded 400 grs. Curtis and Harvey's No. 6.
Temperature of calorimeter before explosion . 53°·19 F.
„ „ after „ . 69·26 „
Hence difference . . . . . 16°·07+1°·31=17·38 „
Hence heat evolved=750·6 gramme-units centigrade.

Experiment 186.—Exploded 400 grs. Curtis and Harvey's No. 6.
Temperature of calorimeter before explosion . 55°·80 F.
„ „ after „ . 72·19 „
Hence difference. . . . . 16°·39+1°·33=17·72 „
Hence heat evolved=765·3 gramme-units centigrade.

Experiment 187.—Exploded 400 grs. Curtis and Harvey's No. 6.
Temperature of calorimeter before explosion . 53°·48 F.
„ „ after „ . 69·58 „
Hence difference. . . . . 16°·10+1°·32=17·42 „
Hence heat evolved=752·3 gramme-units centigrade.

Experiment 189.—Exploded 400 grs. F. G.
Temperature of calorimeter before explosion : 55°·61 F.
„ „ after „ . 71·26 „
Hence difference. . . . . 15°·65+1°·28=16·93 „
Hence heat evolved=731·1 gramme-units centigrade.

Experiment 191.—Exploded 400 grs. F. G.
Temperature of calorimeter before explosion . 53°·04 F.
„ „ after „ . 68·49 „
Hence difference. . . . . 15°·45+1°·27=16·72 „
Hence heat evolved=722·1 gramme-units centigrade.

Experiment 192.—Exploded 400 grs. F. G.
Temperature of calorimeter before explosion . 53°·95 F.
„ „ after „ . 69·59 „
Hence difference. . . . . 15°·64+1°·28=16·92 „
Hence heat evolved=730·7 gramme-units centigrade.

Experiment 193.—Fired 5960 grs. = 386·2 grms. mining powder in cylinder No. 6, containing 14,900 grs.

Temperature of gas = 55°·5 F. = 13°·1 C. Bar. 30·025 = 762·35 millims.
Amount of gas = 27″·94 − 8″·35 = 19″·59.
= 444·8 × 19·59 + 18 cub. inches.
= 8731·63 cub. inches.
= 143,076·49 cub. centims. at 13°·1 C., and 762·35 millims.
= 143,518·9 cub. centims. at 13°·1 C., and 760 millims.
= 136,944·2 cub. centims. at 0° C., and 760 millims.
= 354·6 times original volume.

Experiment 194.—Fired 4650 grs. (301·3 grms.) mining powder in cylinder containing 15,500 grs.

In letting the gas escape, found for the first time that the gas lighted, giving rise to an intensely suffocating smell of sulphurous acid, showing, as was indeed otherwise apparent, that sulphydric acid was present in large quantities. Sealed up gases for examination.

Did not take out the deposit as usual, but after the gases had escaped, filled the cylinder, by displacement, with distilled water, entirely freed from air by long boiling.

On the water touching the deposit it decrepitated with considerable sharpness. When the cylinder was full it was entirely sealed, and reopened after an interval of about 48 hours. The solution was then decanted into bottles, freed from oxygen, and sealed for examination.

δ.	A.	a.	Crush.	Pressure.
·30	·0833	·0417	·015	5·04 tons per square inch.

Experiment 195.—Fired 5960 grs. = 386·2 grms. CURTIS and HARVEY's No. 6, in cylinder No. 6.

Temperature of gas = 60°·8 F. = 16°·0 C. Bar. 30″·430 = 772·9.
Amount of gas = 444·8 × 13·10 + 18 cub. inches.
= 5,844·88 cub. inches.
= 95,774·2 cub. centims. at 772·9, and 16°·0 C.
= 97,399·8 cub. centims. at 16°·0 C. and 760 millims.
= 92,004·6 cub. centims. at 0° C. and 760 millims.
= 238·23 vols.

Experiment 196.—Fired 4650 grs. = 301·3 grms. CURTIS and HARVEY's No. 6, in cylinder containing 15,500 grs. Took all the precautions described in experiment 194; observed gas issuing from vessel would not light.

δ.	A.	a.	Crusher.	Pressures.
·30	·0833	·0417	·015	5·04 tons per square inch.

Experiment 197.—Fired 10,000 grs. of mining powder in cylinder containing 15,500 grs. of water. A good deal of gas escaped past crusher-plug.

δ.	A.	a.		Crush.		Pressure.
·700	·0833	·0417	A	·220	=	20·8 tons per square inch.
			B	·221	=	20·8 „
			C	·226	=	21·2 „

Deposit approximately=2025 grs. water.

Experiment 198.—Fired 10,000 grs. CURTIS and HARVEY'S No. 6.

The greater proportion of the gases escaped, the gas getting between the steel barrel and the coil, by the screw of the crusher-plug, causing the coil to crush and indenting the steel and the coil in a very remarkable manner.

δ.	A.	a.		Crush.		Pressure.
·70	·0833	·0417	A	·214	=	19·95 tons per square inch.
			B	·197	=	19·11 „
			C	·197	=	19·11 „

Experiment 199.—A series of experiments in guns of various calibres with mining powder.

Experiment 200.—Fired cylinder hooped with B. R. iron No. 3, and with a capacity of 11,000 grs. water, with a charge of 8750 grs. pebble, and 2250 grs. F. G., 11,000 grs. in all. Copper in crusher-plug crushed beforehand to 35 tons.

Head of crusher-plug broke off by the explosion, and gas escaped, taking, as nearly as could be guessed, from one to two seconds to escape.

Outside diameter of cylinder before firing . . 10″·368
„ „ after „ . . 10″·393

Pressure developed over 39 tons, but not reliable, owing to the escape of the gases.

Experiment 201.—Fired cylinder hooped with SIEMENS' mild steel No. 2, and with a capacity of 11,200 grs. water, with a charge of 11,200 grs. powder, consisting of 8750 grs. pebble and 2450 grs. F. G.

Gas escaped with great rapidity past the firing cone, which was of course destroyed; great difficulty found in extracting the crusher-piston, which had been jammed by the compression of the chamber in which it was placed. Its record was therefore valueless.

Experiment 202.—Fired same cylinder with a charge of 13,640 grs. powder, of which 8375 grs. were pebble, the rest F. G. Crusher-plug blew out before charge fully fired.

Experiment 225.—Fired 9000 grs. pebble and 3000 grs. F. G. in cylinder No. 2, containing 12,680 grs. water, less 670 grs. occupied by internal crusher-gauge. One internal crusher used; gas escaped slowly.

δ.	A.	a.		Crush.		Pressure.
1·0	·0417	·0833	B crusher	·193	=	42·52 tons per square inch.
			C „	·193	=	42·72 „

Experiment 230.—Fired in No. 2 cylinder (SIEMENS'), containing 12,680 grs. 11,360 grs. mining powder, same as that tested in the 2·5-inch B. L. gun on 4th September, 1878. The gas escaped through the insulated cone, almost at once. It did not escape with any violence.

Pressures indicated were as follow:—

δ.	A.	a.		Crush.		Pressure.	
1	·0417	·0833	A	·165	=	36·8 tons per square inch.	A, doubtful; piston being jammed.
			B	·200	=	43·9 „	
			C	·200	=	43·9 „	

Experiment 233.—Fired 9000 grs. pebble, and 4000 grs. F. G., total 13,000 grs. (842·4 grms.), in No. 2 SIEMENS' cylinder, cubic contents 12,680 grs.—2000 grs. for two internal crusher-gauges; total contents, 10,680 grs. The pressure forced out the closing-plugs by shearing the threads.

ε.	A.	a.		Crush.		Pressure.
1·21	·0417	·0833	A crusher	·256	=	55·6 tons per square inch.
			B „	·256	=	55·6 „
			C „	·260	=	56·8 „

NOTE.

(Added 9th March, 1880.)

Since this memoir was submitted to the Society, we have been led, in consequence of a communication made to us by Dr. DEBUS, to modify considerably our views with regard to the formation of hyposulphite.

The experiments rendered necessary by Dr. DEBUS' discovery are fully described and discussed in a note submitted to the Royal Society,[*] but as the facts there given have led us to the conclusion "that although it would seem that in certain cases and under certain exceptional circumstances potassium hyposulphite does exist as a secondary, it exists in no case as a primary product, and should not, therefore, be reckoned among the normal constituents of powder residues," we have recalculated the whole of our analytical results, and we append two tables, Nos. XII. and XIII., giving for each experiment the products of decomposition calculated on the hypothesis that prior to removal from the explosion vessel the whole of the hyposulphite found was in the form of mono or polysulphides.

---

[*] Proc. Roy. Soc., vol. xxx., p. 198.

TABLE XII.—Showing the mean analytical results obtained from an examination of the solid and gaseous products of decomposition of Pebble, R. L. G., and F. G. powders; showing also the same particulars with respect to four other powders.

No. of experiments.	Nature of powder.	Mean density of products of combustion.	Percentage composition by volume of the gas.							Percentage composition by weight of the solid residue.								
			Carbonic anhydride.	Carbonic oxide.	Nitrogen.	Sulphydric acid.	Marsh gas.	Hydrogen.	Oxygen.	Potassium carbonate.	Potassium sulphate.	Potassium monosulphide.	Potassium sulpho-cyanate.	Potassium nitrate.	Potassium oxide.	Ammonium sesqui-carbonate.	Sulphur.	Charcoal.
8	Pebble, W. A. . . .	·10	46·66	14·76	32·75	3·13	..	2·70	..	58·56	15·84	20·50	0·09	0·51	..	0·17	4·33	..
7		·20	44·78	16·09	31·31	4·23	..	3·59	..	58·01	13·85	20·41	0·06	..	..	0·09	7·58	..
9		30	47·03	15·51	31·71	2·90	..	2·84	..	60·09	12·74	19·24	0·21	0·03	..	0·17	7·52	..
12		·40	49·52	13·95	32·16	1·70	0·32	2·85	..	57·25	13·69	18·52	0·25	0·08	..	0·07	8·74	1·40
14		·50	49·82	13·36	32·19	1·96	0·58	2·08	..	57·04	12·12	23·02	0·23	0·20	..	0·08	7·31	..
37		·60	49·48	13·75	31·83	2·24	0·55	2·15	..	59·00	13·82	17·68	0·36	0·32	..	0·06	8·76	..
38		·70	49·93	12·51	32·08	3·18	0·35	1·95	..	54·64	13·91	22·72	0·41	0·26	..	0·06	8·00	..
43		·80	51·54	11·88	32·61	1·96	0·34	1·67	..	62·35	10·94	16·84	0·06	0·33	..	0·08	9·40	..
77		·90	51·75	10·87	32·72	2·13	0·68	1·85	..	66·43	9·45	11·92	0·59	0·44	..	0·12	11·05	..
		Means	48·95	13·63	32·15	2·60	0·31	2·35	..	59·26	12·93	18·93	0·25	0·24	..	0·10	8·08	0·16
		Highest	51·75	16·09	32·75	4·23	0·68	3·59	..	66·43	15·84	23·02	0·59	0·51	..	0·17	11·05	1·40
		Lowest	44·78	10·87	31·31	1·70	..	1·67	..	54·64	9·45	11·92	0·06	0·03	..	0·06	4·33	..
1	R. L. G., W. A. . . .	·10	49·00	8·98	35·60	4·06	0·29	2·07	..	55·41	21·58	16·68	..	0·59	..	0·06	4·93	0·75
3		·20	46·56	11·47	35·13	3·58	0·07	2·62	0·57	55·47	24·44	13·08	0·05	0·12	..	0·06	6·76	0·02
4		·30	49·35	11·60	32·96	3·11	..	2·98	..	54·16	25·03	13·76	0·05	0·03	..	0·04	6·93	..
11		·40	50·25	10·84	34·23	1·93	0·28	2·47	..	51·82	24·35	17·00	0·17	0·13	..	0·04	6·49	..
70		·50	47·21	17·04	30·29	1·61	0·84	3·01	..	64·77	4·96	19·47	0·30	0·53	..	0·11	9·86	..
39		·60	46·29	14·52	32·40	4·29	0·36	2·14	..	66·43	10·90	11·85	0·28	0·46	..	0·09	9·99	..
96		·60	50·22	13·93	31·74	1·62	0·35	2·14	..	64·88	11·16	13·91	0·26	..	..	0·11	9·68	..
41		·70	49·75	13·38	31·94	2·85	0·55	1·53	..	68·25	11·04	15·34	0·51	0·44	..	0·08	9·34	..
44		·80	51·62	12·16	32·16	1·56	0·77	1·72	..	67·00	8·88	10·92	0·25	0·18	..	0·11	12·66	..
68		·90	52·65	10·73	32·65	1·90	0·80	1·27	..	67·16	8·71	12·50	0·38	0·20	..	0·15	10·90	..
		Means	49·29	12·47	32·91	2·65	0·43	2·19	0·06	61·03	15·10	14·45	0·22	0·27	..	0·08	8·74	0·08
		Highest	52·65	17·04	35·60	4·29	0·84	3·01	0·57	67·16	25·03	19·47	0·51	0·59	..	0·15	12·66	0·75
		Lowest	46·29	8·98	30·29	1·56	0·07	1·27	..	51·82	4·96	10·92	0·05	0·03	..	0·04	4·93	..
16	F. G., W. A.	·10	44·76	16·25	32·57	2·26	0·18	3·83	0·15	52·43	19·00	18·30	..	0·21	5·74	0·07	4·25	..
17		·20	47·41	12·35	32·35	3·76	..	4·13	..	60·20	24·55	8·30	0·02	0·08	..	0·15	6·70	..
18		·30	50·45	11·33	32·22	2·21	..	3·51	0·28	47·17	23·24	19·23	0·07	0·10	..	0·01	10·18	..
19		·40	51·79	10·31	32·54	2·00	..	3·36	..	48·37	23·46	21·50	0·08	0·10	..	0·04	6·45	..
75		·50	51·04	10·38	33·15	2·20	0·27	2·96	..	57·97	21·45	12·55	0·07	0·09	..	0·08	7·79	..
40		·60	52·00	9·60	33·28	2·26	0·18	2·68	..	45·55	24·15	20·12	0·17	0·18	3·49	0·01	6·38	..
42		·70	53·02	7·91	34·26	2·03	0·50	2·13	0·15	48·39	23·61	20·90	0·26	0·21	..	0·03	6·60	..
47		·80	51·80	8·32	34·64	2·61	0·41	2·04	0·18	47·80	23·15	21·98	0·26	0·28	..	0·04	6·49	..
69		·90	53·34	7·71	33·81	2·95	0·16	2·04	..	54·17	19·64	18·88	0·27	0·28	..	0·03	6·73	..
		Means	50·63	10·47	33·21	2·48	0·19	2·96	0·08	51·34	22·47	17·97	0·13	0·17	1·02	0·05	6·83	..
		Highest	53·34	16·25	34·64	3·76	0·50	4·13	0·28	60·20	24·55	21·98	0·27	0·28	5·74	0·15	10·18	..
		Lowest	44·76	7·71	32·22	2·00	..	2·04	..	45·55	19·00	8·30	..	0·08	..	0·01	4·25	..
78	R. F. G., W. A. . . .	·70	52·40	8·86	34·51	1·60	0·12	2·51	..	60·17	22·35	9·14	0·04	0·06	..	0·05	8·19	..
79	Spanish spherical . .	·70	53·34	4·62	37·80	2·74	..	1·29	0·21	35·66	48·55	7·72	0·04	0·95	..	0·04	7·04	..
196	CURTIS & HARVEY, No. 6	·80	50·22	7·52	34·46	2·08	2·46	3·26	..	59·10	21·65	12·42	..	0·29	..	0·09	6·45	..
194	Mining powder . . .	·80	32·15	33·75	19·08	7·10	2·73	5·24	..	41·36	0·59	37·10	2·95	0·09	..	1·73	14·11	2·02

TABLE XIII. Composition by weight of the products of combustion of 1 gramme of fired gunpowder of the under-mentioned natures, and of various gravimetric densities.

No. of experiments.	Nature of powder.	Mean density of products of combustion.	Proportions by weight of gaseous products.							Proportions by weight of the solid residue.									Proportion by weight of total gaseous products.	Proportion by weight of total solid products.	Water.
			Carbonic anhydride.	Carbonic oxide.	Nitrogen.	Sulphydric acid.	Marsh gas.	Hydrogen.	Oxygen.	Potassium carbonate.	Potassium sulphate.	Potassium mono-sulphide.	Potassium sulpho-cyanate.	Potassium nitrate.	Potassium oxide.	Ammonium sesqui-carbonate.	Sulphur.	Charcoal.			
8	Pebble, W. A. . . .	·10	·2634	·0530	·1176	·0137	..	·0007	..	·3174	·0858	·1111	·0005	·0027	..	·0009	·0234	..	·4484	·5418	·0095
7		·20	·2505	·0572	·1114	·0183	..	·0009	..	·3203	·0765	·1127	·0003	..	..	·0005	·0419	..	·4383	·5522	·0095
9		·30	·2609	·0548	·1120	·0124	..	·0007	..	·3303	·0700	·1058	·0012	·0002	..	·0009	·0413	..	·4408	·5497	·0095
12		·40	·2683	·0481	·1109	·0071	·0007	·0006	..	·3176	·0760	·1028	·0014	·0004	..	·0004	·0485	·0077	·4357	·5548	·0095
14		·50	·2768	·0472	·1137	·0084	·0012	·0005	..	·3096	·0658	·1249	·0012	·0011	..	·0004	·0397	..	·4478	·5427	·0095
37		·60	·2695	·0477	·1103	·0094	·0011	·0005	..	·3257	·0763	·0976	·0020	·0018	..	·0003	·0484	..	·4385	·5520	·0095
38		·70	·2748	·0438	·1124	·0135	·0007	·0005	..	·2977	·0758	·1238	·0022	·0014	..	·0003	·0436	..	·4457	·5448	·0095
43		·80	·2785	·0409	·1121	·0082	·0007	·0004	..	·3428	·0601	·0926	·0003	·0018	..	·0004	·0517	..	·4408	·5497	·0095
77		·90	·2743	·0367	·1103	·0087	·0014	·0005	..	·3711	·0528	·0666	·0033	·0025	..	·0007	·0617	..	·4318	·5587	·0095
		Means	·2685	·0477	·1123	·0111	·0006	·0006	..	·3258	·0710	·1042	·0014	·0013	..	·0005	·0445	·0008	·4409	·5496	·0095
		Highest	·2785	·0572	·1176	·0183	·0014	·0009	..	·3711	·0858	·1249	·0033	·0027	..	·0009	·0617	..	·4484	·5587	·0095
		Lowest	·2505	·0367	·1103	·0071	..	·0004	..	·2977	·0528	·0666	·0003	..	..	·0003	·0234	..	·4318	·5418	·0095
1	R. L. G., W. A.	·10	·2653	·0309	·1226	·0170	·0006	·0005	..	·3059	·1191	·0921	..	·0033	..	·0003	·0272	·0041	·4369	·5520	·0111
3		·20	·2497	·0391	·1198	·0148	·0001	·0006	·0022	·3121	·1375	·0736	·0003	·0007	..	·0003	·0380	·0001	·4263	·5626	·0111
4		·30	·2633	·0394	·1119	·0128	..	·0007	..	·3037	·1403	·0772	·0003	·0002	..	·0002	·0389	..	·4281	·5608	·0111
11		·40	·2702	·0371	·1172	·0080	·0006	·0006	..	·2877	·1352	·0944	·0010	·0007	..	·0002	·0360	..	·4337	·5552	·0111
70		·50	·2601	·0581	·1053	·0068	·0017	·0007	..	·3601	·0276	·1083	·0017	·0030	..	·0006	·0548	..	·4327	·5561	·0112
39		·60	·2480	·0495	·1101	·0177	·0007	·0005	..	·3739	·0614	·0667	·0016	·0026	..	·0005	·0562	..	·4265	·5629	·0111
96		·60	·2672	·0471	·1074	·0067	·0007	·0005	..	·3629	·0624	·0778	·0015	..	..	·0006	·0541	..	·4296	·5593	·0111
41		·70	·2655	·0454	·1085	·0118	·0011	·0004	..	·3519	·0614	·0853	·0028	·0024	..	·0004	·0520	..	·4327	·5562	·0111
44		·80	·2651	·0397	·1051	·0062	·0014	·0040	..	·3802	·0504	·0620	·0014	·0010	..	·0006	·0718	..	·4215	·5674	·0111
68		·90	·2760	·0358	·1089	·0077	·0015	·0003	..	·3764	·0488	·0700	·0021	·0011	..	·0008	·0611	..	·4302	·5603	·0111
		Means	·2630	·0422	·1117	·0109	·0008	·0009	·0002	·3415	·0844	·0807	·0013	·0015	..	·0004	·0490	·0004	·4298	·5591	·0111
		Highest	·2760	·0581	·1226	·0177	·0017	·0040	·0022	·3802	·1403	·1083	·0021	·0033	..	·0008	·0718	·0041	·4369	·5674	·0112
		Lowest	·2480	·0309	·1051	·0062	·0001	·0003	..	·2877	·0276	·0620	..	..	..	·0002	·0272	..	·4215	·5520	·0111
16	F. G., W. A.	·10	·2512	·0580	·1163	·0098	·0004	·0010	·0006	·2872	·1042	·1003	..	·0011	·0315	·0004	·0233	..	·4372	·5480	·0148
17		·20	·2490	·0413	·1081	·0153	..	·0010	..	·3434	·1401	·0473	·0001	·0005	..	·0009	·0382	..	·4147	·5705	·0148
18		·30	·2621	·0374	·1065	·0089	..	·0008	·0010	·2683	·1321	·1093	·0003	·0005	..	·0001	·0579	..	·4167	·5685	·0148
19		·40	·2765	·0350	·1105	·0082	..	·0008	..	·2680	·1300	·1192	·0004	·0006	..	·0002	·0358	..	·4310	·5542	·0148
75		·50	·2665	·0344	·1102	·0089	·0005	·0007	..	·3269	·1210	·0708	·0004	·0005	..	·0005	·0439	..	·4212	·5640	·0148
40		·60	·2782	·0327	·1133	·0093	·0003	·0007	..	·2508	·1330	·1108	·0009	·0010	·0192	·0001	·0349	..	·4345	·5507	·0148
42		·70	·2804	·0266	·1152	·0083	·0010	·0005	·0006	·2674	·1305	·1155	·0014	·0012	..	·0002	·0364	..	·4326	·5526	·0148
47		·80	·2752	·0281	·1171	·0107	·0008	·0005	·0007	·2640	·1278	·1214	·0014	·0015	..	·0002	·0358	..	·4331	·5521	·0148
69		·90	·2812	·0259	·1134	·0120	·0003	·0005	..	·2989	·1084	·1042	·0015	·0016	..	·0002	·0371	..	·4333	·5519	·0148
		Means	·2689	·0355	·1123	·0101	·0004	·0007	·0003	·2861	·1252	·0999	·0007	·0009	·0056	·0003	·0381	..	·4282	·5569	·0148
		Highest	·2892	·0580	·1171	·0153	·0010	·0010	·0010	·3434	·1401	·1214	·0015	·0016	·0315	·0009	·0579	..	·4372	·5705	·0148
		Lowest	·2490	·0259	·1065	·0082	..	·0005	..	·2508	·1042	·0473	..	·0005	..	·0001	·0233	..	·4147	·5480	·0148
78	R. F. G., W. A.	·70	·2686	·0289	·1126	·0064	·0002	·0006	..	·3458	·1285	·0525	·0002	·0003	..	·0003	·0471	..	·4173	·5747	·0080
79	Spanish spherical . .	·70	·2457	·0136	·1108	·0097	..	·0003	·0007	·2156	·2975	·0473	·0002	·0058	..	·0002	·0431	..	·3808	·6127	·0065
196	CURTIS & HARVEY, No. 6	·30	·2593	·0247	·1132	·0083	·0046	·0008	..	·3413	·1250	·0717	..	·0017	..	·0005	·0372	..	·4109	·5774	·0117
194	Mining powder . . .	·30	·2279	·1522	·0858	·0389	·0070	·0017	..	·1945	·0028	·1745	·0139	·0004	..	·0084	·0664	·0095	·5135	·4704	·0161

IX. *English Reproduction Table.*

By Dr. W. FARR, *F.R.S.*

Received January 15,—Read January 29, 1880.

IN the office of reproduction there are concerned the male, the female, and their offspring. We represent the males at each year of age by l_x, the females by l_y, the children born of women at each age (*soboles*) by s_y; the children born in wedlock by s'_y, those born out of wedlock by s''_y.

The females are classified into the wives (*uxores*), the spinsters (*filiæ*), the widows (*viduæ*).

We take the wives, spinsters, and widows from the English Census of 1871, and portion out the P_y of the English life table on this basis; the offspring we take from Norwegian returns of 1871 and 1872, after reducing the relative proportions of legitimate and illegitimate births to the same as in England in those years. We take the statistics of children born to women of the several ages for Norway, because in England the age of the mother at the birth of her child is not given at the registry of the birth;* and comparing returns from Scotland, Norway, Sweden, and Denmark, that from Norway commends itself on the whole as the best adapted to our use.

The life table represents a stationary female population; and if we reckon the child-bearing period of life to include all ages between 15 and 55 years, we have a constant population of 11,248,950 to which to look for the sustentation of a population of the uniform strength, including males, of 40,858,184. Of these 11,248,950 women aged 15 to 55 years, 6,278,749 are wives, 4,265,622 are spinsters, and 704,579 are widows.

The wives give birth to 1,395,529 children, the spinsters and widows to 55,381 children; the total annual number of births, therefore, among such a population is 1,450,910.

Then by means of the following formulæ we can determine the number of children born to any given number of women of a specified age: thus

$\frac{s_{20}}{P_{20}}=\cdot08924$, and $\cdot08924\times100{,}000=8{,}924=$ the number of children born in or out of wedlock to 100,000 women aged 20.

$\frac{s'_{20}}{u_{20}}=\cdot40070$, or 100,000 wives aged 20 bear 40,070 children.

---

* It is to be regretted that the mother's age at the birth of her child is not yet included in the particulars recorded in the English birth registers.

$\frac{s''_{20}}{f_{20}+v_{20}}=\cdot01070$, or 100,000 spinsters and widows bear 1,070 illegitimate* children.

$\frac{s_{30}}{P_{30}}=\cdot24634.$ $\frac{s_{40}}{P_{40}}=\cdot16303.$

$\frac{s'_{30}}{u_{30}}=\cdot33652.$ $\frac{s'_{40}}{u_{40}}=\cdot21048.$

$\frac{s''_{30}}{f_{30}+v_{30}}=\cdot02850.$ $\frac{s''_{40}}{f_{40}+v_{40}}=\cdot01024.$

Estimated female population of the child-bearing ages in Norway, 1871.

Ages.	Total women.	Wives.†	Spinsters and widows.†
15-55	475,446	231,644	243,802
15-20	84,855	911	83,944
20-25	74,974	14,701	60,273
25-30	66,583	32,689	33,894
30-35	55,272	37,839	17,433
35-40	55,063	40,932	14,131
40-45	51,170	39,823	11,347
45-50	46,076	34,861	11,215
50-55	41,453	29,888	11,565

Births of children at the various ages of their mothers in Norway, 1871-72.

Ages of mothers.	Births registered.‡ Mean of two years, 1871-72.			After reducing the relative proportions of total legitimate and illegitimate births to the same as in England, 1871-72.		
	Legitimate.	Illegitimate.	Total.	Total.	Legitimate.	Illegitimate.
15-55	47,027	4,649	51,676	51,676	48,826	2,850
15-20	277	210	487	487	358	129
20-25	5,216	1,654	6,870	6,870	5,856	1,014
25-30	11,493	1,539	13,032	13,032	12,089	943
30-35	11,650	717	12,367	12,367	11,927	440
35-40	10,669	348	11,017	11,017	10,804	213
40-45	6,219	145	6,364	6,364	6,275	89
45-50	1,446	32	1,478	1,478	1,458	20
50-55	57	4	61	61	59	2

---

[*] The terms *legitimate* and *illegitimate* in this paper are used in a sense to some extent conventional, since posthumous children, though legitimate, are counted with the illegitimate.

[†] The population at the several ages is here divided into the classes of married and unmarried in the same proportions as existed at the Census of 1865.

[‡] There were 11,750 births of legitimate and 2,550 of illegitimate children the ages of whose mothers were not stated; they have been distributed proportionally over all the ages.

PROPORTIONS of children born annually to women of each age.

Age of mother. y.	Proportion of children born annually to *females* aged y. $\frac{s_y}{P_y}$	Proportion of legitimate children born annually to *wives* aged y. $\frac{s'_y}{u_y}$	Proportion of illegitimate children born annually to *spinsters* and *widows* aged y. $\frac{s''_y}{f_y + v_y}$
15	·00036	·37959	·00010
16	·00137	·38714	·00050
17	·00588	·39297	·00154
18	·02155	·39713	·00449
19	·04684	·39968	·00757
20	·08924	·40070	·01070
21	·12253	·40023	·01381
22	·14776	·39834	·01682
23	·17349	·39510	·01966
24	·19658	·39056	·02225
25	·21589	·38480	·02453
26	·23080	·37786	·02641
27	·24119	·36982	·02782
28	·24570	·35827	·02869
29	·24728	·34725	·02894
30	·24634	·33652	·02850
31	·24332	·32591	·02729
32	·23848	·31520	·02524
33	·23410	·30655	·02348
34	·22881	·29757	·02149
35	·22235	·28779	·01937
36	·21446	·27674	·01721
37	·20484	·26395	·01507
38	·19330	·24896	·01337
39	·17945	·23129	·01174
40	·16303	·21048	·01024
41	·14376	·18606	·00892
42	·12141	·15757	·00784
43	·10156	·13244	·00643
44	·08206	·10762	·00509
45	·06346	·08379	·00385
46	·04631	·06163	·00274
47	·03115	·04182	·00178
48	·02047	·02778	·00110
49	·01221	·01677	·00060
50	·00633	·00880	·00028
51	·00275	·00387	·00014
52	·00137	·00197	·00004
53	·00042	·00062	..
54	·00006	·00009	..

NOTE.—The proportions shown in this table of legitimate births to wives, and of illegitimate births to unmarried women, were derived from the foregoing tables, giving the women of each class living in 1871 and the mean annual number of children born in and out of wedlock to mothers of several quinquennial ages in the two years 1871–72, the proportions being interpolated for each year of age. These proportions applied to the classes of married and unmarried women of the English life table population (so divided according to the Census of 1871) produced the numbers of legitimate and illegitimate children born annually to women of each age. The total births were obtained from the casting of the legitimate and illegitimate births, and the first column of this table was thus derived from a comparison of the total births with the total female population.

PROPORTIONS of children born annually to married and unmarried women of specified ages in Scotland, Norway, Sweden, and Denmark.

Ages of mothers.	Proportions of children born annually to 1,000 women of specified ages.											
	Scotland,* 1855.			Norway, 1871–72.			Sweden, 1871–75.			Denmark,† 1865–74.		
	Total children to 1,000 women of all classes.	Legitimate children to 1,000 wives.	Illegitimate children to 1,000 spinsters and widows.	Total children to 1,000 women of all classes.	Legitimate children to 1,000 wives.	Illegitimate children to 1,000 spinsters and widows.	Total children to 1,000 women of all classes.	Legitimate children to 1,000 wives.	Illegitimate children to 1,000 spinsters and widows.	Total children to 1,000 women of all classes.	Legitimate children to 1,000 wives.	Illegitimate children to 1,000 spinsters and widows.
15–55	120·86	252·22	18·43	108·69	210·78	11·69	110·04	206·65	23·73	114·11	195·17	24·06
15–20	19·55	564·03	7·86	5·74	392·97	1·54	9·00	507·04	4·04	10·15	530·22	5·02
20–25	144·85	475·80	34·41	91·63	398·34	16·82	104·84	495·12	32·98	117·98	478·28	38·52
25–30	227·07	406·60	36·36	195·73	369·82	27·82	200·97	382·23	51·59	221·72	380·79	52·17
30–35	223·63	336·52	23·78	223·75	315·20	25·24	227·95	327·03	50·61	240·93	312·48	43·03
35–40	185·28	268·68	13·84	200·08	263·95	15·07	201·62	263·71	38·60	197·70	235·29	31·60
40–45	83·86	125·84	4·05	124·37	157·57	7·84	116·86	151·74	17·74	101·55	119·41	13·02
45–50	12·81	19·96	·51	32·08	41·82	1·78	17·55	23·58	1·66	11·92	14·24	1·54
50–55	·63	1·12	..	1·47	1·97	·17	·30	·42	·06	·06	·08	..

* The proportions for Scotland are derived from data relating to the year 1855, published in the supplement to the Scotch Registrar-General's Annual Reports for the ten years 1861–70.

† Exclusive of Copenhagen.

ENGLISH Life and Birth Table.

Age. y	Females living at age y by the English life table out of 1,000,000 children born alive, of whom 488,255 are girls.					Children born alive annually, in and out of wedlock, to women living at age y (*soboles*).			Age. y
	Females living at each age. l_y	Female population in each year of age. $P_y = l_{y+\frac{1}{2}}$	Classes of the population.			Legitimate and illegitimate. s_y	Legitimate. s'_y	Illegitimate. s''_y	
			Wives (*uxores*). u_y	Spinsters (*filiæ*). f_y	Widows (*viduæ*). v_y				
15	340,273	339,344	228	339,104	12	121	87	34	15
16	338,385	337,390	756	336,608	26	461	293	168	16
17	336,356	335,277	3,721	331,512	44	1,973	1,462	511	17
18	334,151	332,977	14,469	318,390	118	7,176	5,746	1,430	18
19	331,751	330,473	33,097	297,111	265	15,479	13,228	2,251	19
20	329,142	327,739	65,997	261,034	708	29,246	26,445	2,801	20
21	326,323	324,895	91,408	232,333	1,154	39,808	36,584	3,224	21
22	323,456	322,005	110,512	209,819	1,674	47,578	44,021	3,557	22
23	320,544	319,073	130,732	185,995	2,346	55,355	51,652	3,703	23
24	317,592	316,102	149,620	163,338	3,144	62,140	58,436	3,704	24
25	314,603	313,095	166,304	142,740	4,051	67,595	63,994	3,601	25
26	311,579	310,056	180,312	124,697	5,047	71,560	68,133	3,427	26
27	308,524	306,985	191,528	109,348	6,109	74,043	70,831	3,212	27
28	305,440	303,887	200,095	96,570	7,222	74,666	71,688	2,978	28
29	302,328	300,762	206,298	86,094	8,370	74,371	71,637	2,734	29
30	299,190	297,611	210,482	77,583	9,546	73,314	70,831	2,483	30
31	296,027	294,437	212,999	70,700	10,738	71,641	69,419	2,222	31
32	292,840	291,238	214,176	65,121	11,941	69,453	67,508	1,945	32
33	289,631	288,017	214,303	60,583	13,131	67,426	65,695	1,731	33
34	286,398	284,773	213,849	56,552	14,372	65,159	63,635	1,524	34
35	283,143	281,506	212,877	52,967	15,662	62,593	61,264	1,329	35
36	279,864	278,216	211,447	49,773	16,996	59,665	58,516	1,149	36
37	276,563	274,903	209,608	46,922	18,373	56,310	55,326	984	37
38	273,237	271,565	207,405	44,370	19,790	52,494	51,636	858	38
39	269,887	268,202	204,875	42,083	21,244	48,129	47,386	743	39
40	266,511	264,813	202,056	40,026	22,731	43,172	42,529	643	40
41	263,109	261,397	198,974	38,174	24,249	37,578	37,021	557	41
42	259,678	257,952	195,657	36,500	25,795	31,318	30,830	488	42
43	256,219	254,478	192,125	34,984	27,369	25,846	25,445	401	43
44	252,729	250,972	188,400	33,605	28,967	20,594	20,276	318	44
45	249,207	247,434	184,495	32,350	30,589	15,701	15,459	242	45
46	245,652	243,861	180,424	31,201	32,236	11,294	11,120	174	46
47	242,061	240,252	176,198	30,147	33,907	7,483	7,369	114	47
48	238,434	236,606	171,826	29,176	35,604	4,844	4,773	71	48
49	234,769	232,922	167,315	28,277	37,330	2,845	2,806	39	49
50	231,064	229,196	162,668	27,448	39,085	1,450	1,431	19	50
51	227,318	225,430	157,891	26,663	40,876	620	611	9	51
52	223,530	221,620	152,984	25,930	42,706	304	301	3	52
53	219,698	217,766	147,949	25,237	44,580	92	92	..	53
54	215,822	213,723	142,689	24,562	46,472	13	13	..	54
55	211,576	209,380	137,136	23,888	48,356	..	..	..	55
56	207,137	204,847	131,373	23,218	50,256	..	..	..	56
57	202,509	200,124	125,405	22,549	52,170	..	..	..	57
58	197,692	195,211	119,245	21,871	54,095	..	..	..	58
59	192,683	190,105	112,900	21,179	56,026	..	..	..	59
60	187,477	184,798	106,381	20,464	57,953	..	..	..	60
61	182,068	179,285	99,705	19,718	59,862	..	..	..	61
62	176,449	173,559	92,889	18,985	61,735	..	..	..	62
63	170,614	167,614	86,007	18,117	63,490	..	..	..	63
64	164,557	161,444	79,302	17,307	64,835	..	..	..	64

ENGLISH Life and Birth Table (continued).

Age. y	Females living at age y by the English life table out of 1,000,000 children born alive, of whom 488,255 are girls.					Children born alive annually, in and out of wedlock, to women living at age y (*soboles*).			Age. y
	Females living at each age. l_y	Female population in each year of age. $P_y = l_{y+\frac{1}{2}}$	Classes of the population.			Legitimate and illegitimate. s_y	Legitimate. s'_y	Illegitimate. s''_y	
			Wives (*uxores*). u_y	Spinsters (*filiæ*). f_y	Widows (*viduæ*). v_y				
65	158,275	155,049	72,797	16,502	65,750	..	..	..	65
66	151,766	148,428	66,510	15,700	66,218	..	..	..	66
67	145,035	141,588	60,457	14,900	66,231	..	..	..	67
68	138,088	134,538	54,654	14,097	65,787	..	..	..	68
69	130,939	127,295	49,112	13,293	64,890	..	..	..	69
70	123,607	119,881	43,845	12,485	63,551	..	..	..	70
71	116,118	112,325	38,865	11,672	61,788	..	..	..	71
72	108,505	104,664	34,183	10,854	59,627	..	..	..	72
73	100,807	96,941	29,814	10,015	57,112	..	..	..	73
74	93,071	89,204	25,753	9,183	54,268	..	..	..	74
75	85,347	81,508	22,009	8,362	51,137	..	..	..	75
76	77,694	73,913	18,591	7,553	47,769	..	..	..	76
77	70,173	66,480	15,507	6,763	44,210	..	..	..	77
78	62,844	59,273	12,757	5,995	40,521	..	..	..	78
79	55,773	52,352	10,339	5,258	36,755	..	..	..	79
80	49,018	45,777	8,245	4,555	32,977	..	..	..	80
81	42,636	39,601	6,463	3,892	29,246	..	..	..	81
82	36,677	33,869	4,972	3,278	25,619	..	..	..	82
83	31,181	28,616	3,740	2,716	22,160	..	..	..	83
84	26,178	23,868	2,784	2,236	18,848	..	..	..	84
85	21,688	19,637	2,050	1,825	15,762	..	..	..	85
86	17,716	15,924	1,491	1,477	12,956	..	..	..	86
87	14,258	12,717	1,071	1,183	10,463	..	..	..	87
88	11,296	9,992	758	936	8,298	..	..	..	88
89	8,802	7,719	529	730	6,460	..	..	..	89
90	6,739	5,857	363	561	4,933	..	..	..	90
91	5,066	4,361	245	423	3,693	..	..	..	91
92	3,735	3,183	162	313	2,708	..	..	..	92
93	2,698	2,275	105	226	1,944	..	..	..	93
94	1,908	1,592	69	160	1,363	..	..	..	94
95	1,320	1,088	45	110	933	..	..	..	95
96	892	726	29	73	624	..	..	..	96
97	588	473	19	47	407	..	..	..	97
98	378	300	13	29	258	..	..	..	98
99	236	185	7	18	160	..	..	..	99
100	144	111	5	10	96	..	..	..	100
101	85	65	3	6	56	..	..	..	101
102	49	37	2	3	32	..	..	..	102
103	27	20	1	2	17	..	..	..	103
104	15	11	..	1	10	..	..	..	104
105	8	5	..	1	4	..	..	..	105
106	4	3	..	..	3	..	..	..	106
107	2	1	..	..	1	..	..	..	107
108	1	1	..	..	1	..	..	..	108
109	..	..	..	..	..	..	..	..	109

ENGLISH Life and Birth Table (continued).

Age. y	Total female population at and above each age y. Q_y	Of the total female population at and above each age y, the number of			Children born annually to women of the age y and upwards.			Age. y
		Wives (*uxores*). U_y	Spinsters (*filiæ*). F_y	Widows (*viduæ*). V_y	Legitimate and illegitimate. S_y	Legitimate. S'_y	Illegitimate. S''_y	
15	14,936,770	7,957,456	4,660,311	2,319,003	1,450,910	1,395,529	55,381	15
16	14,597,426	7,957,228	4,321,207	2,318,991	1,450,789	1,395,442	55,347	16
17	14,260,036	7,956,472	3,984,599	2,318,965	1,450,328	1,395,149	55,179	17
18	13,924,759	7,952,751	3,653,087	2,318,921	1,448,355	1,393,687	54,668	18
19	13,591,782	7,938,282	3,334,697	2,318,803	1,441,179	1,387,941	53,238	19
20	13,261,309	7,905,185	3,037,586	2,318,538	1,425,700	1,374,713	50,987	20
21	12,933,570	7,839,188	2,776,552	2,317,830	1,396,454	1,348,268	48,186	21
22	12,608,675	7,747,780	2,544,219	2,316,676	1,356,646	1,311,684	44,962	22
23	12,286,670	7,637,268	2,334,400	2,315,002	1,309,068	1,267,663	41,405	23
24	11,967,597	7,506,536	2,148,405	2,312,656	1,253,713	1,216,011	37,702	24
25	11,651,495	7,356,916	1,985,067	2,309,512	1,191,573	1,157,575	33,998	25
26	11,338,400	7,190,612	1,842,327	2,305,461	1,123,978	1,093,581	30,397	26
27	11,028,344	7,010,300	1,717,630	2,300,414	1,052,418	1,025,448	26,970	27
28	10,721,359	6,818,772	1,608,282	2,294,305	978,375	954,617	23,758	28
29	10,417,472	6,618,677	1,511,712	2,287,083	903,709	882,929	20,780	29
30	10,116,710	6,412,379	1,425,618	2,278,713	829,338	811,292	18,046	30
31	9,819,099	6,201,897	1,348,035	2,269,167	756,024	740,461	15,563	31
32	9,524,662	5,988,898	1,277,335	2,258,429	684,383	671,042	13,341	32
33	9,233,424	5,774,722	1,212,214	2,246,488	614,930	603,534	11,396	33
34	8,945,407	5,560,419	1,151,631	2,233,357	547,504	537,839	9,665	34
35	8,660,634	5,346,570	1,095,079	2,218,985	482,345	474,204	8,141	35
36	8,379,128	5,133,693	1,042,112	2,203,323	419,752	412,940	6,812	36
37	8,100,912	4,922,246	992,339	2,186,327	360,087	354,424	5,663	37
38	7,826,009	4,712,638	945,417	2,167,954	303,777	299,098	4,679	38
39	7,554,444	4,505,233	901,047	2,148,164	251,283	247,462	3,821	39
40	7,286,242	4,300,358	858,964	2,126,920	203,154	200,076	3,078	40
41	7,021,429	4,098,302	818,938	2,104,189	159,982	157,547	2,435	41
42	6,760,032	3,899,328	780,764	2,079,940	122,404	120,526	1,878	42
43	6,502,080	3,703,671	744,264	2,054,145	91,086	89,696	1,390	43
44	6,247,602	3,511,546	709,280	2,026,776	65,240	64,251	989	44
45	5,996,630	3,323,146	675,675	1,997,809	44,646	43,975	671	45
46	5,749,196	3,138,651	643,325	1,967,220	28,945	28,516	429	46
47	5,505,335	2,958,227	612,124	1,934,984	17,651	17,396	255	47
48	5,265,083	2,782,029	581,977	1,901,077	10,168	10,027	141	48
49	5,028,477	2,610,203	552,801	1,865,473	5,324	5,254	70	49
50	4,795,555	2,442,888	524,524	1,828,143	2,479	2,448	31	50
51	4,566,359	2,280,220	497,081	1,789,058	1,029	1,017	12	51
52	4,340,929	2,122,329	470,418	1,748,182	409	406	3	52
53	4,119,309	1,969,345	444,488	1,705,476	105	105	..	53
54	3,901,543	1,821,396	419,251	1,660,896	13	13	..	54
55	3,687,820	1,678,707	394,689	1,614,424	..	..	..	55
56	3,478,440	1,541,571	370,801	1,566,068	..	..	..	56
57	3,273,593	1,410,198	347,583	1,515,812	..	..	..	57
58	3,073,469	1,284,793	325,034	1,463,642	..	..	..	58
59	2,878,258	1,165,548	303,163	1,409,547	..	..	..	59
60	2,688,153	1,052,648	281,984	1,353,521	..	..	..	60
61	2,503,355	946,267	261,520	1,295,568	..	..	..	61
62	2,324,070	846,562	241,802	1,235,706	..	..	..	62
63	2,150,511	753,673	222,867	1,173,971	..	..	..	63
64	1,982,897	667,666	204,750	1,110,481	..	..	..	64

ENGLISH Life and Birth Table (continued).

Age. y	Total female population at and above each age y. Q_y	Of the total female population at and above each age y, the number of			Children born annually to women of the age y and upwards.			Age. y
		Wives (*uxores*). U_y	Spinsters (*filiæ*). F_y	Widows (*viduæ*). V_y	Legitimate and illegitimate. S_y	Legitimate. S'_y	Illegitimate. S''_y	
65	1,821,453	588,364	187,443	1,045,646	..	..	..	65
66	1,666,404	515,567	170,941	979,896	..	..	..	66
67	1,517,976	449,057	155,241	913,678	..	..	..	67
68	1,376,388	388,600	140,341	847,447	..	..	..	68
69	1,241,850	333,946	126,244	781,660	..	..	..	69
70	1,114,555	284,834	112,951	716,770	..	..	..	70
71	994,674	240,989	100,466	653,219	..	..	..	71
72	882,349	202,124	88,794	591,431	..	..	..	72
73	777,685	167,941	77,940	531,804	..	..	..	73
74	680,744	138,127	67,925	474,692	..	..	..	74
75	591,540	112,374	58,742	420,424	..	..	..	75
76	510,032	90,365	50,380	369,287	..	..	..	76
77	436,119	71,774	42,827	321,518	..	..	..	77
78	369,639	56,267	36,064	277,308	..	..	..	78
79	310,366	43,510	30,069	236,787	..	..	..	79
80	258,014	33,171	24,811	200,032	..	..	..	80
81	212,237	24,926	20,256	167,055	..	..	..	81
82	172,636	18,463	16,364	137,809	..	..	..	82
83	138,767	13,491	13,086	112,190	..	..	..	83
84	110,151	9,751	10,370	90,030	..	..	..	84
85	86,283	6,967	8,134	71,182	..	..	..	85
86	66,646	4,917	6,309	55,420	..	..	..	86
87	50,722	3,426	4,832	42,464	..	..	..	87
88	38,005	2,355	3,649	32,001	..	..	..	88
89	28,013	1,597	2,713	23,703	..	..	..	89
90	20,294	1,068	1,983	17,243	..	..	..	90
91	14,437	705	1,422	12,310	..	..	..	91
92	10,076	460	999	8,617	..	..	..	92
93	6,893	298	686	5,909	..	..	..	93
94	4,618	193	460	3,965	..	..	..	94
95	3,026	124	300	2,602	..	..	..	95
96	1,938	79	190	1,669	..	..	..	96
97	1,212	50	117	1,045	..	..	..	97
98	739	31	70	638	..	..	..	98
99	439	18	41	380	..	..	..	99
100	254	11	23	220	..	..	..	100
101	143	6	13	124	..	..	..	101
102	78	3	7	68	..	..	..	102
103	41	1	4	36	..	..	..	103
104	21	..	2	19	..	..	..	104
105	10	..	1	9	..	..	..	105
106	5	..	..	5	..	..	..	106
107	2	..	..	2	..	..	..	107
108	1	..	..	1	..	..	..	108
109	..	..	..	..	..	..	..	109

X. *Agricultural, Botanical, and Chemical Results of Experiments on the Mixed Herbage of Permanent Meadow, conducted for more than Twenty Years in succession on the Same Land.*—Part I.

By J. B. Lawes, *LL.D., F.R.S., F.C.S.,*
and J. H. Gilbert, *Ph.D., F.R.S., F.C.S., F.L.S.*

Received June 16,—Read June 19, 1879.

INTRODUCTION.

In the experiments at Rothamsted with different manures, wheat has now been grown for 36 years in succession on the same land, barley for 28 years, and oats for nine years. Somewhat in like manner, but with some breaks, beans have been grown over a period of more than 30 years, clover for many years, and "root crops" (turnips, sugar-beet, or mangel-wurzel) also for more than 30 years. Each of these individual crops has exhibited certain distinctive characters under this unusual treatment. But, withal, those of the same natural family—wheat, barley, and oats, for example—have shown certain characters in common; those of the Leguminous family characters widely different; whilst the so-called root-crops, belonging to the Cruciferous and Chenopodiaceous families, have exhibited characteristics differing from those of either the Gramineæ or the Leguminosæ.

Compared with the conditions of growth of any one of these individual crops grown separately, those of the mixed herbage of grass land are obviously extremely complicated. Thus, it comprises, besides numerous genera and species of the gramineous and leguminous families, representatives also of many other natural orders, and of some of great prominence and importance as regards their prevalence and distribution in vegetation generally. And if, under the influence of characteristically different manuring agents, as has been the case, there have been observed notable differences in the degree of luxuriance of growth, and in the character of development, even between closely allied plants when each is grown separately, and much greater differences between the representatives of different families when so separately grown, might we not expect very remarkable variations of result when different manures are applied to an already established mixed herbage of perhaps some 50 species growing together, representing nearly as many genera, and more than 20 natural orders?

Such—far beyond what could have been anticipated—has been the case in the experiments to be described. So complicated, indeed, have been the manifestations of the "struggle" that has been set up, that even after more than 20 years of laborious experiment, both in the field and in the laboratory, and following up both the botany and the chemistry of the subject, we can hardly claim to have yet done much more than reach the threshold of a very comprehensive enquiry. Still, we hope to establish some points of general interest, and possibly to indicate promising paths of future research.

From the title of our paper, it will at once be concluded that the experiments were originally undertaken and arranged from an agricultural point of view. But, as experimenting on the feeding of animals soon led us into lines of enquiry of even more interest to the chemist, the animal physiologist, and the dietetician than to the agriculturist, so the investigation of the effects of different manures on the mixed herbage of grass land has led us far beyond the limits of a purely agricultural problem, and has afforded results of more interest to the botanist, the vegetable physiologist, and the chemist, than to the farmer. Indeed, agriculture—the most primitive, and commonly esteemed the rudest of the arts—requires for the elucidation of the principles involved in its various practices a very wide range of scientific enquiry; and the investigation of them may, in its turn, contribute facts of interest to the student of various and very distinct branches of natural knowledge.

It will be readily understood that, as a necessary foundation for the discussion of the botany and the chemistry of the subject, it will be essential first to put on record, and call attention to, what may be distinguished as the agricultural data. It is proposed, then, to arrange and consider the results obtained under the following heads:—

Part I.—The Agricultural Results.
Part II.—The Botanical Results.
Part III.—The Chemical Results.

It will facilitate the understanding of the very voluminous and very various experimental details which are to follow, if the scope of the enquiry, and the general character of the results, are somewhat further, but still only briefly, indicated in this place.

About seven acres in the Park at Rothamsted have been set apart for the experiments, and divided into plots. Two of these have been left without manure from the commencement; two have received ordinary farmyard manure; whilst the remainder have each received a different description of artificial or chemical manure, the same being, except in special cases, applied year after year on the same plot.

Referring first to the *agricultural results*, it may be premised that, *without manure*, the produce of hay has varied from year to year, according to season, from about 8 cwts. to nearly 39 cwts. per acre, and the average yield has been about 23 cwts. per acre per annum. On the other hand, the plot the most heavily artificially manured, and yielding the highest amount of produce, has given an average of about 64 cwts. of hay per acre per annum, with a variation from year to year from under 40 cwts. to nearly

80 cwts. Intermediate between the extremes here quoted, very great variation in the amount of produce has been exhibited on the other differently manured plots.

With these great differences in the amounts of produce the botanical character of the herbage has varied most strikingly. Thus, starting with perhaps 50 species on the unmanured land, any kind of manure—that is, anything that increases the growth of any species—induces a struggle, greater or less in degree, causing a greater or less diminution, or a disappearance, of some other species, until, on some plots, and in some seasons, less than 20 species have been observable; and on some, after a number of years, no more than this are ever traceable.

Even in the first years of the experiments, it was noticed that those manures which are the most effective with wheat, barley, or oats, grown on arable land—that is, with gramineous species grown separately—were also the most effective in bringing forward the grasses proper in the mixed herbage; and, again, those manures which were the most beneficial to beans or clover most developed the leguminous species of the mixed herbage, and *vice versâ*. It was further observed, that there was great variation in the predominance of individual species among the grasses, and also among the representatives of other orders. And even in the second year the differences in the flora, so to speak, were so marked that a first attempt at a botanical analysis of carefully taken samples of the produce of some of the plots was then made. In the third year more detailed separations were made; and, taking advantage of the experience thus gained, pretty complete botanical analyses have since been conducted four times, at intervals of five years, during the course of the now 24 years of the experiments; and on several other occasions partial separations have been undertaken. The character and tendency of the results so obtained may be very briefly indicated as follows :—

In the produce grown continuously without manure the average number of species found has been 49. Of these, 17 are grasses, 4 belong to the order Leguminosæ, and 28 to other orders. The percentage, by weight, of the grasses has averaged about 68, that of the Leguminosæ about 9, and that of species of other orders 23.

In the produce of the plot already referred to as the most heavily manured, and yielding the heaviest crops, the average number of species found has been only 19, of which 12–13 are grasses, 1 only (or none) leguminous, and 5–6 only represent other orders; whilst the average proportions by weight have been—of grasses about 95 per cent., of Leguminosæ less than 0·01 per cent., and of species representing other orders less than 5 per cent.

On the other hand, a plot receiving annually manures such as are of little avail for gramineous crops grown separately in rotation, but which favour beans or clover so grown, has given, on the average, 43 species. Of these, 17 in number are grasses, 4 Leguminosæ, and 22 belong to other orders; but, by weight, the percentage of grasses has averaged only 65–70, that of the Leguminosæ nearly 20, and that of species belonging to other orders less than 15.

With such very great variations, not only in the amount, but in the botanical character, of the produce of a crop under any circumstances so complex, it might be anticipated that there would be very great differences in its chemical composition, partly due directly to the supply of constituents by manure, partly to variation in the description of plants encouraged, and partly to the character and degree of development, and ripeness, of the varying components of the mixed herbage, according to the season and to the manure employed.

With a view to the elucidation of this part of the subject, the dry matter and the ash have been determined in the produce of every plot in every season; the nitrogen in that of all the plots for many of the seasons, and in some cases the amount of it existing as albuminoids has been determined. In selected cases, also, comparative determinations of "crude woody fibre" and of crude fatty matter have been made. About 120 complete ash-analyses have been executed. And, lastly, samples of the soil of every plot—in some cases at different periods, and in most cases representing the first, second, third, fourth, fifth, and sixth depths of nine inches, or, in all, to a total depth of 54 inches—have been collected, and these have been chemically examined in various ways.

It is found that there is a considerable difference in the percentage of dry substance in the produce, and very considerable difference in the percentage of mineral matter (ash) in that dry substance. There is still greater difference in the percentage of nitrogen in the dry matter, and, again, a greater difference still in the percentage of individual constituents of the ash. When, indeed, it is remembered that a plot may have from 20 to 50 different species growing upon it, each with its own peculiar habit of growth, and consequent varying range and power of food-collection, it will not appear surprising that different species are developed according to the manure employed; and, this being so, that the character and amount of the constituents taken up from the soil by such a *mixed herbage* should be found much more directly dependent on the supplies of them by manure than is the case with a crop of a single species growing separately.

In further illustration it may be mentioned that, not only does the percentage of nitrogen in the dry substance of the produce of the different plots vary considerably, but the average annual amount of it assimilated over a given area is more than three times as much in some cases as in others. Again, the percentage of potass in the dry substance is three times as much in some cases as in others; whilst the difference in the average annual amount of it taken up over a given area is more than five times as much on some plots as on others—dependent on the supplies of it by manure, and the consequent description of plants, and amount, and character, of growth induced. The percentage and acreage amounts of phosphoric acid also vary very strikingly; and so again it is with other mineral constituents, but in a less marked degree.

The foregoing summary statements will be sufficient to indicate the general scope, objects, and results, of the inquiry.

It will be seen that in the history of so many of what may be called natural rotations, we can hardly fail to learn not only much that is of interest in reference to the growth of the mixed herbage of permanent grass land, but also something of the relative positions of the different plants that are grown separately, in alternation with one another, in the artificial rotations adopted on arable land.

The botanical results are, moreover, of much independent interest, both by the facts which they already contribute, and by the incentive and direction they may give to future research.

Lastly, the chemistry of the subject will be found to offer many points of interest, in regard—to the variation in the percentage composition of the produce according to the manure applied, to the description of plants developed, and to the character of their development; to the availableness of the constituents artificially supplied, and to the amount and limit of the natural resources of the soil, both actually and compared with the results obtained when individual species are grown in arable culture.

It will be readily understood that the record, and the discussion, of the agricultural, the botanical, and the chemical history of about 20 plots, in 20 different seasons, must involve much detail; and although it is obvious that facts special to anyone of the three main divisions of the subject may require for their elucidation reference to those of one or both of the others, it is still believed that it will conduce to clearness, and reduce unavoidable repetition, to maintain the divisions proposed as far as possible. It may further be explained that, in order to simplify the discussion, and as far as is consistent with clearness to relieve it of embarrassing details, the whole of the numerical results are systematically arranged in tables given in the Appendix, to which reference will be made, and only such quotations or summaries will be embodied with the text as are necessary for illustration.

PART I.—THE AGRICULTURAL RESULTS.

In entering upon the discussion of the so-designated *agricultural* results, it is necessary to premise that almost from the commencement of the experiments it became apparent that more would be learnt, even of purely agricultural value, by continuing them in such manner as to obtain data bearing upon the important questions of the annual assimilation of constituents, the resources and the exhaustion of the soil, and the description of plants, and general characters of the herbage developed, under different conditions of season and of manuring, than by simply adapting the manures to obtain, as a direct result, produce which, considered either in regard to quantity, to quality, or to both, would be, in an economical sense, the most valuable. In fact, in some of the experiments conditions have been maintained which, though yielding very large amounts of produce, have done so not only at a great sacrifice of the quality of the hay, but at an entirely unremunerative cost; whilst, on the other hand, in some cases very high quality has been obtained, but again at far too high a pecuniary cost. In other words, the design of the arrangements has not been to secure results which should serve as direct models for practical agriculture, but such as should provide the data both for interesting scientific generalisation, and for useful practical deduction.

Accordingly, the present section of our report will be devoted mainly to a description of the influence of different manures, and of different seasons, on the amounts of produce obtained—that is, on the activity of accumulation and growth—with only such general references to the botanical, the chemical, or the economic character of the herbage, as may be necessary to prevent erroneous, and lead to correct interpretations and conclusions.

The experiments were commenced in 1856, so that the season of 1875 completed the twentieth, and the present season (1879) completes the twenty-fourth year of their continuance. They have been conducted on a portion of the Park at Rothamsted, where the land has probably been in grass for some centuries. No fresh seed has been artificially sown within the last 50 years certainly, nor is there record of any previous sowing. For many years prior to 1851 the plan was to manure occasionally with farmyard dung, road scrapings, and the like; and in the later years sometimes with guano, or other purchased manures. One crop of hay was removed annually, averaging from 1¼ to 1¾ tons per acre. The second crop was always eaten off by sheep. In the spring of 1851, and again in 1852, four separate acres of the afterwards selected area were appropriated to the consumption by sheep of as many lots of differently-manured turnips, 10 tons of the roots being consumed upon each acre. Neither of these four acres was manured in any other way, nor was the remainder of the land manured at all in those two seasons (1851 and 1852), nor was any of it manured at all in either of the three next succeeding seasons—that is, in the three immediately preceding the commencement of the experiments in 1856. It may be added that the consumption of the different turnips on the land for two consecutive seasons did not in any case increase the produce over the five years, 1851–5 inclusive, by more than about 2 cwts. of hay per acre per annum. The land is a somewhat heavy loam, with red clay subsoil, resting upon chalk, and although it is not artificially, it is thus naturally, well drained. Lastly, the area selected is very level, and at the time the experiments were commenced the character of the herbage appeared fairly uniform over the whole of it.

At first, about five-and-a-half acres, divided into 13 plots, most of half, but some of a quarter of an acre each, were devoted to the purpose. In 1858 four more plots, comprising together two-thirds of an acre, were brought in; in 1865 one plot of half an acre; and in 1872 two plots of an eighth of an acre each; making, in all, about seven acres. Thus, the respective plots were commenced as follows:—

Plots 1–13 in 1856.
Plots 14–17 in 1858.
Plot 18 in 1865.
Plots 19 and 20 in 1872.

Two of the plots (3 and 12) have been left entirely unmanured from the commencement. One plot (1) received farmyard manure, with the addition of some ammonia-salts, for eight years in succession, since which time the application of the dung has been discontinued, and the ammonia-salts alone have been annually applied. One plot (2) received farmyard manure alone, each year, for eight years, and has since been left unmanured. The remaining plots have received, respectively, different descriptions or combinations of artificial or chemical manuring substances; the same, with some special exceptions, being applied year after year on the same plot. The most convenient forms in which the different constituents are supplied in commerce were selected. Thus, the different "mineral"* or ash-constituents were supplied in the substances designated in commerce as follows:—

Potass—as sulphate of potass (and in nitrate of potass).
Soda—as sulphate of soda (and in nitrate of soda).
Magnesia—as sulphate of magnesia.
Lime—as sulphate, phosphate, and "superphosphate."
Phosphoric acid—as bone ash, mixed with sulphuric acid in quantity sufficient to convert most of the insoluble phosphate of lime into sulphate and soluble "superphosphate" of lime.
Sulphuric acid—in the above phosphatic mixture, in sulphates of potass, soda, and magnesia, and in sulphate of ammonia.
Chlorine—in "muriate of ammonia."
Silica—as silicate of soda and silicate of lime (also in cut wheat straw).

Other constituents have been artificially supplied as under:—

Nitrogen—as sulphate of ammonia, as "muriate of ammonia," and as nitrate of soda.
Non-nitrogenous organic matter, yielding, by decomposition, carbonic acid and other products—in sawdust, and in cut wheat straw.

The artificial manures were mixed with ashes (burnt soil and turf) in quantity sufficient to make up a convenient measure for equal distribution over the land, and the mixtures so prepared were sown broadcast by hand, as it has been found that the application of an exact amount of manure over a limited area of land can be best accomplished in that way.

During the first 19 years, the first crop only, each year, was mown, made into hay, removed from the land, and weighed; but in the twentieth season, 1875, a second crop was removed and weighed. As a rule, the second crop was fed off by sheep having no other food, the object being not to disturb the condition of the manuring. A given number was allotted to each plot, according to the amount of produce, and penned upon a portion of it, the area being extended day by day, until the whole was eaten down. Frequently, however, the animals suffered so much, sometimes by the change of food, and sometimes by the combined influence of this and bad weather, that the plan was finally abandoned; and in the years 1866, 1870, 1873, and 1874, the second crops (and third if any) were mown, evenly spread on the respective plots, and left to decay and manure the land. The estimated amount of the second crops, and the influence of the treatment of them as above described, on the produce of the succeeding years, will be considered further on. But, in the first instance, our illustrations will chiefly have reference to the produce of hay of the first crops only.

---

* With regard to the use of the term "mineral," see 'Journal of the Royal Agricultural Society of England,' vol. xxiv., pp. 506–8 (foot-note); vol. xxv., p. 101 (and context); also vol. xvi., pp. 447–8 (and context).

A detailed description of the kinds and quantities of the manures applied to, and of the amounts of produce obtained from, each of the experimental plots, in each of the first twenty (and three subsequent) years, is given in Appendix-Table I., pp. 406–407. There is also given, in the same table, the average produce on each plot, over the first 10, the second 10, and the total period of 20 years, or over the period before, and the period after, any important change in the manures. It will, however, be convenient to give a general description of the manuring of each plot in this place.

Plot 1.—Farmyard manure, and ammonia-salts,* 8 years, 1856–63; ammonia-salts only 1864, and each year since.

Plot 2.—Farmyard manure, 8 years, 1856–63; unmanured 1864, and each year since.

Plot 3.—Unmanured every year, 1856, and since.

Plot 4-1.—Sawdust (without effect), 3 years, 1856–58; superphosphate of lime,† 1859, and each year since.

Plot 4-2.—Sawdust (without effect), 3 years, 1856–58; superphosphate of lime and ammonia-salts, 1859, and each year since.

Plot 5.—Ammonia-salts alone, every year.

Plot 6.—Ammonia-salts 13 years, 1856–68 (with sawdust 7 years, without effect); sulphates of potass, soda, and magnesia, and superphosphate of lime, 1869, and each year since.

Plot 7.—Sulphates of potass, soda, and magnesia, and superphosphate of lime, every year.

Plot 8.—Sulphates of potass, soda, and magnesia, and superphosphate of lime (as plot 7), 6 years, 1856–61; sulphates of soda and magnesia (without potass), and superphosphate of lime, 1862, and each year since; also sawdust the first 7 years, but without effect.

Plot 9.—Sulphates of potass, soda, and magnesia, and superphosphate of lime (as plot 7), and ammonia-salts, every year.

---

* "Ammonia-salts," in all cases equal parts, sulphate and muriate of ammonia of commerce.

† "Superphosphate of lime," always composed as under, per acre: 200 lbs. bone-ash, 150 lbs. sulphuric acid (sp. gr. 1·7), and water.

Plot 10.—Sulphates of potass, soda, and magnesia, and superphosphate of lime (as plots 7 and 8), and ammonia-salts, 6 years, 1856–61; sulphates of soda and magnesia (without potass) and superphosphate of lime (as plot 8), and ammonia-salts, 1862, and each year since; also sawdust the first 7 years, but without effect.

Plot 11–1.—Sulphates of potass, soda, and magnesia, and superphosphate of lime (as plots 7 and 9), every year; twice as much ammonia-salts as plot 9, 3 years, 1856–58; same ammonia-salts as plot 9, 3 years, 1859–61; double quantity again, 1862, and each year since.

Plot 11–2.—The same as plot 11–1, with the addition of a mixture of silicate of lime and silicate of soda, 9 years, 1862–1870, and of silicate of soda only, 1871, and each year since.

Plot 12.—Unmanured, every year, 1856, and since.

Plot 13.—Sulphates of potass, soda, and magnesia, superphosphate of lime, and ammonia-salts (as plot 9), and cut wheat straw, every year.

Plot 14.—Sulphates of potass, soda, and magnesia, and superphosphate of lime (as plots 7, 9, 11, and 13), and nitrate of soda containing nitrogen equal that in the ammonia-salts of plots 5, 9, and 13, 1858 and each year since.

Plot 15.—Nitrate of soda alone, same quantity at plot 14, 18 years, 1858–75; no nitrate, but sulphates of potass, soda, and magnesia, and superphosphate of lime (as plots 7, 9, 11, 13, and 14), 1876, and each year since.

Plot 16.—Sulphates of potass, soda, and magnesia, and superphosphate of lime (as plots 7, 9, 11, 13, and 14), and half as much nitrate of soda as plot 14, 1858, and each year since.

Plot 17.—Nitrate of soda alone, same quantity as plot 16, 1858, and each year since.

Plot 18.—Mixture containing the potass, soda, lime, magnesia, phosphoric acid, silica, and nitrogen, of 1 ton of hay, also sulphuric acid and chlorine, 1865, and each year since.

Plot 19.—Nitrate of soda (same quantity as plots 16 and 17), sulphate of potass, containing the same quantity of potass as the nitrate of potass of plot 20, and superphosphate of lime, 1872, and each year since.

Plot 20.—Nitrate of potass, containing the same quantity of nitrogen, and the same quantity of potass, as the nitrate of soda and sulphate of potass of plot 19, and superphosphate of lime, 1872, and each year since.

It will, perhaps, conduce to clearness of conception of the characteristic actions of the different descriptions of manure, if the results obtained are, in the first instance, discussed as far as possible without reference to the influence of the various seasons. Accordingly, it is proposed to consider the average amounts of produce yielded by characteristically different descriptions of manure, over the first 10, the second 10, and the total period of 20 years, of the continuous application to the same plot; and, in the special cases in which the manure employed has been changed during the course of the experiments, to direct attention to the average produce obtained before and after such change.

As a preliminary to such a consideration of the effects of the different manures, it will be useful to call attention to the amounts of some of the most important constituents removed from a given area of land in a fair average hay-crop, compared with the amounts of the same removed in some other crops. For the purposes of such a comparison, wheat and barley may be selected, and we may assume the following amounts of produce per acre:—

Wheat, 30 bushels=1800 lbs., and 3000 lbs. straw=4800 lbs. total produce.

Barley, 40 bushels=2080 lbs., and 2500 lbs. straw=4580 lbs. total produce.

Meadow hay, 1½ ton, or 3360 lbs.

The following table shows the average amounts of nitrogen, and of most of the mineral constituents, in the above quantities of wheat grain and straw, and barley grain and straw. As already indicated, and as will be very fully illustrated further on, meadow hay may vary so extremely in its botanical, and coincidently in its chemical composition, according to soil, climate, and manuring, that it is not possible to adopt an average composition for such produce with as much confidence as for either wheat or barley. This is well illustrated in the figures given in the four columns of the table relating to meadow hay. Assuming in each case the same weight, 1½ ton, of hay, we have the amounts of the several constituents calculated as follows:—In the first of the four columns according to the average composition, as determined by actual analysis, of the hay grown for eight years successively by farm yard manure; in the second column according to the average composition of the hay grown during the same eight seasons without manure; in the third column is given the mean of the two; and in the fourth are given the amounts according to the average percentage of nitrogen adopted by E. Wolff, and the average composition of the ash adopted by him, founded on the results of 39 analyses by various experimenters.

TABLE I.—Composition of average crops of Wheat, Barley, and Meadow Hay.

	In corn.		In straw.		In total produce.		1½ ton = 3360 lbs. meadow hay.			
	Wheat. (1800 lbs.)	Barley. (2080 lbs.)	Wheat. (3000 lbs.)	Barley. (2500 lbs.)	Wheat. (4800 lbs.)	Barley. (4500 lbs.)	8 years farmyard manure.	8 years without manure.	Mean.	Average E. Wolff.
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Nitrogen. . .	32·0	33·0	13·0	12·0	45·0	45·0	40·8	47·2	44·0	52·0
Lime	1·0	1·5	7·5	9·0	8·5	10·5	27·1	35·2	31·2	33·8
Magnesia . .	3·5	4·0	3·0	2·5	6·5	6·5	7·4	9·4	8·4	12·8
Potass . . .	9·5	11·0	22·0	23·0	31·5	34·0	72·8	40·5	56·6	51·7
Soda	0·2	0·5	1·0	3·0	1·2	3·5	7·9	16·0	11·9	9·0
Phosphoric acid	15·0	16·5	6·0	4·5	21·0	21·0	16·4	11·0	13·7	16·2
Sulphuric acid .	0·2	0·75	4·0	4·0	4·2	4·75	11·3	13·6	12·5	9·2
Chlorine . . .	0·0	0·2	3·0	4·0	3·0	4·2	21·3	12·3	16·8	14·6
Silica	0·5	11·0	95·0	55·0	95·5	66·0	59·9	51·8	55·9	54·6

As, in the ordinary practice of rotation on arable land, most of the constituents of the straw of wheat and barley, which are in the first instance removed from it, are in due course returned to it, the eventual loss of constituents to the land by the growth of such crops will be more nearly represented by the amounts in the grain alone, than by those in the total produce, corn and straw together. In the case of the hay crop the return of constituents is by no means so regular, and the figures show how very variable may be the amount of them in a given weight of the crop, according to the supply of them in the soil, or by manure. Thus, whilst there is more nitrogen in a given weight of the hay grown without manure than with farmyard manure, there is one-and-a-half time as much phosphoric acid, and more than one-and-three-quarters time as much potass, in the hay grown by farmyard manure than in that without manure. On the other hand, there is more lime, more magnesia, much more soda, and more sulphuric acid, but less silica, and much less chlorine, in a given weight of the unmanured than of the manured produce. In spite of these variations the mean of the two gives a composition agreeing very fairly in essential points with the average deduced by E. WOLFF from the results of 39 ash-analyses by different experimenters.

Assuming as the basis of comparison the mean composition of the manured and the unmanured hay, it is seen that a fairly good crop will remove about one-third more nitrogen than the grain of a fairly good crop of wheat or barley, and practically the same amount as the total produce, grain and straw together, of either of the corn crops.

Of phosphoric acid, the hay crop will remove somewhat less than the grain alone, and only about two-thirds as much as the total produce of wheat or barley.

Of potass, the assumed average hay crop will remove five or six times as much as the grain of either the wheat or the barley, and nearly twice as much as the total produce, corn and straw together.

Of lime, soda, sulphuric acid, chlorine, and silica, the hay will remove many times more, and of magnesia much more, than either the wheat or the barley grain. Of lime, soda, sulphuric acid, and chlorine, the hay will also remove much more, and of magnesia more, than both corn and straw together. Of phosphoric acid and silica alone, will the total produce of the corn crops remove more than the hay crop.

To sum up the most important points: it is quite obvious that, in the soil in question, which it should be remembered is a loam with a clay subsoil, the effect of the application of a complex manure such as dung, supplying as it doubtless does much more of all the mineral constituents than the crop takes up, is in a striking degree to increase the assimilation of potass; notably also that of phosphoric acid, and to some degree that of silica; much more chlorine was also taken up. In fact, as will be shown further on, the supply by manure of potass has a more marked effect on the quantity, and on the botanical and the chemical character, of the herbage of the hay crop, than that of any other of the mineral, or ash-constituents. If these few illustrations relating to the composition of the crop be borne in mind, the results relating to the effects of different manures upon it will be the more easily understood.

The next point to be considered is the effects of certain characteristically different conditions as to manuring, each, respectively, continued for many years in succession on the same plot.

1. *Without Manure; Plots* 3 *and* 12.

The following table shows the average produce of hay obtained, per acre per annum, without any manure, over the first 10 years, the second 10 years, and the total period of 20 years, from 1856 to 1875 inclusive; also the amounts of nitrogen, and mineral matter (ash), which the hay contained.

TABLE II.—Average, per acre per annum, without Manure.

	Average per acre per annum.		
	Plot 3.	Plot 12.	Mean.
HAY.			
	lbs.	lbs.	lbs.
First period, 10 years, 1856–1865	2531	2808	2670
Second period, 10 years, 1866–1875	2236	2564	2400
Total period, 20 years, 1856–1875	2383	2686	2535
Second period, per cent. + or – first period . . .	–11·7	–8·7	–10·1
NITROGEN.			
First period, 10 years, 1856–1865	35·1	39·3	37·2
Second period, 10 years, 1866–1875	30·9	35·6	33·3
Total period, 20 years, 1856–1875	33·0	37·5	35·3
Second period, per cent. + or – first period . . .	–12·0	–9·4	–10·5
MINERAL MATTER (ASH).			
First period, 10 years, 1856–1865	148·5	161·7	155·1
Second period, 10 years, 1866–1875	126·1	143·2	134·6
Total period, 20 years, 1856–1875	137·3	152·4	144·9
Second period, per cent. + or – first period . . .	–15·1	–11·4	–13·2

It will be seen that, over both periods, one of the unmanured plots (12) yielded more produce of hay than the other (3). It also yielded more nitrogen, and more mineral matter, whilst, in all three particulars—hay, nitrogen, and mineral matter—its decline in yield over the second period compared with the first was less. For a long time the proper explanation of this superiority was not very obvious; but it was supposed to be due, in part at least, to the fact that plot 12 is earlier sheltered from the evening sun, and therefore the less liable to suffer in dry weather. After the experiments had been continued for 20 years, however, the soil of each of the plots was sampled, in three places, in each case six depths of nine inches, or in all a total depth of 54 inches, being taken; and both the notes made on the sections of the soil and subsoil, and the percentages of nitrogen at the respective depths (determined by the soda lime method), lead to the conclusion that a considerable part, at any rate, of plot 12 must have been made ground, as the mould extended to a greater depth, and, as will be seen further on, the percentage of nitrogen in the lower layers was considerably higher than in the case of the other unmanured plot, and indeed higher than in that of most of the manured plots. After a careful consideration of the facts, it has been decided that the results obtained on plot 3 provide fairer standards with which to compare those yielded on the manured plots than would the mean indications of the two unmanured plots. Accordingly, all such comparisons, unless otherwise stated, have reference to the produce of the unmanured plot 3.

The average annual decline in yield on plot 3, over the second 10 years compared with the first, amounted to nearly 300 lbs. of hay, to rather more than 4 lbs. of nitrogen, and to about 22½ lbs. of total mineral matter. The percentage decline over the second period was slightly greater in the nitrogen than in the hay, and greater in the mineral matter than in either. The analytical results will show that the deficiency was more or less in all of the mineral constituents; but it was the more marked in the cases of the potass and the soda, the phosphoric and the sulphuric acids, and especially of the silica.

It may be concluded that, apart from the influences of season, which were not without effect on the result, the reduction of produce on the continuous growth without manure was due, in part to deficiency of available nitrogen, but probably more still to that of an available supply of potass, of phosphoric acid, and of silica.

Under these conditions of general want of luxuriance—of no marked predominance of either nitrogen or any special mineral constituent, favouring any particular species or families of plants—no artificial struggle was set up. The result was a more complex herbage than on any of the manured plots. In other words, a greater number of species maintained a place. The proportion of the produce consisting of gramineous herbage was comparatively low; that of the leguminous was fairly high; but the most marked characteristic was the large number, and high proportion by weight, of the miscellaneous species.

Under the influence of such great variety in the character and habit of growth of the plants composing the unmanured mixed herbage, accompanied it may be supposed with correspondingly varied powers of food-collection, we have a considerably greater annual assimilation of nitrogen, and of some of the most important mineral constituents, over a given area, than in an unmanured gramineous crop grown separately, on arable land, such as wheat or barley for example. The characteristic differences in the percentage composition of the different kinds of herbage, and in the amounts of constituents they respectively take up, will be further illustrated as we proceed.

2. *Ammonia-Salts alone; Plot* 5.

Over the same periods of 10 years, 10 years, and 20 years, the following results were obtained by the annual application of ammonia-salts alone, at the rate of 400 lbs. per acre per annum, estimated to supply about 100 lbs. of ammonia, corresponding to about 82 lbs. of nitrogen; and, for comparison, the results obtained on plot 3, without manure, are also given.

TABLE III.—Average, per acre per annum, by 400 lbs. Ammonia-Salts alone; Plot 5.

	Average per acre per annum.		
	Plot 3. Without manure.	Plot 5. Ammonia-salts alone.	Plot 5. + or − plot 3.
HAY.			
	lbs.	lbs.	lbs.
First period, 10 years, 1856–1865	2531	3420	+889
Second period, 10 years, 1866–1875	2236	2471	+235
Total period, 20 years, 1856–1875	2383	2946	+563
Second period, per cent. + or − first period	−11·7	−27·7	
NITROGEN.			
First period, 10 years, 1856–1865	35·1	57·9	+22·8
Second period, 10 years, 1866–1875	30·9	47·3	+16·4
Total period, 20 years, 1856–1875	33·0	52·6	+19·6
Second period, per cent. + or − first period	−12·0	−18·3	
MINERAL MATTER (ASH).			
First period, 10 years, 1856–1865	148·5	181·2	+32·7
Second period, 10 years, 1866–1875	126·1	108·9	−17·2
Total period, 20 years, 1856–1875	137·3	145·1	+ 7·8
Second period, per cent. + or − first period	−15·1	−39·9	

Compared with the produce without manure, ammonia-salts alone have given an average annual increase over 20 years of 563 lbs. of hay, or, in all, of 11,260 lbs., or rather more than 5 tons; and it is to be observed that the increased growth was dependent for the necessary mineral constituents (sulphuric acid and chlorine only excepted) on the supplies of the soil exclusively. But, whilst the average annual increase was 889 lbs. over the first 10 years, it was only 235 lbs. over the second

10 years. The falling off of effect is, therefore, very great; and the proportional reduction of produce is more than twice as great as without manure. There is also a considerable reduction in the yield of nitrogen per acre; but as the amount annually supplied in the manure was several times greater than the increase of nitrogen in the crop by its use, and the percentage of nitrogen in the produce was abnormally high, the deficiency of growth was obviously not dependent on a want of nitrogen. The reduction in the yield of mineral matter, on the other hand, is seen to be nearly 40 per cent. over the second compared with the first 10 years.

Leaving out of consideration the amounts of sulphuric acid and chlorine taken up, these being largely supplied in the ammonia-salts, there was, of all the mineral constituents derived from the soil itself, more taken up during the first 10 years under the influence of the ammonia-salts than without manure. Of lime, of potass, and of silica there was, however, considerably less taken up over the second 10 years than without manure; of lime and of potass so much less that of these there was actually less yielded up by the soil in the 20 years with the ammonia-salts than without manure. Of magnesia, and of phosphoric acid, there was, on the other hand, rather more yielded up over both periods with the ammonia-salts than without it. Of lime, of potass, and of silica, indeed, the percentage in the dry substance of the produce of the ammonia plot was abnormally low.

That there should be actually less lime and potass taken up with the ammonia than without manure is probably to be explained by the fact that, under the influence of the purely nitrogenous manure, various species which commanded a more extended range of soil than those which remained, had been displaced. Under the influence of the ammonia-salts the total number of species found was greatly reduced; the herbage became more and more gramineous; *Festuca ovina* sometimes yielded more than half, and *Agrostis vulgaris* more than a quarter of the total produce; there was scarcely any leguminous herbage; and of miscellaneous species there were but few in number, which contributed but a small proportion of the weight, excepting *Rumex acetosa*, which was objectionably flourishing. Thus, the produce consisted chiefly of the poorer grasses, which, moreover, showed very little tendency to form stem and seed, whilst the stunted foliage was of a very dark green colour, indicating a very high percentage of nitrogen in its dry substance, or rather a deficient assimilation of carbon in proportion to the nitrogen taken up.

The next illustrations also show the effects of a continuous supply of nitrogenous without mineral manures.

3. *Nitrate of Soda alone; Plots* 15 *and* 17.

Plot 15 received, for 18 years in succession, approximately the same amount of nitrogen per acre per annum as plot 5, but as nitrate of soda instead of as ammonia-salts; and plot 17 has received annually half the quantity of nitrate. The experiments with the nitrate commenced two years later than most of the others; and that on plot 15 ended in 1875; whilst that on plot 17, like that on plot 5 with the ammonia-salts, has been continued up to the present time. The average produce of hay, &c., cannot, therefore, be taken over precisely the same, or as many years, with the nitrate as with the ammonia-salts.

The following table shows the average results obtained over the first 8, the next 10, and the total period of 18 years, of the application of the nitrate on plot 15; also those obtained, over the same periods, on plot 17, with only half the quantity of nitrate. For comparison, there are also given the results yielded, over the same periods, on plot 3 without manure, and on plot 5 with the ammonia-salts.

TABLE IV.—Average, per acre per annum, by 550 lbs. and 275 lbs. Nitrate of Soda alone; Plots 15 and 17.

	Average per acre per annum.								
	Plot 3. Without manure.	Plot 5. 400 lbs. ammonia-salts.	Plot 15. 550 lbs. nitrate soda.	Plot 17. 275 lbs. nitrate soda.	Plot 15.		Plot 17.		
					+ or − plot 3.	+ or − plot 5.	+ or − plot 3.	+ or − plot 5.	+ or − plot 15.
HAY.									
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
First period, 8 years, 1858-1865	2492	3300	4038	3837	+1546	+ 738	+1345	+ 537	−201
Second period, 10 years, 1866-1875	2236	2471	3918	3755	+1677	+1442	+1519	+1284	−158
Total period, 18 years, 1858-1875	2350	2839	3968	3792	+1618	+1129	+1442	+ 953	−176
Second period, per cent. + or − first period	−10·3	−25·1	−3·1	−2·1					
NITROGEN.									
First period, 8 years, 1858-1865	33·9	57·2	63·1	55·5	+29·2	+ 5·9	+21·6	−1·7	−7·6
Second period, 10 years, 1866-1875	30·9	47·3	61·5	53·2	+30·6	+14·2	+22·3	+5·9	−8·3
Total period, 18 years, 1858-1875	32·2	51·7	62·2	54·2	+30·0	+10·5	+22·0	+2·5	−8·0
Second period, per cent. + or − first period	−8·9	−17·3	−2·5	−4·1					
MINERAL MATTER (ASH).									
First period, 8 years, 1858-1865	145·8	168·2	225·9	223·8	+80·1	+57·7	+78·0	+55·6	−2·1
Second period, 10 years, 1866-1875	126·1	108·9	205·4	205·0	+79·3	+96·5	+78·9	+96·1	−0·4
Total period, 18 years, 1858-1875	134·9	135·3	214·5	213·3	+79·6	+79·2	+78·4	+78·0	−1·2
Second period, per cent. + or − first period	−13·5	−35·3	−9·1	−8·4					

As the experiments with the nitrate commenced two years later than those without manure and those with ammonia-salts, the results are, so far, not strictly comparable; but a careful consideration of them shows that, if the exact figures cannot, the main indications can, without doubt, be thoroughly relied upon.

We will first refer to the results on plot 15, receiving 550 lbs. of nitrate of soda per acre per annum, estimated to contain the same amount of nitrogen as the 400 lbs. ammonia-salts applied to plot 5, namely, 82 lbs. It is remarkable that, whilst with a given amount of nitrogen applied as ammonia-salts there was an average annual increase of produce over the 18 years of only 489 lbs., there was with the same amount applied as nitrate of soda 1618 lbs., or 1129 lbs. more. Or, to put it in another way, whilst the ammonia-salts gave an average annual increase of little more than one-fifth over the unmanured produce, the nitrate of soda gave more than one-and-two-thirds as much. Further, whilst over the second period compared with the first the produce declined 10·3 per cent. without manure, and 25·1 per cent. with ammonia-salts, it only fell off 3·1 per cent. with the nitrate of soda.

The yield of nitrogen, again, was one-fifth more under the influence of the nitrate than of the ammonia-salts; and whilst, over the second period compared with the first, it declined nearly 9 per cent. without manure, and more than 17 per cent. with ammonia-salts, the reduction was only 2½ per cent. with the nitrate of soda.

But it was in the amounts of mineral matter taken up that the difference in the effects of the ammonia-salts and the nitrate of soda was the most marked. Thus, whilst over the 18 years the quantities of mineral matter removed in the crops were almost identical without manure and with ammonia-salts, with the nitrate of soda more than one-and-a-half time as much was taken up; and the increase, which amounted to an average of nearly 80 lbs. per acre per annum, was more than two-and-a-half times as much as is accounted for by the increased amount of soda taken up, due to the nitrate of soda used. It is remarkable, too, that whilst the decline in the amount of mineral matter taken up over the second period compared with the first was 13·5 per cent. without manure, and 35·3 per cent. with the ammonia-salts, it was only 9·1 per cent. with the nitrate of soda. Of individual mineral constituents, not supplied in either manure, more lime, more magnesia, and considerably more of potass, phosphoric acid, and silica, were removed in the produce by the nitrate than in that by the ammonia-salts.

To attempt to explain these results, even as fully as the data at command will allow, would be to go beyond the facts more special to this section, and to trench upon those specially belonging to the botanical and chemical sections, and would, indeed, involve a long discussion. It must suffice in this place briefly to indicate the main points. Under the influence of the nitrate of soda a greater total number of species maintained a place than under that of the ammonia-salts. Among the grasses a greater variety contributed to the bulk of the herbage, and they showed a greater tendency to form stem and seed. There was also somewhat more leguminous herbage, and considerably more referable to miscellaneous species, of which, moreover, a greater number were fairly prominent.

It will be readily conceived that, under the conditions here described, there would be a greater variety, and better development, and consequently a wider and deeper distribution of the roots of the collective mixed herbage growing under the influence of the nitrate, giving it a command of the stores of a correspondingly increased range of soil and subsoil. Again, it is well known that (in whatever combination it does so in either case) the nitrogen of nitrate of soda distributes much more rapidly, not only in the upper but also in the lower layers of the soil, than does that of the ammonia-salts. Direct observation has further shown that, with this, there is a greater development of some of the deeper rooting species, and consequently a greater distribution of roots in the lower layers. Thus, the mixed herbage growing under the influence of the nitrate draws more upon the deeper layers; it has, therefore, a greater supply of mineral constituents at its disposal; and hence, though taking more from the soil itself, it shows exhaustion much more tardily. How much the action of the nitrate of soda, or of its products of decomposition, in liberating or rendering available otherwise locked up constituents of the soil and subsoil, may have contributed to the result is obviously a question worthy of consideration.

The results of plot 17 show the effects of nitrate of soda applied in quantity supplying only half as much nitrogen as that of the ammonia-salts of plot 5, or of the nitrate of soda of plot 15.

The figures show much more produce and increase over the first, second, and total periods, by the use of nitrate of soda in this smaller quantity, than by that of ammonia-salts containing twice as much nitrogen; indeed, there was, over the first period only 201 lbs., over the second only 158 lbs., and over the total period only 176 lbs., less hay per acre per annum, than was obtained by double the amount of nitrate of soda. Further, the yield is only 2 per cent. less over the second period compared with the first.

Of nitrogen in the produce, the smaller quantity of nitrate of soda gave somewhat less over the first period, but considerably more over the second, and more over the total period, than twice the quantity applied as ammonia-salts; and there was, at the same time, comparatively little decline in yield over the second period compared with the first. There was, however, an average of about 8 lbs. per acre per annum less nitrogen in the produce by the smaller than in that by the double amount of nitrate of soda; but there was, nevertheless, a much larger proportion of that supplied recovered in the increase of crop where the smaller quantity was used.

It has been seen how strikingly greater was the amount of mineral matter taken up under the influence of a quantity of nitrate containing the same amount of nitrogen as the ammonia salts; and it is even more remarkable that here, with only half the quantity of nitrate, there is, over the whole period, within about 1 lb. per acre per annum as much mineral matter removed in the crop as with the double quantity, whilst the falling off in amount over the second period is even rather less. There was, indeed, even more lime, more potass, and more silica, taken from the soil in the crops grown by the smaller than in those by the larger amount of nitrate, and at the same time nearly as much magnesia and phosphoric acid. The deficiency was mainly due to much less soda being taken up.

In reference to the striking effects of this smaller quantity of nitrate of soda alone, over a period of 18 years, during which, excepting in the item of soda, the growth was entirely dependent for mineral constituents on the supplies of the soil itself, it may be remarked that the same circumstances which were pointed out as serving to explain the better result with the larger amount of nitrate than with the ammonia-salts, obtained in even a rather greater degree where the smaller quantity of nitrate was used. Thus, a still greater total number of species grew; a somewhat greater number of the grasses contributed to the bulk of the gramineous herbage; the tendency to form stem and seed was greater; the proportion in the produce of the non-gramineous herbage was greater, and it was made up of a greater variety of species. It is to be supposed, therefore, that the roots would have more complete possession of the soil and the higher layers of the subsoil. It has been seen, indeed, that they had sufficient command of them to take up nearly as much total mineral matter, and even rather more of the important constituents—lime, potass, and silica—than with the larger amount of nitrate.

It would appear that the limit of growth under the influence of nitrate of soda alone—that is to say without the aid of any artificial supply of mineral constituents—was nearly reached with the smaller quantity used; and a comparison of the results obtained by it with those by the double quantity led us to stop the application of this after the twentieth year of the experiments, that is to say, after 18 years of its use. A mixed mineral manure, comprising sulphates of potass, soda, and magnesia, and superphosphate of lime, has now been applied instead for four years, and it is proposed to continue the application with a view to determine what proportion, if any, of the hitherto unrecovered nitrogen of the excess of nitrate supplied will be recovered, and also to observe the changes in the flora, so to speak, under this change of treatment. So far, the indications are not in favour of the supposition that any material proportion of the previously unrecovered nitrogen of the nitrate will be recovered in subsequent crops; indeed, it would appear probable that much has passed away by drainage. The general character of the herbage, is, however, rapidly changing. The grasses show a much lighter colour, and much more tendency to form stem, and to mature; leguminous species (especially *Lathyrus pratensis*) are gaining ground; and the character of the miscellaneous herbage is also changing.

4. *Mixed Mineral Manure alone ; Plot* 7.

We have now to show the effects of a mixed mineral manure, such as just referred to, when applied to the complex herbage year after year without any nitrogenous manure. This experiment was commenced in the first year of the series, 1856, and has been continued up to the present time.

The "mixed mineral manure" used, was composed, per acre per annum, as follows :—

300 lbs. sulphate of potass.
100 lbs. sulphate of soda (200 lbs. the first 8 years).
100 lbs. sulphate of magnesia.
200 lbs. bone ash
150 lbs. sulphuric acid, sp. gr. 1·7 } superphosphate of lime.

The following table shows the results obtained over the first 10, the second 10, and the total period of 20 years, 1856–75 inclusive, also, for comparison, those without manure over the same periods.

TABLE V.—Average, per acre per annum, by Mixed Mineral Manure alone ; Plot 7.

	Average per acre per annum.		
	Plot 3. Without manure.	Plot 7. Mixed mineral manure.	Plot 7. + or − plot 3.
HAY.			
	lbs.	lbs.	lbs.
First period, 10 years, 1856–1865	2531	3797	+1266
Second period, 10 years, 1866–1875	2236	4118	+1882
Total period, 20 years, 1856–1875	2383	3958	+1575
Second period, per cent. + or − first period	−11·7	+8·5	
NITROGEN.			
First period, 10 years, 1856–1865	35·1	55·2	+20·1
Second period, 10 years, 1866–1875	30·9	58·0	+27·1
Total period, 20 years, 1856–1875	33·0	56·6	+23·6
Second period, per cent. + or − first period	−12·0	+5·1	
MINERAL MATTER (ASH).			
First period, 10 years, 1856–1865	148·5	246·3	+ 97·8
Second period, 10 years, 1866–1875	126·1	261·6	+135·5
Total period, 20 years, 1856–1875	137·3	254·0	+116·7
Second period, per cent. + or − first period	−15·1	+6·3	

Here then, with a *mixed mineral manure alone,* we have considerably higher amounts of produce than with nitrogenous manure alone applied as ammonia-salts, and, taking the whole period, nearly as much as when applied as nitrate of soda. Not only so, instead of a reduction in hay, in nitrogen, and in mineral matter, removed over the second compared with the first 10 years, there is an increase of all three over the second period.

It is, nevertheless, sufficiently established that nitrogenous manures are specially effective in increasing the growth of gramineous crops grown separately on arable land, such as wheat, barley, or oats, all of which contain a comparatively small percentage of nitrogen, and assimilate a comparatively small amount of it over a given area when none is supplied in manure. The highly nitrogenous leguminous crops, on the other hand, such as beans, peas, clover, &c., are not characteristically benefited by the use of direct nitrogenous manures, though nitrates do act more favourably on them than ammonia-salts. Again, whilst, under equal conditions of soil and seasons, mineral manures alone increase comparatively little the gramineous crops grown separately, such manures, and especially potass manures, do in a striking degree increase the growth of crops of the leguminous family so grown, and coincidently increase the amount of nitrogen they assimilate over a given area.

Consistently with this, as will be fully illustrated in the proper place, the application to the mixed herbage of the mineral manure, containing potass, as above described, did very considerably increase the growth of leguminous species. But, more or less from the beginning, and especially in the later seasons, it increased that of some gramineous species very much also—indeed, much more than our experience in the growth of gramineous crops by mineral manures on arable land led us to anticipate.

Table VI. gives approximate estimates of the average amounts, per acre per annum, of hay, nitrogen, and mineral matter, referable respectively to gramineous, leguminous, and miscellaneous species, in the produce of plot 3 without manure, and of plot 7 with the mixed mineral manure, over the first 10, the second 10, and the 20 years. As the botanical separations were not made so frequently in the earlier as in the later years, and as the determinations of nitrogen and mineral matter, and of the composition of the ash, in the separated gramineous, leguminous, and miscellaneous herbage, have only been made in recent years, the estimates are obviously only approximate; but they are doubtless sufficiently near the truth to bring clearly to view the characteristic distinctions between the produce of the two plots, and therefore to afford important data for judging of the effects of the manure. The following are the mean percentages of nitrogen, and of mineral matter, in the different descriptions of herbage, as adopted in the calculations, in each case reckoned on the herbage in the condition of dryness of hay :—

	Gramineous herbage.		Leguminous herbage.		Miscellaneous herbage.	
	Plot 3.	Plot 7.	Plot 3.	Plot 7.	Plot 3.	Plot 7.
Per cent. nitrogen	1·25	1·18	2·31	2·31	1·45	1·32
Per cent. mineral matter (ash) . . .	5·11	5·64	6·32	6·81	7·30	8·74

TABLE VI.—Hay, Nitrogen, and Mineral Matter, referable to Gramineous, Leguminous, and Miscellaneous Herbage respectively, without Manure, and by "Mixed Mineral Manure;" Plots 3 and 7.

	Average per acre per annum.								
	Gramineous herbage.			Leguminous herbage.			Miscellaneous herbage.		
	Plot 3. Without manure.	Plot 7. Mineral manure.	Plot 7. + or − plot 3.	Plot 3. Without manure.	Plot 7. Mineral manure.	Plot 7. + or − plot 3.	Plot 3. Without manure.	Plot 7. Mineral manure.	Plot 7. + or − plot 3.
HAY.									
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
First period, 10 years, 1856–1865	1668	2324	+ 656	186	754	+568	677	719	+ 42
Second period, 10 years, 1866–1875. . . .	1523	2643	+1120	178	813	+635	535	662	+127
Total period, 20 years, 1856–1875	1596	2484	+ 888	182	784	+602	606	691	+ 85
Second period, per cent. + or − first period	−8·7	+13·7		−4·3	+7·8		−21·0	−7·9	
NITROGEN.									
First period, 10 years, 1856–1865	20·9	27·8	+ 6·9	4·3	17·7	+13·4	9·9	9·7	−0·2
Second period, 10 years, 1866–1875. . . .	19·1	30·8	+11·7	4·1	18·6	+14·5	7·7	8·6	+0·9
Total period, 20 years, 1856–1875	20·0	29·3	+ 9·3	4·2	18·2	+14·0	8·8	9·1	+0·3
Second period, per cent. + or − first period	−8·6	+10·8		−4·7	+5·1		−22·2	−11·3	
MINERAL MATTER (ASH).									
First period, 10 years, 1856–1865	86·6	131·7	+45·1	11·9	51·5	+39·6	50·0	63·1	+13·1
Second period, 10 years, 1866–1875. . . .	76·7	148·7	+72·0	11·1	55·1	+44·0	38·3	57·8	+19·5
Total period, 20 years, 1856–1875	81·6	140·2	+58·6	11·5	53·4	+41·9	44·2	60·4	+16·2
Second period, per cent. + or − first period	−11·4	+12·9		−6·7	+7·0		−23·4	−8·4	

It is seen that, over the 20 years, more than one-and-a-half time as much gramineous herbage was grown by the purely mineral manure as without manure; and that, whilst without manure there was a decline in the second 10 years compared with the first, there was with the mineral manure a considerable increase over the later period.

Of leguminous herbage there was not nearly so much actual increase of weight by the mineral manure as there was of the gramineous; but the proportional increase was much greater, there being more than four times as much grown with the mineral manure as without manure, and there was also more over the second period than over the first.

Of miscellaneous herbage there was very little increase by the manure, and a falling off in actual amount over the later years.

The facts relating to the nitrogen bring out more strikingly the characteristic effects of the mixed mineral manure in favouring the development of leguminous species. The figures given on page 309 show that whilst the percentage of nitrogen in the miscellaneous herbage is not much higher than that in the gramineous, the percentage in the leguminous herbage is very much higher; in fact, not far from twice as high. The result is that, although there is by the mineral manure a greater increase in gramineous than in leguminous hay, a larger proportion of the increase of nitrogen is due to the leguminous than to the gramineous herbage; whilst the miscellaneous herbage contributes practically none of the increased yield of nitrogen. Again, whilst in the unmanured produce the leguminous herbage has taken up little more than one-fifth as much nitrogen as the gramineous, in the mineral manured produce there is nearly two-thirds as much in the leguminous as in the gramineous herbage. Or, to put it in another way, whilst the average annual amount of nitrogen in the gramineous herbage was 20 lbs. without, and 29·3 with, the manure, or 9·3 lbs. more with than without it, it was in the leguminous herbage only 4·2 lbs. without, but 18·2 lbs. with the manure, or 14·0 lbs. more with than without it.

As already said, it is entirely consistent with experience in the growth of individual leguminous crops on arable land, that the amount, both of the produce and of the nitrogen assimilated, should be considerably increased under the influence of such a manure; and although the source of the increased yield of nitrogen under such circumstances is not conclusively proved, the weight of evidence is in favour of the conclusion that it is derived from the soil; and, in the case of clover and beans, at any rate, more probably from the lower than the upper layers. In the unmanured mixed herbage the deeper-rooting *Lotus corniculatus* and *Trifolium pratense* contributed most of the leguminous produce; but in the manured herbage it is the comparatively surface-rooting *Lathyrus pratensis* that mainly contributes the leguminous increase.

Again, although experience in the growth of gramineous crops on arable land did not lead us to expect so much increase in the growth of gramineous species in the mixed herbage, yet even among the grain crops there is a notable difference in the effect of mineral manures, according to the habit of development of the plant, and to the circumstances of its growth. Thus, the spring-sown barley, which has but a short time in which to extend its roots, and to gain command of the resources of the soil, throws out a large amount of fibre near the surface, and is more benefited by the application of direct mineral manures than is the autumn-sown wheat, which has four or five months longer for root-distribution, and is less dependent on the stores of the surface soil. In accordance with this, the greatly increased yield of gramineous produce in the mixed herbage by the purely mineral manure was almost exclusively due to the much denser growth, and much greater tendency to form stem and seed, of the creeping, and surface-rooting, *Festuca ovina, Agrostis vulgaris,* and *Holcus lanatus.*

The question still remains—what is the source of the greatly increased yield of nitrogen, both in the gramineous and the leguminous herbage, under the influence of the purely mineral, that is non-nitrogenous, manure?

The annual yield of nitrogen per acre, in the gramineous herbage alone, of the unmanured plot, is approximately the same as that obtained in wheat or barley grown on arable land of somewhat similar character for many years in succession; and the leguminous and miscellaneous herbage together contribute about two-thirds as much; making, therefore, in all a much greater total annual yield in the mixed herbage than in the cereal crop. That it should be greater, at any rate for a considerable period of time, would seem only natural to expect, when it is borne in mind that the surface-soil of permanent meadow land is much richer in nitrogen, due to vegetable débris and previous accumulations, than arable land of otherwise corresponding character; and, further, that many of the components of the mixed herbage have possession of the soil, and are more or less growing, almost the year round. But, under the influence of the mineral manure, the total yield of nitrogen per acre per annum averaged one-and-two-thirds as much as without manure, and about two-and-a-half times as much as would be yielded in a cereal crop similarly manured on arable land; and, without direct evidence on the point, it would hardly be concluded that this largely increased yield could be accounted for in a similar way to that of the lesser amount without manure.

In seeking for an explanation, it had to be borne in mind, that such mineral manures did also considerably increase the growth of leguminous crops grown on arable land; and, further, that when a leguminous crop is grown on arable land it not only removes much more nitrogen from a given area than a gramineous one grown under equal conditions as to soil and season, but it leaves the upper layers of the soil so much richer in nitrogen, and in such condition that the increase may be determined by the soda-lime method, and the growth of the succeeding cereal is considerably augmented. The question, therefore, suggested itself—how far the increased growth of the grasses proper in the mixed herbage was due to an increased accumulation of combined nitrogen available to them, in the upper layers of the soil, the result of the increased growth of, and accumulation by, the Leguminosæ, induced by the mineral manure? Whether, in fact, where the plants are thus growing in association, there is a parallel action to that which takes place when they are grown in alternation?

This explanation obviously itself left unsolved the question of the source of the nitrogen of the Leguminosæ, besides involving other difficulties. Still, it seemed to accord with other known facts, and to be at any rate the best that could be offered in defect of more satisfactory or countervailing evidence. However, as already referred to, after the experiments had been continued for 20 years, samples of the soils and subsoils were taken, from each plot, at different depths, and determinations of nitrogen have been made in them. The results show a considerably lower percentage in the first nine inches of depth of the mineral manured plot 7 than in that of the unmanured plot 3; and so far as the calculation of such results into quantities per acre can be relied upon, the difference is more than sufficient to account for the increased yield of nitrogen over the 20 years in both the gramineous and the leguminous herbage of plot 7. The details will be fully considered further on. But, in the meantime, a clue to the source of the increased yield of nitrogen in the mixed herbage would seem to be obtained. It would appear that, under the influence of the mixed mineral manure, both the powers of collection, assimilation, and transformation, of the plants themselves, have been considerably augmented, and the accumulated stores of nitrogen within the soil have been rendered more available. They have, in fact, served, either directly or indirectly, as manure for both the Gramineæ and the Leguminosæ. We have, too, in the facts, an addition to the previously existing evidence in favour of the supposition that the Leguminosæ derive the large amounts of nitrogen they assimilate from the stores of the soil itself.

Turning to the amounts of mineral matter taken up under the influence of the liberal supply of it by manure, Table V., page 308, shows that, over the first period nearly one-and-two-thirds as much, over the second period more than twice as much, and over the 20 years nearly twice as much, was taken up on plot 7 as without manure. The analytical data further show that, whilst of lime and silica only about one-fourth more, of magnesia about one-half more, and of sulphuric acid and chlorine about twice as much, was taken up as without manure, of phosphoric acid there was nearly three times, and of potass nearly three-and-a-half times as much; and, with the liberal supply, and the greatly increased assimilation, of potass, there was, of soda alone less taken up and retained in the manured than in the unmanured produce; indeed, there was less than half as much, notwithstanding that soda was annually applied in the manure. Again, whilst the percentages of lime, of magnesia, and of silica, were considerably less in the dry substance of the manured than of the unmanured produce, those of the sulphuric acid and chlorine were rather higher, that of the phosphoric acid was nearly twice, and that of the potass more than twice as high; the percentage of soda, on the other hand, was little more than one-fourth as much in the dry substance of the manured as of the unmanured produce.

The general result is, that a mixed mineral manure alone has, in a very marked degree, influenced the description of plants developed, and the character of their development, greatly favouring the tendency to form stem and seed—that is, to mature; it has also much increased the amount of total produce grown, and of nitrogen as well as mineral matter taken up. The evidence, so far, points to the potass of the manure as mainly conducive to these effects, the phosphoric acid probably coming second in order of importance. A comparison of the results obtained by superphosphate of lime alone, and by superphosphate of lime and sulphates of soda and magnesia without potass, with those by the "mixed mineral manure" which, besides these, contained potass, will throw further light on these points.

5. *Superphosphate of Lime alone; Plot* 4–1.

Plot 4–1 has been manured with superphosphate of lime alone every year from 1859 up to the present time, or for a period of 21 years. In each of the three preceding years, however (1856–57–58), it received sawdust, at the rate of 2000 lbs. per acre per annum, the application being made to test the validity of LIEBIG'S assertion that it produced great effects by virtue of the solvent action, upon the mineral constituents of the soil, of the carbonic acid which it yields in its decomposition. Nevertheless, and notwithstanding that the sawdust annually applied contained nitrogen equal to between 4 lbs. and 5 lbs. per acre, the result was even less produce than without manure in each of the three seasons.

The following table shows the average annual yield of hay, nitrogen, and mineral matter, on plot 4–1, over the first 7, the next 10, and the total period of 17 years, from 1859–1875 inclusive, by the annual application to it of superphosphate of lime alone; and for comparison there are also given the results, over the same periods, without manure, and with the mixed mineral manure (including potass).

TABLE VII.—Average, per acre per annum, by Superphosphate of Lime alone; Plot 4–1.

	Average per acre per annum.				
	Plot 3. Without manure.	Plot 7. Mixed mineral manure.	Plot 4–1. Superphosphate lime.	Plot 4–1. + or – plot 3.	Plot 4–1. + or – plot 7.
HAY.					
	lbs.	lbs.	lbs.	lbs.	lbs.
First period, 7 years, 1859-1865	2495	3828	2732	+237	–1096
Second period, 10 years, 1866-1875 . . .	2236	4118	2384	+148	–1734
Total period, 17 years, 1859-1875. . . .	2343	3999	2527	+184	–1472
Second period, per cent. + or – first period	–10·4	+7·6	–12·7		
NITROGEN.					
First period, 7 years, 1859-1865	33·8	54·5	36·3	+2·5	–18·2
Second period, 10 years, 1866-1875 . . .	30·9	58·0	31·6	+0·7	–26·4
Total period, 17 years, 1859-1875. . . .	32·1	56·5	33·5	+1·4	–23·0
Second period, per cent. + or – first period	–8·6	+6·4	–12·9		
MINERAL MATTER (ASH).					
First period, 7 years, 1859-1865	146·5	247·8	170·8	+24·3	– 77·0
Second period, 10 years, 1866-1875 . . .	126·1	261·6	140·4	+14·3	–121·2
Total period, 17 years, 1859-1875. . . .	134·5	255·9	152·9	+18·4	–103·0
Second period, per cent. + or – first period	–13·9	+5·6	–17·8		

Thus, over both periods, the plot with the superphosphate gave more produce than that without manure; but the increase was only small, and averaged less over the second period than over the first, whilst the actual produce per acre declined more than that without manure. On the other hand, the superphosphate alone gave, over the total period of 17 years, less than two-thirds as much hay as the mixed mineral manure.

Of nitrogen, again, the increase by the superphosphate was insignificant, and both it and the actual yield declined considerably over the later period. The average annual yield of it over the 17 years was 23 lbs. less than, and did not amount to three-fifths as much as, by the mixed mineral manure.

Of mineral matter taken up, the increased amount was proportionally greater than that of either the hay or the nitrogen; but here, again, both the actual yield and the increase declined considerably over the later years. But the quantity taken up was much less, and decreased as the experiment proceeded, compared with that yielded by the mixed mineral manured plot.

The result is, that the percentage reduction in annual yield of hay, of nitrogen, and of mineral matter, over the second period compared with the first, was greater with the superphosphate than without manure; whilst with the mixed mineral manure there was, instead of a reduction, an increase of all three over the second period.

The botany of the plot was comparatively little affected by the superphosphate of lime. The number and the relative predominance of species were much the same as without manure. Perhaps the most characteristic changes were a rather greater weight of total gramineous herbage, due mainly to more of ripening tendency; a reduction in weight of Leguminosæ, with a prevalence of *Lathyrus pratensis* rather than of *Lotus corniculatus;* and an increase in weight of miscellaneous herbage, with a greater prevalence especially of *Ranunculus repens* and *bulbosus, Achillea millefolium* and *Rumex acetosa.*

Of the constituents supplied in the superphosphate of lime, a little more of lime and magnesia, one-and-a-half time as much sulphuric acid, and nearly twice as much phosphoric acid, were taken up under its influence as without manure. Of constituents not supplied in the manure, but derived from the soil itself, there was of potass and silica very little more taken up than without manure, but the increase of soda was somewhat greater. There was a decline in the amount taken up of every one of the mineral constituents over the second period; and this was proportionally the greatest with the soda, the silica, and the sulphuric acid, and greater in the magnesia than in either the potass or the lime.

It would appear that the superphosphate had enabled the plants to draw somewhat more largely on the resources of the soil, especially in the earlier years, but that the small increased available supply under its influence was rapidly diminishing. Lastly, the comparison of the produce by the superphosphate of lime alone, with that by the mixed mineral manure, clearly shows that it was neither exclusively, nor even mainly, to the phosphoric acid which the latter contained that the results it yielded were due.

6. *Mixed Mineral Manure, with, and without, Potass; Plot* 8.

In the next table are compared the results obtained by mixed mineral manure alone in both cases, but in one including and in the other excluding potass. During the first six years of the experiments (1856–61) plots 7 and 8 each received the "mixed mineral manure," comprising superphosphate of lime, and sulphates of potass, soda, and magnesia, the only difference being that during those years, and in 1862, plot 8 also received 2000 lbs. of sawdust per acre per annum. From 1862 inclusive up to the present time, plot 7 has continued to receive the same mixed mineral manure, and plot 8 the same, excluding the sulphate of potass, but with an increased amount of sulphate of soda. The table shows the results yielded on the two plots, over the first six years, during which (with the exception of the sawdust on plot 8) both were manured alike, over the next 14 years, when the only difference was the omission of the potass and increase of the soda on plot 8, and over the total period of 20 years, 1856–1875 inclusive.

Table VIII.—Average, per acre per annum, by Mixed Mineral Manure, with and without Potass; Plots 7 and 8.

	Average per acre per annum.		
	Plot 7. Mineral manure, including potass, every year.	Plot 8. Mineral manure, including potass 6 years, without potass 14 years.	Plot 8. + or − plot 7.
Hay.			
	lbs.	lbs.	lbs.
First period, 6 years, 1856–1861	3835	4027	+192
Second period, 14 years, 1862–1875	4011	3098	−913
Total period, 20 years, 1856–1875	3958	3377	−581
Second period, per cent. + or − first period .	+4·6	−23·1	
Nitrogen.			
First period, 6 years, 1856–1861	56·6	60·9	+ 4·3
Second period, 14 years, 1862–1875	56·6	40·3	−16·3
Total period, 20 years, 1856–1875	56·6	46·5	−10·1
Second period, per cent. + or − first period .	0·0	−33·8	
Mineral Matter (Ash).			
First period, 6 years, 1856–1861	254·4	271·8	+17·4
Second period, 14 years, 1862–1875	253·7	181·9	−71·8
Total period, 20 years, 1856–1875	253·9	208·8	−45·1
Second period, per cent. + or − first period .	−0·3	−33·1	

It is to be observed that there was an average of rather more hay, and of rather more nitrogen and mineral matter taken up, on plot 8 than on plot 7, during the first six years, the only difference in the treatment being that plot 8 received sawdust in addition to the other manures. It has been seen that when the sawdust was used alone it yielded no increase. Whether the result here is due to any reaction with the associated mineral manures, or to some accidental difference in the otherwise duplicate plot, it is not easy to say; but it may be remarked, in passing, that the increased amount of nitrogen taken up corresponded very closely with that contained in the sawdust.

During the next 14 years the relations are strikingly reversed in every particular. There is an average of nearly one-fourth less hay produced, and of more than one-fourth less nitrogen and mineral matter taken up, on plot 8 without, than on plot 7 with potass. In fact, the falling off in amount of produce on the exclusion of potass from the manure was very great. The number of species has remained about the same on the two plots. The percentage by weight, in the total produce, of gramineous species was, during the earlier of the 14 years, and has been on the average, greater in the produce of plot 8 without potass, but, as already referred to, it has increased on plot 7 in the later years. The percentage of the Leguminosæ has diminished exceedingly on plot 8 since the exclusion of the potass, and over the 14 years has averaged less than half as much as on plot 7. The proportion of species of other orders has, on the other hand, been higher in the produce without potass. In weight per acre, however, the yield of gramineous herbage has averaged only about four-fifths as much, that of the leguminous herbage not two-fifths as much, and that of the miscellaneous herbage also has been somewhat less, without than with the potass.

There is no very striking difference between the two plots in the relative proportion of the different species making up the gramineous herbage; but whilst with the potass there was a very marked tendency to form stem and seed, without it there was much less of this character, the herbage being more leafy, less dense, and having much less tendency to mature, some of it, indeed, appearing to dry up from exhaustion rather than to ripen.

Of leguminous herbage, there was a decline of both *Trifolium repens* and *Trifolium pratense* in the later years, both with and without the potass; but the reduction was greater without than with it. Of the deep-rooting and hardy *Lotus corniculatus* there was the more without the potass. But of the comparatively surface-rooting *Lathyrus pratensis* there was very much less without the potass. In fact, it was to the greatly increased growth of this with the potass, and to the greatly reduced growth of it without it, that the great difference in the amount of total leguminous herbage on the two plots was chiefly due.

Among the miscellaneous species those of *Ranunculus* thrived rather more without the potass, as also did *Pimpinella saxifraga*, *Centauria nigra*, and *Plantago lanceolata;* and in a greater degree *Achillea millefolium*. *Conopodium denudatum* was rather more prominent with the potass. *Rumex acetosa* was, on both plots, the most prominent of the weeds, but yielded somewhat more on plot 7 with, than on plot 8 without potass.

The analytical results, which will be fully discussed in Section III., provide the data for determining what changes in the taking up of individual mineral constituents have taken place coincidently with the great decline in the yield of hay, and the much less development of leguminous herbage, on the exclusion of potass from the manure of plot 8. Over the 14 years of this exclusion, an average of only about half as much potass was annually taken up as during the preceding six years when it was supplied, and less than half as much as on plot 7 during the 14 years of the continued application. With this reduction in the amount of potass there was considerably more, but not correspondingly more, soda taken up. Of lime, magnesia, phosphoric acid, and sulphuric acid, although all were liberally supplied, considerably less was taken up on plot 8 without than on plot 7 with the potass. Of chlorine also much less was taken up on plot 8. Of silica, which was not supplied, even rather more was taken up without than with the potass, but with this exception, and that of soda already referred to, less of every mineral constituent was taken up on plot 8 without than on plot 7 with the potass. Of percentage in the dry substance of the produce, however, that of lime, magnesia, phosphoric acid, sulphuric acid, and chlorine, was nearly the same without and with the potass; that of soda and of silica was much higher, but that of potass was not two-thirds as high, without the potass, though it was still much higher than without manure, as also was the percentage of phosphoric acid; but that of the other constituents was, for the most part, much the same as without manure, excepting the lime, which was considerably less both without and with the potass.

A consideration of the facts enumerated can leave no doubt that it was to a relative deficiency of available potass that the falling off, not only in the total weight of produce, but also in the description, and in the character of development, of the herbage, is to be attributed.

Notwithstanding the great reduction in the amount of produce, and in the amount of potass taken off in the crop, after the cessation of its application, there was still, in every one of the 14 years, much more potass taken up than without manure. There was, however, something like a gradual reduction of the excess from year to year, and it averaged not much more than half as much over the second as over the first seven of the 14 years. It is obvious, therefore, that the unexhausted residue of the potass applied during the first six years has, in a considerable proportion at any rate, remained in the soil within the range of the roots, but that only a limited and gradually decreasing proportion has been available each year. The questions arise—how much unexhausted residue did remain at the end of the six years of the application? and whether the whole of it has as yet been yielded up? In estimating the amount of unexhausted residue, we have first to consider whether the whole of the potass of the crop was derived from that which was artificially supplied, or whether only so much of it as was in excess of that taken up without manure?

If we assume, for the sake of illustration, that the potass artificially supplied was the most readily available, and that the crop relied entirely upon it so long as it lasted, the result would be that only about 376 of the 900 lbs. applied, remained as residue after the first six years, and that nearly the whole of this was taken out during the next seven years. That it was not all recovered by that time would appear from the fact that in every one of the following seven years there was still considerably more potass in the crop of plot 8 than in that without manure.

If, on the other hand, we suppose that the soil yielded up as much potass from its own stores to the crop of plot 8 as to that without manure, there would remain at the end of the six years of the application 567 lbs., or nearly two-thirds of the whole supplied still to be accounted for. On the same supposition, at the end of the next 14 years there would still remain more than 300 lbs. of the supplied potass as yet unaccounted for. That some did, and does still, remain in an available condition may be concluded from the fact that there was a considerable excess of produce, of mineral matter, and of potass, removed from plot 8 compared with plot 3 in the two succeeding years, 1876 and 1877, and there was again an excess of produce in 1878.

Whether, however, the soil did actually yield up from its own resources more or less to this manured than to the unmanured produce, we have not the means of determining. It has been seen that the superphosphate of lime applied alone on plot 4–1 enabled the herbage to take up very little more potass from the soil. But in that case there was extremely little increased growth. On plot 8, on the other hand, there was, besides superphosphate, the residue of potass previously applied, an annual supply of sulphates of soda and magnesia also, and with these considerably increased luxuriance of growth. Under such conditions the roots of the herbage would doubtless have possession of an increased range of soil and subsoil, and as it is obvious that notwithstanding the considerable residue of previously supplied potass the growing plants nevertheless suffered from a deficient available supply of it relatively to other constituents, it would appear probable that they would extract from the soil, at any rate as much as the unmanured herbage. Our knowledge, or rather our ignorance, of the condition in which the available portion of the potass of the soil itself exists within it, and is taken up by the roots, or of the condition in which the unexhausted residue of the previously supplied potass is retained, and becomes available in so piece-meal a manner, is such that it is impossible to come to any definite conclusion on the point. It will be seen that the facts suggest an interesting and useful line of experimental enquiry.

That certain soils do, as a matter of fact, retain a residue of supplied potass, and in a slowly available condition, for very many years, is conclusively proved by evidence of very different kinds. Thus, in the experiments at Rothamsted in which wheat has now been grown for 36 years in succession on the same land, the effects of the residue of potass supplied more than 28 years ago are still traceable in an increased produce compared with that where there had not been such supply, and also in an increased amount of potass in the crops grown. Further, on the examination of the soils of some of the plots of that same field, Baron LIEBIG's son, HERMANN VON LIEBIG, found the amounts of both potass and phosphoric acid considerably greater, especially in the upper layers, the greater had been the previous supplies by manure. Lastly, Dr. VOELCKER's analyses of the drainage waters from the different plots of the same field showed very much less loss of potass in that way than of either soda, lime, or magnesia, and also very much less of phosphoric acid than of sulphuric acid or of chlorine. There was in fact comparatively little loss by drainage of either potass or phosphoric acid.

To sum up in regard to the effects of the different mineral manures when used without artificial nitrogenous supply: the comparison of the results obtained without manure, with superphosphate of lime alone, and with a mixed mineral manure with, and without, potass, brings clearly to view the much more striking influence of an increased supply of potass, than of lime, magnesia, soda, phosphoric acid, or sulphuric acid; and it will be observed that this is quite consistent with the facts adduced in Table I., and context, in regard to the mineral composition of the hay crop. It is true that when there was either a direct supply, or an available residue, of potass at command, more of the other constituents enumerated above were taken up, and even when superphosphate of lime was used alone, considerably more phosphoric acid was taken up, but coincidently with this there was but little increase of produce, and very little increase in the amount of nitrogen assimilated.

The more special effects of the mineral manures, and particularly of the artificial supply of potass, on the botanical and chemical characters of the herbage, will be discussed in Parts II. and III.

Having now considered the results obtained by nitrogenous manures alone, and by mineral manures alone, we have next to call attention to those yielded by mixtures of the two. Following somewhat the same order of illustration as in regard to nitrogenous manures used alone, we will discuss—first the effects of a given amount of nitrogen applied as ammonia-salts, then of double the quantity, then of the same (the single) amount applied as nitrate of soda, and then of half the quantity, also as nitrate, in each case used in conjunction with the same mixed mineral manure, including potass, as that applied to plot 7; and, following the same order as with the mineral manures used alone, comparison will then be made of the results obtained by the ammonia-salts alone, by the same with superphosphate of lime, with the mixed mineral manure including potass, and with the mixed mineral manure excluding potass.

7. 400 *lbs. Ammonia-Salts with Mixed Mineral Manure containing Potass; Plot* 9.

The next table shows the results obtained by this mixture, side by side with those, over the same periods, by the same quantity of ammonia-salts used alone, and by the same mixed mineral manure used alone.

TABLE IX.—Average, per acre per annum, by 400 lbs. Ammonia-Salts, with Mixed Mineral Manure containing Potass; Plot 9.

	Average per acre per annum.				
	Plot 5. Ammonia-salts alone.	Plot 7. Mineral manure alone.	Plot 9. Ammonia-salts and mineral manure.	Plot 9. + or − plot 5.	Plot 9. + or − plot 7.
HAY.					
	lbs.	lbs.	lbs.	lbs.	lbs.
First period, 10 years, 1856–1865	3420	3797	6002	+2582	+2205
Second period, 10 years, 1866–1875 . . .	2471	4118	5421	+2950	+1303
Total period, 20 years, 1856–1875	2946	3958	5711	+2765	+1753
Second period, per cent. + or − first period	−27·7	+8·5	−9·7		
NITROGEN.					
First period, 10 years, 1856–1865	57·9	55·2	75·7	+17·8	+20·5
Second period, 10 years, 1866–1875 . . .	47·3	58·0	70·7	+23·4	+12·7
Total period, 20 years, 1856–1875	52·6	56·6	73·2	+20·6	+16·6
Second period, per cent. + or − first period	−18·3	+5·1	−6·6		
MINERAL MATTER (ASH).					
First period, 10 years, 1856–1865	181·2	246·2	368·3	+187·1	+122·1
Second period, 10 years, 1866–1875 . . .	108·9	261·6	301·0	+192·1	+ 39·4
Total period, 20 years, 1856–1875	145·1	254·0	334·6	+189·5	+ 80·6
Second period, per cent. + or − first period	−39·9	+6·3	−18·3		

By this combination of ammonia-salts and mixed mineral manure supplying all the mineral constituents of the crop, except silica, we have, over 20 years, considerably more than twice as much hay as without manure, nearly twice as much as by the same amount of ammonia-salts used alone, and nearly one-and-a-half time as much as by the same mineral manure used alone. There was, however, a falling off in the produce of the last 10 years compared with the first 10, of nearly 10 per cent., or in nearly as great a proportion as without manure, but in very much less proportion than with the ammonia-salts without the mineral manure; whilst, on the other hand, there was an increase instead of a reduction in the later years with the mineral manure alone.

The average annual yield of nitrogen per acre was more than twice as much as without manure, more than one-and-one-third time as much as by the same amount of ammonia-salts alone, and nearly one-and-one-third time as much as by the same mineral manures used alone. There was a falling off in the yield over the second 10 years compared with the first 10, but in less proportion than in that of the hay, in less proportion than without manure, and in much less than where the ammonia-salts were used alone; whilst, as with the produce, so with the nitrogen, there was an *increased* yield over the second 10 years with the mineral manures used alone.

Of total mineral constituents about two-and-one-third times as much were taken up as either without manure, or with the ammonia-salts alone, and about one-and-one-third time as much as when the same mineral manure was used without ammonia-salts. With the mineral manures alone, however, there was a slight increase in the amount taken up over the second 10 years; but when used with the ammonia-salts there was a reduction of 18 per cent., or in greater proportion than without manure, but in only about half the proportion as when the ammonia-salts were used alone.

Thus, with the ammonia-salts and the mixed mineral manure together, there was a great increase in the produce of hay, and in the amounts of nitrogen and of mineral matter taken up, compared either with the result without manure, with the ammonia-salts alone, or with the mineral manure alone. But whilst with the mineral manure alone there was an increase in all three items of yield over the second compared with the first 10 years, there was with the ammonia-salts and mineral manure together a reduction in all three, but in much less proportion than when the ammonia-salts were used alone.

In explanation of these great differences in the amounts of produce yielded, and of nitrogen and mineral matter taken up, it is essential to take into consideration the very different character of the herbage in the several cases.

Without manure the herbage was very complex, and though diminutive, a relatively large proportion of the plants developed stem, showed tendency to seed formation, and would be comparatively mature at the time of cutting. The complex herbage contained a comparatively large proportion of other than gramineous species, of which the Leguminosæ would be nearly twice as rich in nitrogen as the grasses, and the miscellaneous species would also be somewhat richer. The result is that without manure there was, as already pointed out, an average over the 20 years of nearly twice as much nitrogen as would be obtained in a gramineous crop growing separately, such as wheat or barley.

With the mixed mineral manure, including potass, used alone, there was, it will be remembered, besides a considerable increase of produce due to gramineous species, a proportionally much greater increase of leguminous herbage, and also some increase due to miscellaneous species. Under these conditions, there was a considerably increased amount of nitrogen taken up without any being supplied in the manure, and it was concluded that it was derived from the stores of the soil itself.

With ammonia-salts alone, on the other hand, there was scarcely a trace of leguminous herbage, and very little due to any miscellaneous species, except *Rumex acetosa*. The produce was very characteristically gramineous. In the later years nearly one-half consisted of one species, *Festuca ovina*, and about a quarter of *Agrostis vulgaris*. This almost exclusively, and very simple, gramineous herbage showed, moreover, very little tendency to form stem and seed, or to mature, but consisted chiefly of stunted, and very dark green, leafy growth. This colour indicates a high percentage of nitrogen in the dry substance, or more properly speaking a deficient assimilation of carbon in proportion to the nitrogen taken up, owing to deficient mineral supply. The result was a considerable increase in the amount of nitrogen taken up compared with that without manure, but still, less than with the mineral manure alone without nitrogen.

With the mixture of the nitrogenous and mineral manures, on the other hand, the produce was still more gramineous than with the ammonia-salts alone; in fact, the weight per acre of gramineous herbage averaged more than twice as much; the proportion of the produce due to both leguminous and miscellaneous species was also less, though the average weight per acre of both was rather more. But here, the still more characteristically gramineous herbage was made up in much less degree of a few species only. Thus, whilst *Festuca ovina* and *Agrostis vulgaris* contributed by far the larger proportion of the total herbage grown by ammonia-salts alone, and *Anthoxanthum odoratum, Holcus lanatus,* and *Dactylis glomerata* were only very moderately, and every other grass very sparingly, represented, with the ammonia-salts and mineral manure together both *Festuca ovina* and *Agrostis* were much less prominent, *Holcus* and *Dactylis* were nearly as much so, and *Poa pratensis* was the most prominent of all. The character of development of the grasses was, moreover, totally different; there being, when mineral manures, including potass, as well as ammonia-salts, were used, a great tendency to form stem and seed, and to mature. Under these conditions a much larger proportion of the supplied nitrogen was taken up than when the ammonia-salts were used alone.

Owing, however, to the totally different proportion of gramineous, leguminous, and other species in the herbage without manure, or with mineral manures alone, on the one hand, and with ammonia-salts alone, or ammonia-salts with mineral manure on the other, it is very difficult to decide on what basis to estimate how much of the nitrogen taken up where it was artificially supplied was due to natural sources, and how much therefore is to be attributed to the supplies by manure when the ammonia-salts were used alone, and how much more when these were used in conjunction with the mineral manure? It is probable that, even when used in conjunction with the mineral manure, not much more than half the supplied nitrogen was taken up, so that the reduction in produce over the second 10 years would not seem to be connected with a deficiency of available nitrogen.

The reduction in the amount of total mineral matter taken up was greater than in that of the nitrogen. The falling off in produce would, so far, seem more probably connected with a deficiency of available supply of one or more of the mineral constituents. Calculation shows, however, that more, and generally much more, of each constituent, with the one exception of silica, was applied each year than was removed, so that there would be a constantly increasing residue accumulated during the later years in all cases in which there was no material loss by drainage. Again, as will be seen further on, when the same mineral manures were used with an increased amount of ammonia-salts, a larger amount of every one of the mineral constituents, excepting lime, was taken up; and of this, very much more was annually supplied than was taken up. Nor can the falling off be attributed wholly to less favourable seasons during the second 10 years; for, as the next illustrations will show, some other combinations of manure yielded more hay, containing more nitrogen, and more mineral matter, over the second period than over the first. It should perhaps be noted that one end of the plot was apparently to the eye to some extent injured by the roots of a large tree, and this injury was the greater in the later years.

The actual falling off in the amount of mineral constituents taken up over the second period was considerable, and it was pretty equal in the soda, the chlorine, the lime, and the sulphuric acid; it was about twice as great in the silica, but very slight indeed in either the potass or the phosphoric acid. The decreasing amount of soda, chlorine, and sulphuric acid, taken up and retained, with at the same time an all but maintained assimilation of potass and phosphoric acid, would indicate increased tendency to stem and seed formation, and to maturation; whilst, the deficiency of lime, and of silica, is also consistent with a less amount of leaf and greater amount of stem. In fact, the percentage in the dry matter—of lime, magnesia, soda, sulphuric acid, and silica, was lower than in the produce either without manure or with the ammonia-salts alone; whilst that of the potass was nearly twice as high as without manure, and more than twice as high as with the ammonia-salts alone, and that of the phosphoric acid was also very much higher than either without manure, or with ammonia-salts alone.

The probability is, that the reduced assimilation of both nitrogen and most of the mineral constituents over the second period was less connected with any real deficiency of either within the soil, in a condition capable of being yielded up, than with the description of plants favoured, and the character both of their underground and their aboveground development, with which would vary, not only the range and power of collection from the resources of the soil, but also the requirement for this or that individual constituent. And it is probable that, in addition to these conditions affecting the whole plot, some of the reduced assimilation is to be explained by the action of the tree-roots above referred to.

8. 400 *lbs. Ammonia-Salts, with Mixed Mineral Manure containing Potass, and 2000 lbs. cut Wheat Straw; Plot* 13.

Table X. shows the results obtained over the same 10, 10, and 20 years, on plot 13, with the same quantity of ammonia-salts, and the same description and amount of mineral manure, applied every year, as on the last plot considered (9), but with 2000 lbs. of cut wheat straw per acre per annum in addition. The chief object of the application of the straw-chaff was to supply silica, and carbonaceous organic matter, somewhat in the same condition as they are provided in dung. It obviously also provides some other mineral constituents, and a little nitrogen, but all presumably in an only slowly available condition.

TABLE X.—Average, per acre per annum, by 400 lbs. Ammonia-Salts, with Mixed Mineral Manure containing Potass, and 2000 lbs. cut Wheat Straw; Plot 13.

	Average per acre per annum.		
	Plot 9. Mineral manure and 400 lbs. ammonia-salts.	Plot 13. As plot 9, and 2000 lbs. cut wheat-straw.	Plot 13 + or − plot 9.
HAY.			
	lbs.	lbs.	lbs.
First period, 10 years, 1856–1865	6002	6186	+ 184
Second period, 10 years, 1866–1875	5421	6679	+1258
Total period, 20 years, 1856–1875	5711	6432	+ 721
Second period, per cent. + or − first period . . .	− 9·7	+8·0	
NITROGEN.			
First period, 10 years, 1856–1865	75·7	79·5	+ 3·8
Second period, 10 years, 1866–1875	70·7	87·3	+ 16·6
Total period, 20 years, 1856–1875	73·2	83·4	+ 10·2
Second period, per cent. + or − first period . . .	− 6·6	+9·8	
MINERAL MATTER (ASH).			
First period, 10 years, 1856–1865	368·3	389·2	+ 20·9
Second period, 10 years, 1866–1875	301·0	382·8	+ 81·8
Total period, 20 years, 1856–1875	334·6	386·0	+ 51·4
Second period, per cent. + or − first period . . .	−18·3	−1·6	

There is here considerably more average produce of hay, and more of nitrogen and mineral matter removed, over the 20 years, than by the same manures without the straw. There is also, instead of a reduction, a considerable increase, in the yield of hay and of nitrogen, and a nearly equal yield of mineral matter, over the second 10 years compared with the first.

The exact explanation of the much greater productiveness where the cut straw was used is not very obvious, but it may be well be enumerate some of the coincidents of this result.

Plot 13, like plot 12, the duplicate unmanured plot, is earlier shaded from the afternoon sun than the remaining plots, and plot 12 gave, every year of the 20, excepting the first two and the twelfth, more produce than the other unmanured plot (3). Plot 13 again, though it gave less produce than plot 9 during the first three years, has, with one slight exception, given more every year since, and in the latter half of the period very much more. But, besides any advantage from position and shade of plot 12 over plot 3, the examination of, and the determinations of nitrogen in, the subsoil of plot 12 showed it to be much more mouldy and much richer than that of plot 3, and it was concluded that the plot consisted, in great part at any rate, of made or moved ground, the site possibly of a filled-up hollow, or of a removed and levelled plantation or hedge green. Neither the sections nor the analyses of the subsoil of plot 13 showed any such characters, however, so that, independently of manure, the only known difference between it and plot 9 was in its earlier afternoon shade.

The herbage of plot 13 has become even more prominently gramineous than that of plot 9. Leguminosæ have almost equally disappeared on both; and the Miscellaneæ have also greatly reduced on both, *Rumex acetosa* being the only very prominent weed on either, *Conopodium denudatum* coming next. The increase of produce on plot 13 was, in fact, almost exclusively due to an increase in gramineous herbage, and this, compared with that of plot 9, contained very much more *Dactylis g.*, which here becomes the most prominent species; about the same proportion of *Agrostis v.*, considerably less *Holcus l.*, very much less of *Festuca o.*, and especially of *Poa prat.*; but, on the other hand, a fair proportion of *Alopecurus p.*, which is scarcely represented on plot 9; whilst, *Avena pub.* and *flav.*, and *Poa triv.* have gone down very much on both plots. Thus, compared with that of plot 9, the produce of plot 13 (with the straw) was somewhat differently made up, but it comprised a considerable proportion of several of the freer-growing grasses. These, too, manifested considerable tendency to form strong seed-stems.

These characters indicate liberal nitrogenous and mineral supply within the soil, and as they predominated more on plot 13 than on plot 9, it would be supposed that these conditions prevailed on it the more.

Supposing the whole of the nitrogen of the staw became available, it would supply about 9 lbs. per acre per annum, and presumably a smaller proportion of it at first, and more and more each year. Consistently with this, there was either no excess of yield of hay and of nitrogen, or but little, on plot 13 compared with plot 9, in the earlier years, but a considerable excess over the latter half of the period; and there was an increase of about 10 lbs. of nitrogen per acre per annum over the 20 years. It is probable, however, that, owing to various conditions favourable to luxuriance, the growing plants were enabled to avail themselves of more also of the other supplies of nitrogen within the soil.

Of mineral matter, the crop of plot 13 took up, over the first 10 years rather more than 20 lbs., over the second 10 rather more than 80 lbs., and over the 20 years rather more than 50 lbs., per acre per annum, more than that of plot 9. There was some increase in the amounts taken up of all the mineral constituents, but the most marked was—of potass an average of about 23 lbs., of phosphoric acid about 4 lbs., and of chlorine about 8 lbs., with in each case more over the second period than the first.

There was, too, an average of about 8 lbs. more silica annually taken up on the straw plot, but less over the second period than the first. Of lime, magnesia, and phosphoric acid, the straw applied would contain considerably more, and of silica about eight times more, than the increased amount of them taken up where it was used. But of potass the amount contained in the straw would be much less than the increased amount of it taken up. There is evidence here, again, that owing to the increased activity of growth from whatever cause, the growing herbage had availed itself of more of the other supplies within the soil than those actually contributed by the straw itself.

Something may be due to the "mulching" effect, or protection given by the straw to the young shoots, and something (though the effects obtained by sawdust would not lead us to expect much) to a slightly increased supply of carbonic acid to the young plants, and to the soil. At any rate the herbage shows a somewhat better colour in the early spring.

Upon the whole, there can be little doubt that some of the increased luxuriance under the influence of the straw-manuring is to be attributed to an increased supply of manurial constituents, and some may be due to its mechanical effects. But the question suggests itself—whether something may not be also due to the position of the plot as above referred to, and something possibly also to an unknown difference in the character of the soil and subsoil? Finally, it must not be overlooked that if the combination of the conditions of manure, soil, and local circumstances, was more favourable to certain freer-growing species, these, as a coincident of the character of freer growth, would necessarily be able to avail themselves more than others could do, of the same supplies within the soil. In fact, something is to be attributed to the greater powers of food-collection of the species favoured, as well as to the somewhat increased supplies by manure. With an increased faculty to take up food, more of it is taken up than would result from the mere increase of supply; that is, more than would be the case were one and the same species, with one and the same habit and functional capacity, growing on the two differently manured plots.

9. 800 *lbs. Ammonia-Salts, and Mixed Mineral Manure containing Potass; Plots* 11–1 *and* 11–2.

The next illustrations show the effects of a considerably increased supply of ammonia-salts, used in conjunction with the same mixed mineral manure, including potass, as on plot 9. During the first three years double the amount, or 800 lbs. per acre, of ammonia-salts, containing about 200 lbs. of ammonia, equivalent to about 164 lbs. of nitrogen, was annually applied. This quantity, from its effects, appearing excessive, was then reduced to one-half, or to the same as on plot 9, for three years. But, as it then appeared that the maximum growth possible was not attained, the quantity was again doubled, and so has remained up to the present time. At the period of this change, too, that is, after the first six years, the plot (11) was divided, and to one-half, in addition to the ammonia-salts and the usual mixed mineral manure, artificial silicates were thenceforward applied; for nine years (1862–1870), 200 lbs. of a crude silicate of lime, and 200 lbs. of a crude silicate of soda, and afterwards 400 lbs. of the silicate of soda, per acre per annum, were employed.

The following table shows the produce of hay, of nitrogen, and of mineral matter, over the first 10, the second 10, and the total period of 20 years, on each of the two portions of the plot—11–1 continuously without, and 11–2 for the last 14 years with, the silicate. For comparison there is also again given, the results obtained on plot 9, with the same mixed mineral manure and only 400 lbs. of ammonia-salts, per acre per annum, over the whole period.

TABLE XI.—Average, per acre per annum, by 800 lbs. (or 400 lbs.) of Ammonia-Salts, and Mixed Mineral Manure, without, and with, the addition of Silicates; Plots 11–1 and 11–2.

	Average per acre per annum.				
	Plot 9. Mineral manure, and 400 lbs. ammonia-salts.	Plot 11–1. Mineral manure, and 800 lbs.* ammonia-salts.	Plot 11–2. As plot 11–1, and silicates.	Plot 11–1. + or − plot 9.	Plot 11–2. + or − plot 11–1.
HAY.					
	lbs.	lbs.	lbs.	lbs.	lbs.
First period, 10 years, 1856–1865	6002	6913	7077	+911	+164
Second period, 10 years, 1866–1875 . . .	5421	6006	6909	+585	+903
Total period, 20 years, 1856–1875	5711	6459	6993	+748	+534
Second period, per cent. + or − first period	−9·7	−13·1	−2·4		
NITROGEN.					
First period, 10 years, 1856–1865	75·7	102·7	103·7	+27·0	+1·0
Second period, 10 years, 1866–1875 . . .	70·7	103·5	111·7	+32·8	+8·2
Total period, 20 years, 1856–1875	73·2	103·1	107·7	+29·9	+4·6
Second period, per cent. + or − first period	−6·6	+0·8	+7·7		
MINERAL MATTER (ASH).					
First period, 10 years, 1856–1865	368·3	420·7	435·9	+52·4	+15·2
Second period, 10 years, 1866–1875 . . .	301·0	339·7	392·1	+38·7	+52·4
Total period, 20 years, 1856–1875	334·6	380·2	414·0	+45·6	+33·8
Second period, per cent. + or − first period	−18·3	−19·3	−10·1		

Thus, an increased supply of ammonia-salts, with the same mineral manures as on plot 9, has considerably increased the annual produce of hay, but in a much greater degree the assimilation of nitrogen, over a given area; and it has increased, in about the same proportion as the increase of hay, the amount of total mineral matter taken up over that on plot 9, where the same amount was applied by manure. The addition of silicates during the last 14 years of the 20 has further increased the amounts of hay yielded, and of nitrogen and mineral matter taken up.

---

* 400 lbs. only in 1859, 1860, and 1861.

Bearing in mind that in the fourth, fifth, and sixth, of the first 10 years, only the same amount of ammonia-salts was applied on the plots 11 as on plot 9, and that the double amount was applied in each of the last 10 years, it will be observed that there was nevertheless a considerable falling off in the produce of hay, and in the quantity of mineral matter taken up, during the second 10 years, where the silicates were not also applied. Where the silicates were applied, however, the falling off in the produce was very trifling, and that of the mineral matter taken up much less than where they were not applied; whilst, under their influence, there was also a considerably greater increase in the amount of nitrogen taken up over the second 10 years.

With regard to the nitrogen, it is remarkable that, with the enormous amount of about 200 lbs. of ammonia, corresponding to about 164 lbs. of nitrogen, applied per acre per annum, for so many years, there was, so far as can be judged, as large if not a larger proportion of that supplied taken up by the growing herbage as where only half the quantity was employed. There was, in fact, a proportionally much greater increase in the assimilation of nitrogen than in the amount of total growth—that is, produce of hay—by doubling the application. In other words, the percentage of nitrogen in the produce was very much increased. It was, indeed, very abnormally high; and, as already pointed out, such a condition indicates a deficient assimilation of other constituents in proportion to the nitrogen taken up. This result may be due to a deficiency of available mineral constituents, or to the limitation of the climatic characters of the seasons for the assimilation of a larger amount of carbon by the quantity and quality of the leaf-surface presented over a given area; or the effect may be due to a combination of these causes.

The further increased total yield of hay, of nitrogen, and of mineral matter, where the silicates of soda and lime were used, and at the same time a decrease over the second period compared with the first in the amount of mineral matter, but an increase in that of the nitrogen, taken up, would seem to indicate a relative deficiency of available supply within the soil of mineral constituents compared with that of nitrogen. Still, a limitation of the climatic characters of the seasons may have had some share in the limitation of the amount of carbon assimilated, or, in other words, of the amount of produce grown.

With a view, if possible, of further elucidating the conditions and the results of the growth induced, let us briefly consider—what was the difference in the botanical character of the herbage developed by the single and by the double application of ammonia-salts, and without and with silicates, respectively? and what were the chief differences in the chemical composition of the hay?

In the unmanured hay there are nearly 50, and in that grown by mineral manures alone (including potass) more than 40, species represented. In that grown by the mineral manure and the smaller amount of ammonia-salts (plot 9) there are, however, scarcely 30; and in that by the double amount of ammonia-salts and the mixed mineral manure, both without and with silicates, there have been, in recent years, less than 20, and even as few as 16. The reduction in the number of species where mineral manures and ammonia-salts are used together is in the gramineous, the leguminous, and the miscellaneous, but chiefly in the miscellaneous species; and, comparing the produce of plot 9 with the single, and of the plots 11 with the double amount of ammonia-salts, the reduction, both in number, percentage, and actual weight, is again very prominently in the miscellaneous species. In fact, the produce on the plots 11 has become almost exclusively gramineous, and considerably more so than on plot 9 with the smaller quantity of ammonia-salts.

The most prominent grass on the plots 11 with the excessive amount of ammonia-salts (as was the case on plot 13 with half the quantity and the straw chaff) has been, both actually and compared with plot 9, the very free-growing *Dactylis glomerata;* but, of late years, it has appeared to be declining, especially on plot 11–2, with the silicates, where it seems to be giving place largely to *Alopecurus pratensis,* with which occur in considerable quantity, and each increasing on both plots, *Agrostis vulgaris, Holcus lanatus,* and *Avena elatior.* The five grasses enumerated make up a very large proportion of the total produce, whilst *Poa pratensis* has gone down much, and *Poa trivialis* has almost disappeared. On plot 11–1, without the silicates, the *Dactylis* has also lost the first place; but instead of *Alopecurus,* which is somewhat going down, *Agrostis* is coming forward the most prominently; *Holcus* and *Avena elatior* coming next, and increasing. On the other hand, *Festuca ovina,* though still occurring in only small proportion, seems to be rather gaining ground on both plots. The grasses which are the most strikingly decreasing, both without and with the silicates, are the two Poas, especially the *trivialis,* and, without the silicates, perhaps the *Alopecurus.*

On all three plots, 9, 11–1, and 11–2, there has been either no leguminous herbage found, or only a small fraction of 1 per cent. of it.

The most prominent species belonging to the "miscellaneous" orders is the *Rumex acetosa,* which, however, seems to be going down on all, but the most on the plots 11. Next in order of prevalence are *Conopodium denudatum* and *Achillea millefolium,* both of which are decreasing, and on the plots 11 have nearly disappeared.

Thus, with the combination of mineral manures and ammonia-salts, the greater the quantity of the latter the more nearly does the produce become exclusively gramineous, and the more does it consist of a few of the most freely-growing grasses. These, too, take possession of the ground very much in tufts or patches, and grow coarse, strong seed-stems, and broad, flaggy, dark-green leaves. The herbage is, in fact, very coarse, often laid, and dead at the bottom before it is ripe; indeed, it generally matures irregularly and imperfectly, yielding hay of low quality. Though, if the grass were fed off young, or cut green for feeding, it would probably be fairly good food.

Coincidently with these botanical and other characteristics of the herbage grown by mineral manures and ammonia-salts, we have, as already stated, an abnormally high percentage of nitrogen in the produce when an excessive amount of it is supplied in the manure. There is also a somewhat high percentage of mineral matter in the dry substance of these large, coarse, unevenly ripened, and almost exclusively gramineous crops.

Further, the results of the ash-analyses, which will be fully considered subsequently, show some striking differences in the mineral composition of the produce. Thus, there is a strikingly lower percentage of lime, a lower percentage of magnesia, and, notwithstanding the produce is so prominently gramineous, a lower percentage of silica, in the dry substance of the hay, than where the growth is less forced, and the herbage is at the same time more mixed. Of potass and phosphoric acid, but especially of potass, there is, on the other hand, a considerably higher percentage in the dry substance of these coarse gramineous crops, as also there is of chlorine. Again, whilst the percentage in the dry substance, of lime, magnesia, soda, sulphuric acid, and silica, has decreased over the later compared with the earlier years, that of the potass and phosphoric acid (and also that of chlorine) has increased over the later years. The percentage of lime especially has decreased the more, but that of the soda, sulphuric acid, and silica, the less, the greater the amount of the ammonia-salts applied, and the coarser the herbage.

Comparing plots 11–1 and 11–2, both with the mixed mineral manure and the large amount of ammonia-salts, but 11–1 without, and 11–2 with silicates, the only noticeable difference in the percentage mineral composition of the herbage is, that there is a slightly higher percentage of silica in that where it was applied, and with this there is a less percentage of potass, and a less increased percentage of it over the later years. There is, at the same time, a somewhat less excessive percentage of nitrogen where the silicates were used. The increased actual amount per acre of mineral matter removed where the silicates were applied was, however, much greater than is represented by the increased amount of silica taken up (which was only about 7 lbs. per acre per annum out of the 400 lbs. of crude silicates used). Nor is this result to be explained by supposing that there was a deficiency of available soda and lime where the silicates were not applied, since but a small proportion of that otherwise supplied of either had been utilised; and there was, therefore, unless lost by drainage, an annually increasing residue of them accumulating within the soil. There can, indeed, be little doubt that, in the presence of the excessive supply of nitrogen, and the increased activity of growth induced by it, the silicates of soda and lime employed were effective in other ways than merely as supplies of either silica, soda, or lime, to the plant. The effect was probably due in part to reactions of the alkaline silicates within the soil; for, under their influence there was more nitrogen, magnesia, potass, phosphoric acid, sulphuric acid, and chlorine, as well as more silica, lime, and soda, taken up; and, with these, more carbon assimilated.

In reference to the differences in the mineral composition of the hay grown under different manurial conditions, and accordingly possessing very different botanical and other characteristics, it is to be borne in mind that the dry matter of the species of the "Miscellaneous" orders generally contains a higher percentage of mineral matter than that of the leguminous herbage, and the leguminous a higher percentage than that of the gramineous herbage; and, further, that the less matured the produce, the higher, as a rule, will be the percentage of mineral matter in its dry substance. Again, the ash of the leguminous herbage contains the highest, that of the miscellaneous a lower, and that of the gramineous the lowest percentage of lime. On the other hand, the ash of the gramineous herbage is richer in potass than that of either the miscellaneous or the leguminous, and it will be the richer in potass, and the less rich in lime, the greater the proportion of stem to leaf. Hence, we should expect, as we find, less lime and more potass in the coarse, almost exclusively gramineous, and stemmy herbage.

In conclusion, although with the mixed mineral manure and the double amount of ammonia-salts the large average amount of about 3 tons of hay has been annually obtained over a period of 20 years and more, and an even somewhat larger proportion of the supplied nitrogen was taken up than when only half the quantity was applied, there was not a corresponding increase in the amount of vegetable matter grown. There was, nevertheless, annually much more nitrogen, and much more of every one of the mineral constituents, supplied in the manure than was contained in the increase of crop. That the produce was not greater would appear, therefore, to be due to other conditions than a deficiency of any of the constituents derived from the soil, unless, indeed, of silica in an available form. It was more probably due to defective atmospheric, that is climatic, conditions, limiting the assimilation of carbon; in other words, to a limitation in the amount, and adaptation, of the light, heat, and moisture, necessary for the assimilation over a given area of a larger amount of that substance. And, as there has been, not only a less amount of growth in proportion to the nitrogen taken up, but, notwithstanding an annually increasing manurial residue within the soil, a more or less declining amount over the later years, it would appear not improbable that, with the characters of the herbage described, there has been a less capability of the plants, either to gather up the accumulating stores within the soil, or to take full advantage of such atmospheric conditions as did exist.

10. 550 *lbs. Nitrate of Soda, with Mixed Mineral Manure containing Potass; Plot* 14.

We have already compared the effects of a given amount of nitrogen applied as ammonia-salts with those of the same amount applied as nitrate of soda, when each was used alone. We have now to compare their effects when each is employed in conjunction with the usual "mixed mineral manures." The comparison will be between the results on plot 9, with the mineral manures and the nitrogen applied as ammonia-salts, and those on plot 14, with the same mineral manures and the same quantity of nitrogen applied as nitrate of soda. As, however, the experiments with the nitrate were not commenced until two years later than those with the ammonia-salts, the periods selected for illustration will be 8, 10, and 18, instead of 10, 10, and 20 years.

TABLE XII.—Average, per acre per annum, by 550 lbs. Nitrate of Soda, and Mixed Mineral Manure, including Potass; Plot 14.

	Average per acre per annum.		
	Plot 9. Mineral manure and ammonia-salts.	Plot 14. Mineral manure and nitrate soda.	Plot 14 + or − plot 9.
HAY.			
	lbs.	lbs.	lbs.
First period, 8 years, 1858–1865	5904	5944	+ 40
Second period, 10 years, 1866–1875	5421	6777	+1356
Total period, 18 years, 1858–1875	5636	6407	+ 771
Second period, per cent. + or − first period . . .	− 8·2	+14·0	
NITROGEN.			
First period, 8 years, 1858–1865	75·4	67·6	− 7·8
Second period, 10 years, 1866–1875	70·7	70·6	− 0·1
Total period, 18 years, 1858–1875	72·8	69·3	− 3·5
Second period, per cent. + or − first period . . .	− 6·2	+ 4·4	
MINERAL MATTER (ASH).			
First period, 8 years, 1858–1865	356·1	357·2	+ 1·1
Second period, 10 years, 1866–1875	301·0	373·2	+ 72·2
Total period, 18 years, 1858–1875	325·5	366·1	+ 40·6
Second period, per cent. + or − first period . . .	−15·5	+ 4·4	

When 550 lbs. of nitrate of soda were used alone there was considerably more yield of hay, of nitrogen, and of mineral matter, than when 400 lbs. of ammonia-salts were employed, and there was also a much less reduction of yield over the later compared with the earlier years. Here, again, when each of the nitrogenous manures is used in conjunction with the mixed mineral manure, we have a much greater produce of hay when the nitrogen is applied as nitrate of soda than when as ammonia-salts. There is also more mineral matter taken up under the influence of the nitrate, but rather less nitrogen. And, whereas with the ammonia-salts there is a decline in annual yield of hay, and in the quantity of nitrogen and mineral matter taken up, over the second period compared with the first, there is, under the influence of the nitrate of soda, an increase in each of the three items over the later period.

When considering the effects of the two nitrogenous manures used alone, attention was called to the fact that the nitrogen of the nitrate of soda distributed much more rapidly through the soil than that of the ammonia-salts (p. 306), and that, coincidently, not only were those plants favoured by the nitrate which could the most rapidly take up the supplies near the surface, but those also which had a tendency to distribute their roots in the deeper layers; and hence, under their influence, there was a greater development of root in the lower layers, and with this the growing herbage acquired possession of wider and deeper ranges of soil and subsoil.

This was strikingly illustrated in the results obtained on the two plots now under consideration (14 and 9) in the season of drought of 1870. In that year the produce of hay on plot 9, with the mixed mineral manure and ammonia-salts, was nearly 23 cwts. below its average amount, whereas on plot 14 it was less than 1½ cwt. below its average. In view of this extraordinary difference of result from the same mineral manure and the same amount of nitrogen applied, not only the description of plants grown, but the distribution of the roots was examined; and samples of the soils were taken, every nine inches, down to a depth of 54 inches, on these and on the unmanured plot, for the purpose, among others, of determining the amount of moisture remaining in the soil at the different depths.

The following table shows the percentage of moisture (as determined by drying at 100° C., and including the loss by evaporation during preparation for analysis) in the samples of the different soils at the different depths.*

TABLE XIII.—Samples of soil collected July 25–6, 1870.

Depth of sample.	Per cent. moisture.		
	Plot 3. Without manure.	Plot 9. Mineral manure and ammonia-salts.	Plot 14. Mineral manure and nitrate soda.
First nine inches . . .	10·83	13·00	12·16
Second nine inches . .	13·34	10·18	11·80
Third nine inches . . .	19·23	16·46	15·65
Fourth nine inches . .	22·71	18·96	16·30
Fifth nine inches . . .	24·28	20·54	17·18
Sixth nine inches . . .	25·07	21·34	18·06
Means	19·24	16·75	15·19

Now, if we assume, as is sufficiently near the truth for the purpose of illustration, that down to the total depth of 54 inches an acre of the soil would weigh (exclusive of stones) 18,000,000 lbs., it would result that, down to that depth, the soil of plot 9 (with the ammonia-salts) contained 200 tons, and that of plot 14 (with the nitrate) 325 tons, less water per acre, than that of the unmanured plot to the same depth—quantities which correspond respectively, to about two, and three-and-a-quarter, inches of rain. And, from the great difference in the percentage of moisture at the lower depths, it may be concluded that the difference extended deeper still.

---

* *See* "Effects of the Drought of 1870 on some of the Experimental Crops at Rothamsted." (Jour. Roy. Ag. Soc. Eng., vol. vii., S.S. Part I.)

It would thus seem that the subsoil had contributed more water to the growing vegetation on the manured than on the unmanured land, and much more where the nitrate was applied than where the ammonia-salts were used. But the questions arise—if the unmanured subsoil retained so much more water, why did the crop suffer from the drought? and why did the crop manured with the ammonia-salts suffer so much more than that with the nitrate? The answer, briefly stated, is that different plants, of different habits of growth, prevailed on the respective plots, according to the conditions of manuring.

On the unmanured plot there was the greatest number of species, but those which prevailed had finer and less vigorous roots, which penetrated comparatively little below the surface soil; the raw clay of the subsoil was consequently much less changed, and it had yielded up very much less moisture to the growing crop.

On the plot manured with the mineral manure and ammonia-salts free-growing grasses predominated, but chiefly those whose underground habit rendered them dependent for their food and moisture in a great measure on the stores to be found in the surface soil, and in the upper layers of the subsoil. Still, owing to the increased vigour of growth under the influence of the manure, more moisture was obtained from the lower layers; probably, in part directly by the roots, and in part by the aid of capillary action induced by the pumping out of the upper layers; and the results indicated that the action extended beyond the depth to which the samples were taken.

On the plot manured with mineral manure and nitrate of soda, in that year one species, the *Bromus mollis*, contributed nearly half the produce; its wiry roots have a very characteristically downward tendency, and they were found to penetrate deeper than those of any other of the grasses, and, therefore, to have acquired command of lower layers than any associated with them; and this was especially so compared with the herbage on plot 9, with the ammonia-salts. The result was that the lower layers were pumped drier, and, under these circumstances, the drought affected the amount of crop but little.

In accordance with these facts as to the difference in the character of growth, and the amount of water found at different depths, great differences were observed in the characters of the soil and subsoil of the several plots.

The first nine inches of soil of the unmanured plot possessed the character of mould in nearly the same degree as that of the manured plots. The second nine inches was also very much altered from the character of the clay subsoil. Below this point very slight difference was observable, though the next, the third from the surface, perhaps showed slightly the least, and the lowest or sixth depth the most, of the bright clay tinge.

In the soil of the plot manured with the mineral manure and ammonia-salts the roots did not appear to penetrate much deeper than in that of the unmanured, but they were in greater quantity and of larger size. The first nine inches of soil was perhaps rather darker, and more mould-like; the second nine inches was decidedly more changed by vegetation than that of the unmanured plot, and the third and fourth were slightly so. The fifth and sixth were little distinguishable in colour from the raw reddish-yellow clay of the unmanured plot at corresponding depths.

The first and second nine inches of the soil of the nitrate-plot showed but little difference compared with those manured with ammonia-salts. The third, fourth, fifth, and sixth nine inches were, however, very different in appearance from the corresponding layers of either of the other plots, the clay being much mottled or veined, and when the samples were powdered they were of a yellowish-grey instead of reddish-yellow colour, and the lighter or less red the greater the depth. In the samples taken in 1876 the distinctions were by no means so marked; but still the subsoils of the nitrate-plot showed in the second, third, fourth, and fifth depths, a lighter, yellower, and less red tinge.

The percentage of nitrogen in the different soils, at the different depths, will be recorded and considered in detail further on; but it will be of interest briefly to notice the general bearing of the results in this place. Although the percentages (determined by soda lime) in the samples collected after a lengthened continuance of the drought in 1870 differ in some respects from those on the samples of 1876, the two series agree in showing, at every depth, a higher percentage on the ammonia-plot than on the unmanured, and, with one slight exception which may be accidental, a lower percentage in the soils of the nitrate than in those of the ammonia-plot, especially in the lower layers, where it is lower also than in the case of the unmanured plot, and indeed than in that of the majority of the other plots.

That the upper layers of the soil of the ammonia-plot should show higher percentages of nitrogen than those of either of the other plots is consistent with the fact that the nitrogen of ammonia-salts is, in the first instance, in a great degree arrested in the upper layers of the soil, and is so, much more than is that of nitrate of soda; and that the percentage should be also higher in the lower depths than without manure indicates, presumably, a gradual percolation of the supplied nitrogen in some form. With the rapidly distributing nitrate of soda, and with it the more deeply penetrating roots, there is, on the other hand, a less accumulation of nitrogen, not only in the upper but in the lower layers; and that the percentage should decrease so rapidly in the lower layers is probably partly due to passage upwards in dry, and, in greater measure, drainage downwards in wet, weather. Thus, it has been shown that in the drought more water passed upwards from the lower layers, and it would doubtless carry with it nitrate in solution; and with the greater solubility of the nitrate, and the greater disintegration, and consequently greater permeability of the subsoil under its influence, the loss of the supplied nitrogen by drainage would be the greater. Lastly, that the percentage of nitrogen in the lower layers should be even less in the case of the nitrate than of the unmanured plot may perhaps be partly due to more rapid oxidation, and more easy percolation and loss by drainage, of the nitrogen of previous accumulations, in the more disintegrated and more porous subsoil.

The main distinctions in the flora on plot 14 with the nitrate and plot 9 with the ammonia-salts are, that with the nitrate there was more, and an increasing amount, of *Alopecurus*, very much more *Bromus mollis* and *Poa trivialis*, and more of *Lolium perenne;* but, on the other hand, there was very much less *Poa pratensis* and *Festuca ovina*, much less *Agrostis*, and upon the whole less *Holcus*, with, in all, more total Gramineæ.

On neither plot is there, on the average of seasons, anything like 1 per cent. of leguminous herbage in the produce; but there is more on the plot with the nitrate than on that with the ammonia-salts.

Of herbage referable to other orders, the ammonia-salts have generally yielded more than the nitrate, but latterly not so much. The most prominent of these plants under the influence of the ammonia-salts is *Rumex acetosa;* next in order comes the *Conopodium denudatum*, and then the *Achillea millefolium;* others occurring in quite immaterial amounts. The chief of those developed on the nitrate-plot are, again, *Rumex acetosa*, and, in somewhat equal and increasing amount, *Anthriscus sylvestris*, the *Achillea millefolium* and *Conopodium denudatum* coming next in order, but each in less quantity than with the ammonia-salts. All others are in insignificant amount, but *Taraxicum officinale* occurs more plentifully than on plot 9.

With the striking differences in the amount of produce, in the amount of total mineral matter taken up, in the botanical composition of the herbage, in the character and distribution of the roots, and in the influence of the vegetation and the manures on the mechanical condition, and on the chemical composition, of the subsoil, accordingly as the nitrogen is applied as ammonia-salts or as nitrate of soda, it will be of interest briefly to call attention to the chief differences in the chemical composition of the produce.

As shown in Table XII., there was, with much more vegetable matter produced, even less nitrogen taken up and retained in the produce grown by the nitrate than in that by the ammonia-salts. The result was, a much lower percentage of nitrogen in the dry matter of the produce of the nitrate-plot; and, indeed, a much more normal percentage for this almost purely gramineous herbage. This lower percentage of nitrogen, with at the same time increased crop, implies of course an increased assimilation of carbon—that is to say more activity of growth—in proportion to the nitrogen taken up. There was, coincidently, more lime, magnesia, phosphoric acid, and sulphuric acid, considerably more silica, and very much more soda, taken up under the influence of the nitrate; but there was, on the other hand, considerably less potass, and very much less chlorine, taken up.

It has been before observed that, with an increased amount of soda applied in the manure, more was taken up and retained—a result which would be more marked in the case of imperfectly ripened vegetable matter, like hay, than in that of fully ripened products, such as cereal grain. It has also been observed that when there was an increased fixation of silica, it was generally in association with an increased amount of soda rather than of potass taken up. It was strikingly so in the case of the produce of plot 14. Thus, besides the much greater accumulation of soda which was so liberally applied, the plants growing under the influence of the nitrate accumulated not only more of each of the other mineral constituents (except potass) which were equally supplied to both plots, but more also of silica which was not supplied to either. This obviously indicates possession by the roots of the growing herbage of a greater range of soil and subsoil, and is consistent with the facts which have been pointed out as to the character of the root development, and the altered condition of the subsoil, under the influence of the nitrate of soda.

Since the produce grown by the mineral manures and nitrate was riper when cut than that by the mineral manures and ammonia-salts, a somewhat lower percentage in the dry substance, of potass, as well as of other constituents, would be expected; and it might merely indicate a greater accumulation of organic matter at the maturing period, by which, obviously, the percentage in the dry substance of the already accumulated mineral matter would be reduced. Such reduction is known to be the general accompaniment of favourable maturation. But here, the percentage of potass in the dry substance of the produce grown by the nitrate was, taking the average of the 18 years, only about four-fifths as much as in that grown by the ammonia-salts. This reduction is probably greater than can be accounted for merely by a greater accumulation of organic matter during the maturing period. Indeed, there was, not only a lower percentage, but, as already mentioned, a notably less actual amount per acre, of potass fixed under the influence of the nitrate, though it was equally supplied to both plots, and though the herbage was even more gramineous, and more stemmy—a result which is certainly somewhat remarkable. And although there was at the same time much more soda taken up, and the excess was more than equivalent to the actual deficiency of potass per acre, it was not sufficient to compensate for the deficiency of potass per cent. in the dry substance.

In connexion with these results, showing a greater amount of produce by the use of a given amount of nitrogen as nitrate of soda than as ammonia-salts, an increasing instead of a decreasing produce in the later years, a greater independence of drought, and a greater disintegration, increasing the porosity of the clay subsoil, it may be well to refer to some results obtained in experiments with barley. In the field at Rothamsted which is now (1879) growing the twenty-eighth crop of barley in succession, two plots received, in the first season, equal amounts of mineral manure; but in the second, third, fourth, fifth, and sixth years, the one received 550 lbs., and the other 275 lbs., of nitrate of soda per acre per annum. From that time up to the present, each has received only 275 lbs. per acre per annum. Yet, with one exception, in each of the 21 succeeding years, the plot to which the double amount of nitrate had previously been applied has yielded more produce; and in the twentieth year since the double application (1877), it yielded 3¾ bushels more corn, and 3¼ cwts. more straw, per acre. The excess of yield has also been more marked in the drier seasons. The soils and subsoils of these two plots have not been examined. But, judging from the results which have been described, the question suggests itself—whether the explanation be not that where the excessive amount of nitrate was applied in the earlier years, the subsoil was so acted upon, disintegrated, and rendered more porous, that it offered a greater surface for the retention of the otherwise easily washed-out nitrate, a greater surface for the retention of moisture, and greater permeability to the roots; thus increasing the store, both of available food and available moisture, at command of the plant, and facilitating the penetration of the roots in search of them?

To conclude, in regard to the greater effects of a given amount of nitrogen as nitrate of soda than as ammonia-salts when applied to the mixed herbage of grass land: it is obvious that the result is dependent on a great variety of circumstances. The nitrogen of the nitrate distributes much more rapidly through the soil and subsoil. The flora becomes greatly modified. Plants of different habits, both of aboveground and underground growth, are developed. Accordingly, the roots obtain possession of different ranges of soil and subsoil; and according to their range, and to their functional capability of food-collection, they gather up the more. With these favourable soil-conditions, the plant takes up more from the atmosphere within a given time. But, how far it does so simply as a result of a more favourable condition of the cell and sap, due to a more favourably balanced supply of nitrogenous and mineral food, inducing greater activity of a given leaf-surface, or how far it is that, conjointly with greater vegetative activity, there is, as there necessarily must be, also a constantly increasing above-ground surface, and hence a complex and cumulative action, is a question beyond the scope of our observed facts to determine. That the increased assimilation of carbon over a given area cannot be entirely accounted for by the supposition that the particular species of plants developed have a greater assimilative power for a given leaf-surface than those they have replaced, would appear from the fact that, when a given amount of nitrogen, as nitrate of soda and ammonia-salts respectively, is applied for one and the same description of crop—barley, for example—there is also found a greater fixation of carbon—that is more growth over a given area—under the influence of the nitrate.

11. 275 *lbs. Nitrate of Soda, with Mixed Mineral Manure, containing Potass; Plot* 16.

The next selection of results shows the effects of the mixed mineral manure with only 275 lbs. of nitrate of soda per acre per annum; that is, only half as much as was applied on plot 14, and containing, of course, only half as much nitrogen as the ammonia-salts on plot 9. This experiment was made on plot 16, and as in the case of the other nitrate plots it did not commence until 1858, so that, as before, we give the average results for only 8, 10, and 18 years; and for comparison those of plot 14 for the same periods are again brought forward.

TABLE XIV.—Average, per acre per annum, by 275 lbs. Nitrate of Soda, with Mixed Mineral Manure, containing Potass; Plot 16.

	Average per acre per annum.		
	Plot 14. Mineral manure and 550 lbs. nitrate soda.	Plot 16. Mineral manure and 275 lbs. nitrate soda.	Plot 16 + or − plot 14.
HAY.			
	lbs.	lbs.	lbs.
First period, 8 years, 1858–1865	5944	5058	− 886
Second period, 10 years, 1866–1875	6777	5332	−1445
Total period, 18 years, 1858–1875	6407	5210	−1197
Second period, per cent. + or − first period . . .	+14·0	+5·4	
NITROGEN.			
First period, 8 years, 1858–1865	67·6	63·0	− 4·6
Second period, 10 years, 1866–1875	70·6	62·4	− 8·2
Total period, 18 years, 1858–1875	69·3	62·6	− 6·7
Second period, per cent. + or − first period . . .	+ 4·4	−1·0	
MINERAL MATTER (ASH).			
First period, 8 years, 1858–1865	357·2	320·9	− 36·3
Second period, 10 years, 1866–1875	373·2	307·9	− 65·3
Total period, 18 years, 1858–1875	366·1	313·7	− 52·4
Second period, per cent. + or − first period . . .	+ 4·4	−4·1	

There was no plot with the mixed mineral manure and ammonia-salts in quantity containing only as much nitrogen as 275 lbs. of nitrate of soda. The main comparison must, therefore, be made with double the amount of nitrate (plot 14).

As with the double so with the single amount of nitrate (and the mineral manure), there was an average of more produce of hay over the last 10, than over the first eight of the 18 years. But, on the other hand, there is not, as there was with the double quantity, a greater average annual amount of nitrogen and of mineral matter removed over the second period. There was, however, only a slight deficiency; and, what is more remarkable, there is, over the 18 years, an average of only about 6½ lbs. less nitrogen annually removed than where the double amount of nitrate, supplying annually about 41 lbs. more nitrogen, was applied.

On both plots, 14 and 16, *Alopecurus pratensis* is plentiful and increasing. With the mineral manure and the smaller quantity of nitrate (plot 16) there was a larger proportion of *Holcus lanatus*, and much more of *Avena flavescens*, *Agrostis vulgaris*, and *Festuca ovina*, especially of the two latter; but there was a smaller proportion of *Lolium perenne*, considerably less *Dactylis glomerata*, and very much less of both *Poa trivialis* and *Bromus mollis*; in all, a less proportion of total Gramineæ than with the double amount of nitrate. There was, however, much more leguminous herbage, especially in the later years, and chiefly *Lathyrus pratensis*. There was also a larger, but a decreasing, proportion belonging to other orders; more *Ranunculus*, both *acris* and *bulbosus*, more *Conopodium denudatum*, more *Achillea millefolium*, and more *Rumex acetosa*, but much less *Anthriscus* than on 14.

In fact, on plot 16, with the smaller amount of nitrate, a greater variety of grasses contributed the bulk of the produce, but there was a less proportion of the more freely-growing species. The herbage was made up in considerably less proportion of Gramineæ, and it contained both a greater number of species, and a considerably greater percentage by weight, of those belonging to the leguminous and other orders.

As already mentioned, the average percentage of mineral matter in the dry substance is highest in the miscellaneous, lower in the leguminous, and lowest in the gramineous herbage. The dry matter of the gramineous herbage, again, contains the lowest average percentage of nitrogen, that of the miscellaneous a higher, and that of the leguminous much the highest. Hence, we should expect a higher percentage of both mineral matter and nitrogen in the dry substance of the less gramineous mixed herbage grown by the smaller amount of nitrate, and this is actually found. And, as the leguminous, and to some extent the miscellaneous herbage also, seems less influenced by, and less dependent on, artificial supply of nitrogen, the greater yield of nitrogen in proportion to that applied in manure may not indicate the utilisation of a correspondingly larger proportion of that supplied, but only that more has been gathered from natural sources.

It is obvious that, with a greater number of species, comprising those belonging to very characteristically different orders, the representatives of which have widely different habits of growth, not only above but underground, there will be a greater variety of root-distribution, a greater variety in food-collecting capacity, and probably, with these, a greater total capability of such collection. There is, accordingly, a higher percentage of lime, an equal percentage of magnesia, a considerably higher percentage of potass and phosphoric acid, a higher percentage of sulphuric acid, about an equal percentage of chlorine and of silica, but considerably less of soda, in the dry substance of the produce grown by the smaller amount of nitrate of soda. And, reckoned per acre, there is very nearly as much lime, potass, and phosphoric acid, and not much less magnesia, though considerably less sulphuric acid and chlorine, and very much less soda, and at the same time considerably less silica, gathered up under the influence of the smaller amount of nitrate.

It has before been shown, that a given amount of nitrogen applied as nitrate of soda gave more produce than the same amount applied as ammonia-salts, whether these nitrogenous manures were respectively used alone, or in conjunction with a mixed mineral manure supplying in excess all the mineral constituents of the crop, except silica. It was also shown that, when used alone—that is to say, when the whole of the mineral constituents (except soda) had to be obtained from the soil itself—275 lbs. of the nitrate yielded nearly as much produce, containing more of some, and nearly as much of all, of the mineral constituents, as when 550 lbs. were employed.

It is now seen that, when used in each case in conjunction with the mixed mineral manure, the smaller amount of nitrate does not give nearly so large an actual amount of produce as the larger; but the smaller amount does give a greater produce of hay, and a greater yield of nitrogen in the crop, for a given amount of nitrate used. Thus, with the mineral manure and the larger amount of nitrate there was, over the 18 years, an average per acre per annum, of about 57 cwts. of hay, containing a little over 69 lbs. of nitrogen; and with the smaller amount of nitrate there was an average of 46½ cwts. of hay, containing about 62½ lbs. of nitrogen. This larger yield for a given amount of nitrate used, was associated with, and doubtless greatly dependent on, the much greater complexity of herbage than when the more forcing excessive amount was employed. The herbage of plot 16, with the mineral manure and the smaller amount of nitrate, was, indeed, more complex than that of any other plot yielding the same weight of hay; its gramineous herbage was in a less degree made up of a few very freely-growing grasses; it contained a considerable and an increasing amount of leguminous herbage—much more than in the other cases with an equal amount of crop; and finally, it contained a large number, and a considerable percentage by weight, of miscellaneous species. Although the herbage of some of the best pastures of some of the best grazing districts of the country comprises but a small total number of species, it nevertheless includes a considerable proportion of other than gramineous species, and it is especially rich in Leguminosæ. Comparing the produce of the different experimental plots, however, as a rule, the more complex the herbage, the higher is the quality of the hay, and it is especially so when leguminous species are in fair proportion; and, doubtless, the quality of the hay of plot 16 would be higher than that of any other of the plots yielding an equal weight of produce. The result is, then, that with the mixed mineral manure and a not excessive amount of nitrate of soda, we have both a large actual amount of produce, a large amount in proportion to the nitrate used, and a comparatively high quality of the hay.

12. 400 *lbs. Ammonia-Salts, and Superphosphate of Lime; Plot* 4–2.

In the experiments hitherto considered in which nitrogenous and mineral manures have been used together, the mineral manures have consisted of a complex mixture, in some cases supplying an excess of all the mineral constituents of the crop except silica, and in some including silica. We have yet to consider the comparative effects of a given amount of nitrogenous manure, 400 lbs. ammonia-salts—when used alone (plot 5); when in conjunction with superphosphate of lime alone (plot 4–2); with superphosphate of lime, and salts of soda and magnesia, without potass (plot 10); and when with superphosphate of lime, salts of soda and magnesia, and potass also (plot 9).

The first comparison will be between ammonia-salts alone, and in conjunction with superphosphate of lime, plots 5 and 4–2; and as the experiment with the superphosphate did not commence until the fourth year, the periods selected are 7, 10, and 17 years.

TABLE XV.—Average, per acre per annum, by 400 lbs. Ammonia-Salts, and Superphosphate of Lime; Plot 4–2.

	Average per acre per annum.		
	Plot 5. Ammonia-salts, alone.	Plot 4–2. Ammonia-salts, and superphosphate.	Plot 4–2 + or − plot 5.
HAY.			
	lbs.	lbs.	lbs.
First period, 7 years, 1859–1865	3202	4440	+1238
Second period, 10 years, 1866–1875	2471	3414	+ 943
Total period, 17 years, 1859–1875	2772	3837	+1065
Second period, per cent. + or − first period . . .	−22·8	−23·1	
NITROGEN.			
First period, 7 years, 1859–1865	56·0	67·6	+11·6
Second period, 10 years, 1866–1875	47·3	57·8	+10·5
Total period, 17 years, 1859–1875	50·9	61·8	+10·9
Second period, per cent. + or − first period . . .	−15·5	−14·5	
MINERAL MATTER (ASH).			
First period, 7 years, 1859–1865	163·0	250·0	+87·0
Second period, 10 years, 1866–1875	108·9	162·3	+53·4
Total period, 17 years, 1859–1875	131·2	198·3	+67·1
Second period, per cent. + or − first period . . .	−33·2	−35·1	

Over the 20 years, ammonia-salts alone gave about one-fourth more produce than was obtained without manure; and the results quoted above show that, over the 17 years of the comparative trial, the mixture of ammonia-salts and superphosphate of lime gave more than one-third more produce than the ammonia-salts alone. Superphosphate of lime alone, however, gave scarcely any more produce than that without manure, doubtless owing to a deficiency of nitrogen available to such plants as were developed. But when the ammonia-salts were used alone, there was obviously, on the other hand, a deficiency of mineral supply for the amount of available nitrogen. The superphosphate of lime, supplying as it did, besides a considerable excess of phosphoric acid, also an excess of sulphuric acid and lime, and some magnesia, remedied this so far; but, as will be seen from the next comparisons, there was still a considerable deficiency of some other mineral constituent, or constituents, necessary to give anything like full effect to the amount of nitrogen supplied in the ammonia-salts.

The figures show that there was, not only much more total produce, but also much more nitrogen and total mineral matter annually removed, when the superphosphate as well as the ammonia-salts was used; but that, notwithstanding the different actual amounts of yield, there was, under the two conditions of manuring, very nearly the same percentage reduction in yield of hay, of nitrogen, and of mineral matter, over the last 10 compared with the first seven years. There is thus indicated the influence of a larger supply of mineral matter, but at the same time an almost equal rate of decline in the amount available as the experiment proceeded. Calculation of the analytical results shows, however, that the increase in the amount of total mineral matter removed under the influence of the addition of superphosphate was considerably greater than was represented by the increased amount taken up of those mineral constituents which it supplied—namely, lime, magnesia, phosphoric acid, and sulphuric acid. There was also more potass, in a greater proportion more soda, and more silica, taken up—all of which must have been supplied by the soil itself. It has before been pointed out that the produce by ammonia-salts alone removed even less potass than that without manure—a fact supposed to be explained by the greater root-range of the much more varied flora of the unmanured plot; and although the addition of superphosphate to the ammonia-salts causes the removal of considerably more potass, there is still not much more removed than without manure. There was, too, a very great decline in the average annual amount taken up of almost every mineral constituent over the second period compared with the first; the exceptions being phosphoric acid and magnesia, in which the reduction was much less. Further, whilst in the dry substance there was (excepting lime) an abnormally high percentage of the mineral constituents supplied, there was an abnormally low percentage of those not supplied—again indicating that the point of exhaustion of available supply was reached. With this evidence of exhaustion of mineral matter, there was an abnormally high percentage of nitrogen in the dry substance of the produce of both plots, but especially in that by the ammonia-salts alone.

With this chemical evidence of repletion of nitrogenous, and deficiency of mineral supply, the botanical character of the herbage was equally significant. On both plots it became more and more gramineous, and the more so where the superphosphate of lime was also used. On both, the Leguminosæ nearly disappeared, and, again, the more so where the superphosphate of lime was employed. On both, the miscellaneous herbage much declined, excepting that the *Rumex acetosa* was in some seasons very prominent. With the superphosphate and the greater predominance of the grasses, the number of species also declined more than with the ammonia-salts alone.

On both plots the *Festuca ovina* became by far the most prominent grass, constantly increasing, and in 1877 it made up more than half the total crop in each case. *Agrostis vulgaris* was the next in order of predominance, and has also gradually increased, and in 1877 the two grasses, *Festuca ovina* and *Agrostis vulgaris,* made up nearly 83 per cent. of the total produce grown by ammonia-salts alone, and within a fraction of 80 per cent. of that by ammonia-salts and superphosphate. On the plot with ammonia-salts alone, three other grasses, *Anthoxanthum, Holcus,* and *Dactylis,* together contributed about 10 per cent. more, and no other single grass as much as a quarter of 1 per cent. On the plot with the ammonia-salts and superphosphate, *Anthoxanthum, Alopecurus, Holcus, Avena elatior, Poa pratensis,* and *Dactylis,* together contributed an additional 14 per cent., and no other grass more than a small fraction of 1 per cent.

Thus, not only did the herbage consist mainly of two grasses, but the one which took the lead, *Festuca ovina,* is the prevalent plant on poor common-lands; and on both these plots both grasses, but especially the *Festuca ovina,* showed very inferior characters of development. They consisted chiefly of very dark green, leafy herbage, growing in patches or tufts, with very little tendency to produce stem and seed, and particularly in dry seasons, dying at the bottom, without properly ripening, and always yielding a soft, woolly, and very inferior hay.

13. 400 *lbs. Ammonia-Salts, and Mixed Mineral Manure, with and without Potass; Plots* 9 *and* 10.

The next table compares the results obtained on plots 9 and 10. During the first six years of the 20 both received annually 400 lbs. of ammonia-salts, and the "mixed mineral manure," including potass. The only difference was that plot 10 received also 2000 lbs. of sawdust per acre per annum during those six years, and again in the seventh year, but without effect. After the first six years, both plots were manured as before, with the important exception that, from that date, the potass was omitted from the manure of plot 10, and a somewhat increased amount of sulphate of soda was applied instead. The table shows the average produce of hay, and its contents of nitrogen and mineral matter, over the first six years when both plots were manured alike (excepting the sawdust on plot 10); over the next 14 years during which the potass was still applied to plot 9, but omitted on plot 10, and over the total period of 20 years.

TABLE XVI.—Average, per acre per annum, by 400 lbs. Ammonia-Salts, and Mixed Mineral Manure, with and without Potass; Plots 9 and 10.

	Average per acre per anunm.		
	Plot 9. With potass 20 years.	Plot 10. With potass 6 years, without 14 years.	Plot 10 + or − plot 9.
HAY.			
	lbs.	lbs.	lbs.
First period, 6 years, 1856–1861	6349	6223	− 126
Second period, 14 years, 1862–1875	5438	4725	− 713
Total period, 20 years, 1856–1875	5711	5174	− 537
Second period, per cent. + or − first period	−14·3	−24·1	
NITROGEN.			
First period, 6 years, 1856–1861.	78·7	73·3	− 5·4
Second period, 14 years, 1862–1875	70·9	72·7	+ 1·8
Total period, 20 years, 1856–1875	73·2	72·9	− 0·3
Second period, per cent. + or − first period	− 9·9	− 0·8	
MINERAL MATTER (ASH).			
First period, 6 years, 1856–1861	406·8	419·8	+13·0
Second period, 14 years, 1862–1875	303·7	243·8	−59·9
Total period, 20 years, 1856–1875	334·6	296·6	−38·0
Second period, per cent. + or − first period	−25·3	−41·9	

Of hay, the average produce per acre per annum over the 20 years was—without manure, 2383 lbs.; with ammonia-salts alone, 2946 lbs.; with ammonia-salts and superphosphate of lime (17 years), 3837 lbs.; with ammonia-salts and mixed mineral manure, with potass for six years and without potass 14 years, 5174 lbs.; and with ammonia-salts and the mixed mineral manure, including potass every year, 5711 lbs.

The table shows that, even where the application of sulphate of potass was continued, there was a falling off of annual produce amounting to 14·3 per cent. over the last 14 compared with the first six years. But where the potass was discontinued there was a falling off, over the same period, of 24 per cent.

It is remarkable that there was, over the 20 years, almost identically the same amount of nitrogen taken up where the potass was discontinued as where it was continuously applied, and during the period of the omission there was even rather more. There was, moreover, scarcely any reduction in the amount taken up over the second period compared with the first. Indeed, the greater average yield of nitrogen over the whole period in the produce with the more potass applied is due to the greater amount taken up during the earlier years, when both plots were practically manured alike. In regard to the even lower amount with than without the potass over the later period, it should be observed that the potass-plot (9) has probably suffered at one end, for some years, from the roots of a large tree, but to what extent it is difficult to say.

Of mineral matter, on the other hand, about one-fifth less has been taken off, over the 14 years, in the crop grown without than with potass; and the falling off in amount, compared with the earlier period, is nearly 42 per cent. without, against only 25 per cent. with, the potass.

Thus, notwithstanding there was even rather more nitrogen annually taken up on plot 10 over the 14 years during which the potass was excluded from the manure, there was much less hay produced, and much less mineral matter taken up, than on plot 9, where the application of potass was continued. There was an average annual deficiency of more than 700 lbs. of hay, and of about 60 lbs. of mineral matter; the former representing a deficiency of carbon assimilated averaging about 230 lbs. per acre per annum in the removed produce, to say nothing of the second crops, the roots, &c.

The deficiency of total growth on plot 10 compared with plot 9 was not manifested in any marked degree until after the first five years of the 14; in fact, during the first six years of the 20, a total of about 900 lbs. of potass per acre had been applied to each plot, and the total amount taken out in the crop was much less than this. It is to be supposed, therefore, that there was a considerable residue of the supplied potass possibly available for the succeeding crops. How much, however, it is impossible to decide absolutely; for, in the first place, we do not know whether as much, or more, or less, was given up from the soil itself in the presence of the liberal artificial supply than was taken up without manure, or by the ammonia-salts alone, or by the ammonia-salts and superphosphate of lime without potass or soda; nor, in the second place, do we know how much of the residue of the supplied potass would enter into such combinations within the soil as to remain either unavailable, or but slowly available to succeeding crops.

If we assume, by way of illustration, and as as probable an estimate as we are able to make, that the soil of plot 10, during the six years when it received ammonia-salts and mixed mineral manure containing potass, yielded up to the growing crop the same quantity from its own stores as plot 4–2 during the first six years in which it received the same quantities of ammonia-salts and superphosphate of lime, but without potass or soda, then the residue of potass supplied during the six years to plot 10 would be about 412 lbs. at the end of that period. Yet, there was a very marked deficiency of potass taken up on plot 10 compared with plot 9 even in the first year of the discontinuance of the application. But there was, on the other hand, a very marked increase in the amount taken up compared with that on plot 4–2, with the ammonia-salts and superphosphate only; and that increase continued, though diminishing, throughout the 14 years. And if, as above supposed, there were 412 lbs. residue of supplied potass within the soil at the commencement of the 14 years, and during those years the soil itself still yielded up as much as during the same period on plot 4–2, which had not received potass at all, there would still remain to be accounted for more than 200 lbs. of the supplied potass at the end of the 14 years; and it may be observed that, in each of the three succeeding years (1876–77–78), plot 10 gave considerably more produce than plot 4–2.

It would appear, therefore, that there was a considerable residue of potass remaining after the six years' application; that this was yielded up less freely than that from fresh supplies, but more freely in the earlier than in the later years; and that at the end of the 14 years a residue still remained, of which some still continued to be yielded up. That this should be so is quite consistent with results obtained on arable land, which, as already referred to, show, both in the amounts of crop and in its chemical composition, the effects of a residue of potass applied 25 years or more previously. Supposing, however, that in the presence of an artificial supply of potass less was yielded up from the soil itself, the residue remaining would of course be by so much less than 200 lbs. at the end of the 14 years.

About an equal number of species, and nearly the same species, were represented on the two plots 9 and 10—the one with the continuous, and the other with the temporary supply of potass; but on neither was the total number much more than half as many as without manure. As on plots 5 and 4–2, the one with ammonia-salts alone, and the other with ammonia-salts and superphosphate, so now when, with these, salts of potass, soda, and magnesia, are also used, the herbage has become more and more, and very prominently, gramineous. But whilst without the alkali-salts, in the later years about 80 per cent. of the total produce was made up of two grasses, yielding a stunted, dark-green, and almost exclusively leafy, herbage, with them the produce was very much more mixed, and the bulk was made up of many more grasses, few of which are in undue prominence.

Thus, the six grasses which have of late years become the most prominent on plot 9, with, besides the other manures, an excess of potass every year, are (and somewhat in the following order), *Poa pratensis, Agrostis vulgaris, Festuca ovina, Dactylis glomerata, Avena elatior*, and *Holcus lanatus;* and they together average more than 80 per cent. of the produce. Again, the seven which have become, or remain, the most prominent on plot 10, with the discontinued supply of potass, are—*Festuca ovina, Agrostis vulgaris, Alopecurus pratensis, Avena elatior, Poa pratensis*, and (in a less degree than any of the six on plot 9) *Holcus lanatus*, and *Dactylis glomerata*, which together make up about as much as the six on plot 9. The chief differences between the two plots are, that whilst *Alopecurus* has much increased on plot 10, there is scarcely any of it on plot 9; *Holcus* and *Poa pratensis* are the less prominent on plot 10; *Dactylis glomerata* has decreased on plot 10, but increased on plot 9; *Festuca ovina* has considerably increased on both plots, but the most on plot 10. On the other hand, *Avena pubescens*, *Avena flavescens*, *Poa trivialis*, *Bromus mollis*, and *Lolium perenne* have almost disappeared on both plots.

On both plots leguminous species are represented by only a small fraction of 1 per cent. of the total produce, and by less on plot 10 than on plot 9.

Miscellaneous species also have considerably decreased, and more in weight than in number, on both plots—and again the most on plot 10; excepting that the *Rumex acetosa*, which is the most, and very, prominent on both, is upon the whole the more prominent on plot 10. Next, but in much less quantity, come the *Conopodium denudatum*, and the *Achillea millefolium*, both of which are much declining, but are somewhat more prominent on plot 9 than on plot 10.

Thus, there was considerable similarity in the number and description of species on the two plots, and some, but less, in the proportion by weight of individual species; whilst, the weight per acre of total gramineous herbage was much less, and that of both the leguminous and the miscellaneous was also the less, on plot 10. The character of development also was extremely different. On plot 9, with the continuous supply of potass, the grasses showed much tendency to produce stem and seed, and to mature. On plot 10, on the other hand, with the only temporary supply of potass, the proportion of leaf to stem was very much greater, the herbage was patchy, of a much darker green colour, and matured unevenly, and imperfectly. There was, in fact, a relative plethora of nitrogen, and with the deficiency of potass a deficient assimilation of carbon.

With these great differences in the character of development apparent to the eye, there were corresponding differences in the chemical composition quite consistent with them and their cause.

In the dry substance of the produce grown with the deficient supply of potass there was a considerably higher, and an abnormally high, percentage of nitrogen, and a lower percentage of total mineral matter. There was a considerably higher percentage of lime, magnesia, phosphoric acid, and sulphuric acid; a quadruple percentage of soda; with this a considerably increased percentage of silica, and at the same time a very greatly increased percentage of chlorine. There was, on the other hand, in the dry substance of this leafy, highly nitrogenous, and immature produce, over the average of the 14 years, considerably less than half as high a percentage of potass as in that where it had been more liberally supplied, and where the herbage was better developed and more matured.

The actual amounts taken up per acre over the 14 years were, of lime, magnesia, phosphoric acid, sulphuric acid, and silica, much the same on the two plots; of chlorine one-and-a-half time, and of soda three-and-a-half times as much was taken up on plot 10, but of potass little more than one-third as much on plot 10 as on plot 9.

It will be of interest briefly to compare the results obtained on these plots, 9 and 10, with those on plots 7 and 8, on both of which the same "mixed mineral manure," including potass, was also applied during the first six years; then the same was continued on one plot (7), and the same, excluding potass, on the other plot (8), for the next 14 years. But whilst ammonia-salts were liberally applied equally to plots 9 and 10 throughout the whole period of 20 years, no nitrogenous manure was applied to plots 7 and 8.

On the plots with the ammonia-salts there was, both with and without the continuous supply of potass, much more produce, and much more nitrogen and mineral matter taken up, than on the corresponding plots without nitrogenous manure.

With the continuous supply of potass there was, when used with ammonia-salts, a considerable reduction in the produce over the later years; but when without the ammonia there was even a slight increase.

Where the application of potass was discontinued over the later period there was, both with and without the nitrogenous manure, a considerable reduction in the yield of hay over that period; but whilst with the nitrogenous manures there was no reduction in the amount of nitrogen taken up, there was, where there was no artificial supply of nitrogen, a reduction of about one-third in the amount of it taken up, in that case from natural sources, due to the discontinuance of the application of potass.

Both with and without the ammonia-salts the exclusion of the potass caused a great reduction in the amount of mineral matter taken up, and a greater, both actually and proportionally, where the ammonia-salts were used.

On the plots with ammonia, both with the continuous and the temporary supply of potass, the produce was chiefly gramineous, contained scarcely any leguminous herbage, but few prominent miscellaneous species, and the flora of the plot was not very materially affected by the discontinuance of the potass; but, without the potass, the gramineous herbage showed very different, and much inferior, characters of development. Without ammonia, on the other hand, and with the potass, the produce contained a large proportion of leguminous herbage; and the most prominent effect of the discontinuance of the potass was the reduction of the leguminous herbage to a very insignificant amount, whilst, at the same time, the character of development of the grasses was deteriorated.

Thus, there were very different amounts of produce, and the botanical character of the herbage was very different, accordingly as the mixed mineral manure, including potass, was employed with or without nitrogenous manure; and when the application of potass was stopped, the effect on the botany of the plots was, in some important respects, very different. Notwithstanding this, the effect of the exclusion of the potass was, both with and without ammonia, immediately and greatly to reduce the amount of potass taken up (and more or less that of some other constituents), and by degrees greatly to reduce the amount of produce also.

We have in these comparisons remarkably consistent and cumulative evidence of the importance of a liberal available supply of potass within the soil if we would grow large, properly-developed, and well-matured, hay crops. With a fairly mixed herbage, and only moderate nitrogenous manuring, a liberal supply of potass will increase, or a deficiency of it will greatly diminish, the growth of leguminous plants; whilst, at the same time, both the quantity and the character of growth of the gramineous herbage will be affected. Or, with a predominantly gramineous herbage, and full supply of nitrogen, a deficient supply of potass will much diminish the amount of produce, and consequently the efficiency of the nitrogenous manure, and it will, moreover, lead to a deterioration in the character of the growth.

14. *Mixed Mineral Manure alone* 7 *years; succeeding Ammonia-salts alone* 13 *years; Plot* 6.

The results next quoted will illustrate the effects of applying the mixed mineral manure, including potass, for seven years in succession on a plot which had received ammonia-salts, without mineral manure, in each of the 13 preceding seasons (with sawdust in addition the first seven years). The experiment was made on plot 6; and, for comparison, the average results, over the same 13, 7, and 20 years, are given—for plot 5, which received the same quantity of ammonia-salts (without sawdust) in each of the 20 years, as were applied to plot 6 during the first 13 years; and for plot 7, which received the same mixed mineral manure throughout the 20 years, as plot 6 received during the last seven years.

TABLE XVII.—Average, per acre per annum, by Mixed Mineral Manure alone 7 years, after Ammonia-Salts alone 13 years; Plot 6.

	Average per acre per annum.				
	Plot 5. Ammonia-salts alone 20 years.	Plot 6. Ammonia-salts 13 years, mineral manure 7 years.	Plot 7. Mineral manure alone 20 years.	Plot 6. + or − plot 5.	Plot 6. + or − plot 7.
HAY.					
	lbs.	lbs.	lbs.	lbs.	lbs.
First period, 13 years, 1856–1868	3317	3425	3914	+ 108	− 489
Second period, 7 years, 1869–1875	2257	3502	4040	+ 1245	− 538
Total period, 20 years, 1856–1875	2946	3452	3958	+ 506	− 506
Second period, per cent. + or − first period.	− 32·0	+ 2·2	+ 3·2		
NITROGEN.					
First period, 13 years, 1856–1868	57·8	59·1	56·9	+ 1·3	+ 2·2
Second period, 7 years, 1869–1875	43·0	41·9	55·9	− 1·1	− 14·0
Total period, 20 years, 1856–1875	52·6	53·1	56·6	+ 0·5	− 3·5
Second period, per cent. + or − first period.	− 25·6	− 29·1	− 1·8		
MINERAL MATTER (ASH).					
First period, 13 years, 1856–1868	171·8	175·6	254·5	+ 3·8	− 78·9
Second period, 7 years, 1869–1875	95·4	189·2	252·8	+ 93·8	− 63·6
Total period, 20 years, 1856–1875	145·1	180·4	254·0	+ 35·3	− 73·6
Second period, per cent. + or − first period.	− 44·5	+ 7·7	− 0·7		

The sawdust applied on plot 6 during the first seven years being of little or no effect, the two plots, 5 and 6, each annually receiving the same quantity of ammonia-salts, were practically duplicates. On both, the produce was gradually diminishing, or in other words the effects of a given amount of nitrogen supplied was decreasing, and the character of the herbage was very greatly deteriorating. Hence, after 13 years, it was decided to continue the application of ammonia-salts alone on one plot only (5), to discontinue it on the other (plot 6), and to apply to it the mixed mineral manure, including potass, instead. The objects sought in the change were—to determine its effects, not only on the amount of produce, but on the character of the herbage, both as to the description of species developed, and the character of development, also on the chemical composition of the produce; and especially to acquire data in reference to the question whether any, or how much, of the nitrogen of the previous applications which had not been already recovered in the increase of crop, would be so under the influence of liberal mineral manuring?

The table shows that whilst with the ammonia-salts alone every year the average annual produce of hay was nearly one-third less over the last seven than over the first 13 years, it was even rather higher over the last seven years than over the previous 13, where (on plot 6) the mixed mineral manure had been substituted for the ammonia-salts. Still, the amount of produce did not nearly reach that yielded on plot 7, where the same mixed mineral manures had been applied every year of the 20.

In the amount of nitrogen annually taken off there was a considerable reduction over the last seven compared with the first 13 years, both where the application of the ammonia-salts was continued, and where the mixed mineral manure was substituted. But it is significant that there was nearly as much annually taken up where the application of ammonia-salts had been stopped, and mineral manures applied instead, as where the ammonia-salts were still applied. In neither case, however, was there as much nitrogen taken up as over the same seven years where no nitrogenous manure, but the mixed mineral manure alone, had been applied from the commencement.

The reduction in the amount of mineral matter annually taken off was, where the ammonia-salts were continuously applied, very much greater than that of either the hay or the nitrogen. On the other hand, where the mineral manures were substituted there was even rather more mineral matter taken up over the seven years of their application than previously. But, as in the case of the produce yielded, and of the nitrogen removed, so also in that of the mineral matter, there was much less annually taken up than where the same mineral manures had been annually applied over the whole period.

With regard to the nearly equal amount of nitrogen taken up where the mineral manures succeeded the ammonia-salts, as where the nitrogenous manure was still applied, the question suggests itself—whether the result was merely due to the gathering up by the herbage already prevalent, of the unexhausted residue of the nitrogen of the previous applications—or, whether to the development of different plants, having more extended root-ranges, or different powers of collection from natural sources? For certainly it was due to the latter causes that where, as on plot 7, mineral without any nitrogenous manures were applied every year, there was yet yielded, over the 20 years, even rather more nitrogen in the produce than was obtained on plot 5, where 82 lbs. of nitrogen were applied per acre annually. It will be remembered that the surface-soil of the continuously mineral-manured plot showed a lower percentage of nitrogen than that of the continuously unmanured plot, indicating the source whence the increased yield of nitrogen in the mineral manured complex herbage had been derived.

The data at command do not enable us to give a decisive answer on these points, but it will be of interest briefly to notice the facts bearing upon them.

In the first place, not quite three-fourths as much nitrogen was taken off in the total produce of plots 5 and 6, during the 13 years of the application of the ammonia-salts to both, as was contributed by the manure during that period. And if we deduct from the nitrogen in those crops the amount obtained during the same period in the produce without manure, the so reckoned *increased* amount taken off, due to the action of the ammonia-salts, would be little more than one-fourth of that supplied. Again, during the last seven years of the 20, little more than half instead of three-fourths as much nitrogen was removed in the total produce of plot 5 as was supplied in the ammonia-salts during that period; or if as before we deduct the amount taken up without manure, less than 20 per cent. of that supplied would appear to have been recovered.

We have then, according to the mode of reckoning, from one-fourth to three-fourths of the supplied nitrogen unrecovered in the crops during the 13 years; and, as already intimated, there was almost identically the same amount of nitrogen taken up during the next seven years from plot 6, where the application of the ammonia-salts was stopped and mineral manures were applied instead, as on plot 5, where the application of the ammonia-salts was continued. Still, the total amount of nitrogen taken off in the produce of plot 6, over 20 years, was scarcely as much as had been supplied during the first 13 years, and if the yield without manure be deducted, more than 60 per cent. of that supplied during the 13 years remained unrecovered as increase in the 20 years.

Determinations of nitrogen, made by the soda-lime method, in the soils and subsoils of the unmanured plot 3, and of the continuously ammonia-manured plot 5, after the 20 years, did indicate some accumulation of nitrogen, so determinable, in the soil and subsoils of the ammonia-plot, down to the depths examined, namely, 54 inches, but far from sufficient to account for the otherwise unrecovered amount of supplied nitrogen; and, judging from somewhat parallel cases, it would be concluded that at any rate some would be retained by the soil and subsoil in an only slowly available condition, but that a considerable proportion of the supplied ammonia would be oxidated, and pass away in the drainage, as nitrites and nitrates, beyond the reach of the roots.

It would thus appear probable that after the 13 years' application of ammonia-salts to plot 6, some at any rate of the supplied nitrogen remained within the soil, and that some of this was available to the growing plants during the succeeding seven years. But that as much was taken up as where the ammonia-salts were still annually applied would seem to require some further explanation.

In reference to this point we have the significant fact, as will be fully illustrated further on, that the surface soil of plot 6 showed at the end of the 20 years—that is, after the mineral manures had been applied for seven years—a notably lower percentage of nitrogen than the corresponding layer of plot 5; thus pointing to a source of nitrogen to the plants similar to that which is supposed to have contributed to the high nitrogenous yield of plot 7, where the mineral manures alone, including potass, had been applied throughout the 20 years. It remains to consider whether, not only the conditions of manuring, but the characters of the vegetation, were also consistent with, in great measure, a similar source of nitrogen in the two cases.

It will be remembered that on plot 5, with ammonia-salts alone, the herbage became more and more gramineous, contained scarcely any leguminous plants, and, with the exception of *Rumex acetosa*, very little of any miscellaneous species. The grasses became patchy, stunted, very dark-green, and almost exclusively leafy; and in the later years the two species, *Festuca ovina* and *Agrostis vulgaris*, contributed more than four-fifths of the total produce. In the first year of the substitution of the mixed mineral manure, including potass, for the ammonia-salts (on plot 6), the appearance of the plot entirely changed. Although *Festuca ovina* and *Agrostis* were still very prominent, several other grasses began to assert themselves; and now, compared with plot 5, the two grasses mentioned are in much less proportion, and others are becoming characteristic of the plot. The *Agrostis* especially has gone down very much; *Holcus* has very much increased; and *Dactylis glomerata*, *Avena elatior* and *pubescens*, *Lolium perenne*, and *Poa pratensis*, are more favoured than on plot 5. Not only did the gramineous herbage become more mixed under the influence of the substitution of the mineral for the nitrogenous manure, but the character of development was totally different. In the first season the grasses showed a healthy, rather light-green, instead of the dark bluish-green, colour. The patchy character has gradually disappeared, and there is not only a free bottom growth, but there is more and more tendency to form stem and seed.

The Leguminosæ, which together had never contributed half a per cent. to the produce under the influence of the ammonia-salts alone, now began to increase in prominence; and, in 1877, they contributed nearly 7 per cent. to the hay. Neither *Trifolium pratense*, nor *Trifolium repens*, has, however, recovered any prominence in the produce. Nor has the deep-rooting *Lotus corniculatus* increased; but the creeping and comparatively surface-rooting *Lathyrus pratensis*, which is by far the most prominent leguminous plant on plot 7, where there has been a liberal top-dressing of mineral manure, including potass, from the commencement, appears to be rapidly gaining ground.

Of miscellaneous species, too, there has been, since the application of the mineral manure, a greater number fairly represented, and a greater proportion and actual amount by weight in the produce than where the ammonia-salts are still applied. The total number of species of all orders found on the plot has also considerably increased under the influence of the mixed mineral manure.

Thus, since the application of the mixed mineral manure, the flora of the plot has become much more complex. With this, the roots of the herbage would doubtless acquire possession of a more extended range of soil and subsoil, and more varied powers of underground food-collection would come into play. Whilst, therefore, some part of the nitrogen of the increased produce obtained on the substitution of the mineral for the nitrogenous manuring would probably be derived from the residue of the previous applications, it is probable that the greater part would be due to increased power of underground food-collection, by virtue of which not only the immediately preceding, but the earlier accumulations, or what may be called the normal stores of the soil and subsoil, would be drawn upon. That such was really the case may be concluded from the fact of the reduction of the percentage of nitrogen in the surface soil of plot 6, as of plot 7 where mineral manures alone had been applied for 20 years, and where the very complex and highly leguminous herbage accumulated, throughout the period, an otherwise unaccountably large amount of nitrogen.

With the differences—in the amount of produce, in the botany, in the character of development of the herbage, and in the probable sources of the nitrogen of the crop, the change in its chemical composition was also extremely marked.

Although there was very little less nitrogen taken up than where the ammonia-salts were still applied, the percentage of nitrogen in the dry substance of the mixed herbage was reduced by more than one-third. In other words, with the increased supply of mineral matter there was, for a given amount of nitrogen taken up, a greatly increased growth, involving a greatly increased amount of carbon assimilated.

Of total mineral matter there was, with the increased supply of it, an increased instead of a much reduced actual amount of it taken up; in fact, there was twice as much taken up during the seven years as where the application of ammonia-salts alone was continued, and the percentage of it in the dry matter of the produce was also higher. There was, over the seven years, more of every mineral constituent (except soda) taken up on plot 6 with the mineral manure, than on plot 5 with the ammonia-salts. Of potass there was nearly four times, of phosphoric acid about two-and-a-half times, of sulphuric acid about one-and-a-half time, and of silica (of which none was supplied in the manure) more than one-and-a-half time, as much taken up as where the ammonia-salts were still used. There was, however, less of every mineral constituent (except soda) taken up, than where the mixed mineral manure had been applied from the commencement (on plot 7).

But, notwithstanding the greater actual amount of the mineral constituents taken up on plot 6 than on plot 5, there was, with the more healthy character of development and better maturation, an actually lower percentage of lime, magnesia, soda, and chlorine, in the dry substance of the produce. On the other hand, the percentage of potass in the dry substance was considerably more than twice as high, that of phosphoric acid one-and-a-half time as high, and that of silica rather higher, under the influence of the mineral supply, and with it better characters of growth. Still, the percentage of phosphoric acid, lime, and potass, especially of the two latter, was not so high as where the mineral manures had been supplied throughout the whole period (on plot 7); and where, accordingly, there was a still greater proportion of leguminous herbage, and the grasses developed more flower stems.

In conclusion, it is obvious that it was of potass chiefly, of phosphoric acid also notably, but of most of the mineral constituents more or less, that the available supply had become so deficient under the continuous application of the ammonia-salts. It has already been illustrated how ineffective was a supply of phosphoric acid (super-

In the first year of the application of the mixed mineral manure instead of the nitrate, the plot assumed a lighter green colour, the grasses yielded more stem, matured better, and a greater number of them contributed to the bulk of the herbage. *Festuca ovina*, which had greatly increased, and contributed a very large proportion to the produce under the influence of the nitrate, still maintained the first place, but in much diminished quantity. *Holcus lanatus*, and *Agrostis vulgaris*, became much more prominent, and several other grasses gained ground. The leguminous plants, which had almost disappeared, immediately increased on the application of the mixed mineral manure, including potass; *Lathyrus pratensis*, as in other cases with the same manure, at once came forward. The number of miscellaneous species also increased; but the percentage by weight which they yielded to the produce was less, owing to the greatly increased and denser undergrowth of the grasses.

With the changes in the description of the herbage, and the character of its development, due to the change in the manure, there is, so far, a diminution in the average produce of the first crops; but as second crops were removed from the land in 1875 and 1877, the first crops of the three years 1876–77–78 would doubtless be affected by the consequent increased exhaustion. And, as a second crop was also taken in 1878, and will probably be so in future, the first crops may perhaps remain at a lower level. The average total produce, first and second crops together, is, however, considerably more over the last three years with the mineral manure, than that of the first crops only over the previous 18 with the nitrate.

In the total mineral matter, again, there is a somewhat less average amount removed in first crops, but a greatly increased amount taking first and second crops together. In first crops alone, however, there is twice as much potass, and nearly one-third more phosphoric acid, annually removed, since the change than previously, and very much more including the second crops. But, of lime, magnesia, chlorine, and silica, there is less removed in the first crops, and of soda only about one-seventh as much as previously. And, with the improved character of development of the herbage, the percentage of potass in its dry substance is more than twice, and that of phosphoric acid nearly one-and-a-half time, as high as previously.

Lastly, as to the nitrogen: its percentage in the dry substance of the produce (of first crops) has very much diminished, and the average amount of it removed per acre per annum is, in first crops, little more than half as much as, and even including second crops it is considerably less than, during the period of the application of the nitrate. Indeed, although the amount of produce under the influence of the mixed mineral manure succeeding the excess of nitrate is very much greater than without manure, there is, with the less mixed herbage than on the unmanured plot, and better development and maturation, especially of the grasses, a much lower percentage of nitrogen, and not much more total yield of it per acre, than without manure. Further, in the present season, 1879, the herbage of plot 15 has a very light colour, and upon the whole the appearance of restricted growth from deficiency of nitrogenous supply; much more so than is the case on plot 6, where the mixed mineral manures have for a longer period succeeded the ammonia-salts.

So far, therefore, there is little indication that much of the nitrogen previously supplied in the nitrate, and not recovered in the crops grown in the years of its application, will be gathered up under the influence of the mineral manure and the more varied flora it induces.

15. *Equal Nitrogen, and equal Potass, in Nitrate of Soda and Sulphate of Potass, and in Nitrate of Potass; in each case with Superphosphate of Lime; Plots* 19 *and* 20.

The marked effects of nitrate of soda, and of sulphate of potass, pointed to the interest of determining whether nitrate of potass, or a mixture of nitrate of soda and sulphate of potass, containing the same amounts of nitrogen and of potass, would be the most effective? Accordingly, in 1872, two plots (19 and 20) were set apart foı experiments on the point. In each of the seven years, 1872–78 (and the experiment is still in progress), plot 19 has received 275 lbs. nitrate of soda, and plot 20, 327 lbs. nitrate of potass, both containing the same amount of nitrogen. Plot 19 has also received 290 lbs. sulphate of potass, containing the same amount of potass as the 327 lbs. nitrate of potass. Each plot has also received annually 3½ cwts. superphosphate of lime. For comparison with the results so obtained there is also given, in the following table, the average produce over the same seven years, and also over the preceding 14 years, on plot 16, manured annually, during the whole period of 21 years, with 275 lbs. nitrate of soda, 300 lbs. sulphate of potass, 100 lbs. sulphate of soda, 100 lbs. sulphate of magnesia, and 3½ cwts. superphosphate of lime.

TABLE XVIII.—Average, per acre per annum, by Nitrate of Soda and Sulphate of Potass, and by Nitrate of Potass, containing equal Nitrogen and equal Potass, in each case with Superphosphate; Plots 19 and 20.

		Per acre per annum.		
		Plot 16. 275 lbs. nitrate soda, 300 lbs. sulphate potass, 100 lbs. sulphate soda, 100 lbs. sulphate magnesia, 3½ cwts. superphosphate.	Plot 19. 275 lbs. nitrate soda, 290 lbs. sulphate potass, 3½ cwts. superphosphate.	Plot 20. 327 lbs. nitrate potass, 3½ cwts. superphosphate.
HAY.				
		lbs.	lbs.	lbs.
14 years, 1858–1871.	One crop only each year	5451	. .	. .
7 years, 1872–1878.	First crop only each year	4716	4368	4362
	Including second crops, 1875–77–78	5639	5273	5191
NITROGEN.				
14 years, 1858–1871.	One crop only each year	66·1	. .	. .
7 years, 1872–1878.	First crop only each year	54·0	49·1	47·9
	Including second crops, 1875–77–78	69·0	62·5	61·1
MINERAL MATTER (ASH).				
14 years, 1858–1871.	One crop only each year	331·4	. .	. .
7 years, 1872–1878.	First crop only each year	283·8	261·4	254·4
	Including second crops, 1875–77–78	369·9	336·4	322·6

It will be seen that the three plots received the same amount of nitrogen as nitric acid. Plots 19 and 20 received the same amount of potass, and plot 16 about 5 lbs. per acre per annum more. All three received the same amount of superphosphate of lime. But plot 16 received annually 100 lbs. sulphate of soda, and 100 lbs. sulphate of magnesia, in addition. Thus, plot 16 received rather more potass, and more soda, magnesia, and sulphuric acid, than plot 19; and plot 20, with the same amount of nitrogen and potass as 19, received no soda, and less sulphuric acid, but the nitric acid and the potass were applied to the soil in combination.

It is to be borne in mind that, in each of the 14 seasons (1858–1871), only one crop was removed each year; the after-growth being either fed off by sheep having no other food, or mown, spread on the land, and left to decay; but, in 1875, 1877, and 1878, a second crop was removed from these and all the other plots. The table therefore gives the average annual produce of one crop only each year, over the 14 years on plot 16, but over the seven years (1872–1878), for each plot, both excluding and including any second cuttings. It is seen that plot 16, which annually received the same amount of nitric acid, but a fuller mineral manure, and was under experiment 14 years before plots 19 and 20, yielded over those 14 years more produce of hay, containing more nitrogen, and more mineral matter, than the first crops of the succeeding seven years, but less than when both the first and second crops of the later period are taken together. Again, whichever basis of comparison be taken, plot 16 yielded, over the seven years, more hay, containing more of both nitrogen and mineral matter, than either plot 19 or 20. Plot 19 again, with the nitrate of soda and sulphate of potass, which, besides the difference of condition, supplied soda, and more sulphuric acid, yielded rather more in each particular, especially of mineral matter, than plot 20 with the nitrate of potass.

As on plot 16, so on plots 19 and 20, a fairly mixed herbage has been maintained. With only a moderate supply of nitrogen, and this in the form of nitrate, and with a liberal supply of potass, leguminous herbage increased on all three. The increase was, in each case, mainly due to the greatly increased growth of *Lathyrus pratensis;* but on plots 19 and 20 to some increase in *Trifolium repens* also. The percentage of miscellaneous species has gone down considerably since the commencement, on plot 16; but, as yet, it has not done so on plots 19 and 20; and with the greatly increased amount of total produce since the application of the manures, there is an increase in the actual amount of such herbage grown per acre. On all three plots, the bulk of the gramineous herbage is made up of a good many species; and on plot 16, which has been the longest under treatment, the mixture is greater—that is, there is less predominance of individual species than on either of the other plots; *Festuca ovina, Agrostis vulgaris, Alopecurus pratensis, Avena flavescens,* and *Holcus lanatus,* are somewhat equally represented; whilst, *Poa trivialis, Dactylis glomerata,* and *Lolium perenne,* each show moderate growth. On both plots, 19 and 20, *Festuca ovina* is more prominent, as also, and increasing, is *Holcus lanatus.* On both, *Agrostis vulgaris,* though much decreasing, is still in considerable quantity; whilst *Alopecurus pratensis, Avena flavescens* and *pubescens,* and *Poa trivialis,* are each fairly represented, and increasing. There is, on all three plots, a pretty normal character of growth, fair proportion of stem, and tendency to maturation of the grasses.

Although there is a notably smaller yield per acre of dry matter, of nitrogen, and of mineral matter, on plots 19 and 20 than on plot 16, the percentages in the dry substance, of both nitrogen and mineral matter, are very similar in the produce of plots 19 and 16, but in that of plot 20 they are rather lower, whilst the percentage of the dry substance itself in the air-dried hay was rather higher. These conditions would, with the comparatively similar botanical composition of the produce, indicate greater maturation, or ripeness, on plot 20 with the nitrate of potass, than on plot 19 with the nitrate of soda and sulphate of potass.

The analyses of the ash of the produce of the first crops for the seven years, and of the second crops for the three years, show that that of plot 16, with the fuller mineral manure and the longer continuance of the experiment, contained, both per acre and per cent. in its dry substance, more phosphoric acid, considerably more potass, and rather more magnesia, than that of either plots 19 or 20. On the other hand it contained, both per acre and per cent., less lime than that of either of the other plots, and at the same time less soda and chlorine than that of plot 19; but that of plot 20, with nitrate of potass, but without soda in the manure, contained very much less soda than that of either plot 16 or 19 with it. The produce of plot 20 again, contained less of sulphuric acid, chlorine, and silica, than that of plot 19; but more of lime, and especially more of potass, both per acre and per cent. These differences are, so far as they go, also consistent with a more advanced condition of maturity of the produce of plot 20.

There is, then, so far, no marked difference in the amount, or in the botanical composition, of the produce, whether the nitrogen and the potass be supplied as nitrate of soda and sulphate of potass, or as nitrate of potass; but the data at command as to the chemical composition would indicate a somewhat more matured condition of the produce grown by the nitrate of potass.

16. *Mixture, supplying the Ash-constituents, and the Nitrogen, of* 1 *ton of Hay; Plot* 18.

We have the results of one more experiment with artificial or chemical manuring substances to give in this section—those obtained on plot 18. Commencing in 1865, there has been applied annually to this plot, a mixture containing the quantities of potass, soda, lime, magnesia, phosphoric acid, silica, and nitrogen, contained in 1 ton of hay. The mixture also supplied sulphuric acid and chlorine in abundance. The object of the experiment was, in part, to put to the test of direct experiment the principles of manuring put forward by LIEBIG, according to which all the constituents, neither more nor less, which we remove in crops, should be returned to the soil. He says:—

"By an exact estimation of the quantity of ashes in cultivated plants, growing on various kinds of soils, and by their analysis, we will learn those constituents of the plants which are variable, and those which remain constant. Thus also we will attain a knowledge of the quantities of all the constituents removed from the soil by different crops.

"The farmer will thus be enabled, like a systematic manufacturer, to have a book attached to each field, in which he will note the amount of the various ingredients removed from the land in the form of crops, and therefore how much he must restore to bring it to its original state of fertility. He will also be able to express in pounds' weight, how much of one or of another ingredient of soils he must add to his own land, in order to increase its fertility for certain kinds of plants." ("Agricultural Chemistry," 4th ed., pp. 212–213.)

He further says as to the object of manuring:—

"This purpose is the restoration, or an increase of the original fertility, and by manure we must replace all the constituents of the plants which have been taken away in the harvest, or which are contained in the plants which we are desirous to cultivate." (Address—" On the Principles of Artificial Manuring," p. 26.)

Again, in ridicule of the different plan followed in our experiments, he says: " Shall we do as Mr. LAWES did . . . . ?" and in answer he adds: " Truly, in such cases, there is no other guide but logic, that is, sound common sense, which tells us, *to give to the land what we have removed from it, neither more nor less.* Of course, this is on the assumption that the land is *to retain its original fertility* only, not to be rendered more fertile than before." (" Principles of Agricultural Chemistry," p. 63.)

And again: " Our first object will naturally be, to restore to the soil the mineral constituents in the same quantity and in the same proportions as those in which they have been removed in the crops; and *none must be omitted.*" (*Ib.*, pp. 83–84.)

Another of the objects of the experiment was to acquire data as to the proportion in which the several constituents artificially supplied would be recovered in the increase of crop—a question the fuller consideration of which must be postponed to the third or Chemical Section of our report.

The manures actually applied, and the constituents they contained, were as follows :—

76 lbs. commercial chloride potassium . .	38 lbs. potass. 7 lbs. soda. 36·7 lbs. chlorine.
35 lbs. sulphate magnesia	5·6 lbs. magnesia. 11·1 lbs. sulphuric acid.
26 lbs. bone-ash	11·0 lbs. lime. 8·2 lbs. phosphoric acid.
26 lbs. sulphuric acid (sp. gr. 1·7) . . .	16·9 lbs. sulphuric acid.
50 lbs. silicate soda	? lbs. soda. 23·0 lbs. soluble silica.
50 lbs. silicate of lime	? lbs. lime. 8·0 lbs. soluble silica.
164 lbs. " ammonia-salts "	34·0 lbs. nitrogen. 44·4 lbs. sulphuric acid. 46·2 lbs. chlorine.

If the quantity of constituents supplied, as shown in the right-hand column of the above tabular statement, be compared with the figures given on page 298 for the contents of 1½ ton of hay, it will be seen that fully sufficient of all the constituents have been supplied for an increase of 1 ton of hay, whilst of soda, sulphuric acid, and chlorine, much more than sufficient has been applied.

The experiment had been conducted 11 years, from 1865 to 1875 inclusive; and the following table gives the average produce per acre per annum over the first five-and-a-half, the second five-and-a-half, and the total period of 11 years; that is, the produce of the first five years, together with half that of the sixth, is divided by 5·5; and, again, the produce of the last five years and half of the sixth is, in like manner, divided by 5·5. For comparison, the average produce obtained on the unmanured plot 3 is also given for the same periods.

Table XIX.—Average, per acre per annum, by a Mixture supplying the Ash-constituents, and the Nitrogen, of 1 ton of Hay; Plot 18.

	Average per acre per annum.		
	Plot 18. Nitrogen and ash-constituents of 1 ton of hay.	Plot 3. Without manure.	Plot 18. + or − unmanured.
Hay.			
	lbs.	lbs.	lbs.
First period, 5½ years	3908	2514	+1394
Second period, 5½ years	3301	1787	+1514
Total period, 11 years, 1865–75	3604	2151	+1453
Second period, per cent. + or − first period	−15·5	−28·9	
Nitrogen.			
First period, 5½ years	50·1	35·1	+15·0
Second period, 5½ years	39·9	24·3	+15·6
Total period, 11 years, 1865–1875	45·0	29·7	+15·3
Second period, per cent. + or − first period	−20·4	−30·8	
Mineral Matter (Ash).			
First period, 5½ years	222·7	146·7	+76·0
Second period, 5½ years	175·7	96·9	+78·8
Total period, 11 years, 1865–1875	199·2	121·8	+77·4
Second period, per cent. + or − first period	−21·1	−33·9	

The average yield obtained on the application of the mineral constituents and the nitrogen contained in 1 ton of hay was, over the first half of the period, about 35 cwts., over the second half rather less than 30 cwts., and over the whole period rather more than 32 cwts. of hay. There was, therefore, a reduction in the total produce in the later years.

But, obviously, before we can estimate the effects of the manure, we must deduct something for the annual yield of the soil and seasons; and, as plot 18 had, like plot 3, been unmanured from the commencement of the experiments in 1856, and for some years previously, we shall arrive at an approximate estimate of the amount of produce due to the manure, if we deduct from the total amount that obtained on the unmanured plot over the same seasons. Reckoned in this way, we have an average annual increase of produce, due to the manure, of not quite 12½ cwts. during the first period, about 13½ cwts. during the second, and not quite 13 cwts. over the whole period. The increased amount of nitrogen in the produce is only about 45 per cent. of that supplied;

and the increase of mineral matter removed is not much more than half as much as would be contained in 1 ton of hay.

Whilst it is clear from the above comparisons, that the annual supply of not only the mineral constituents, but the nitrogen also, of 1 ton of hay, yielded less than two-thirds instead of 1 ton of increase of produce, it is at the same time pretty certain that the differences in the amounts of hay, nitrogen, and mineral matter, obtained, without and with the manure, do not accurately represent the proportion of the constituents supplied which contributed to the result; for, one effect of the manure was very materially to modify the character of the herbage. The percentage in the produce, and the amount per acre, of gramineous herbage, were much increased; both the percentage, and the actual amounts, of leguminous herbage were much reduced; the percentage of the miscellaneous herbage was also reduced, though the amount of it per acre was increased. Thus, whilst the amount of leguminous herbage, which is to a great extent independent of direct supplies of nitrogen by manure, was reduced, the increase of produce consisted almost exclusively of gramineous herbage, the increased growth of which would doubtless depend—mainly, at any rate—for the nitrogen it required, on that which was artifically supplied; and, therefore, probably a larger amount of that supplied contributed to the increase than the table shows.

As in the case of the nitrogen, so also in that of each of the mineral constituents, there was always very much less, and sometimes more than one-half less, increased amount in the produce removed, than was supplied in the manure; and in the case of some important constituents—potass, for example—a less proportion of that supplied was recovered as increase than when larger amounts of both the mineral and the nitrogenous manures were applied, and when, consequently, greater luxuriance of the grasses was induced. And, notwithstanding potass is very little subject to loss by drainage, and there would therefore be an annually increasing residue of it within the soil, there is even less of it taken up during the second than during the first half of the period of experiment. There is, nevertheless, a higher, but a decreasingly higher, percentage of potass in the manured, than in the unmanured produce. There is also a higher percentage of soda, of phosphoric acid, of sulphuric acid, and of chlorine, but a lower percentage of lime, magnesia, and silica.

These results suggest several points of interest.

It has already been shown, in experiments in which comparatively large quantities of ammonia-salts or nitrate of soda were used, how much greater was the increase of produce, how much more mixed was the herbage, and how much better it was developed, when a given amount of nitrogen was applied as nitrate of soda than when as ammonia-salts; the difference in favour of the nitrate in these respects being much the greater when these nitrogenous manures were respectively used alone—that is, without the conjunction of mineral manures. On plot 18—with nearly one-fifth less nitrogen annually applied as ammonia-salts than on either plots 16, 19, and 20 as nitrate of soda, and therefore with less forced luxuriance, but, it is true, at the same time, with less (though still an excess) of mineral constituents supplied—the description and condition of the herbage (on plot 18) were inferior; a somewhat larger proportion of the produce was made up of a few of the poorer grasses; leguminous herbage was greatly reduced; and among miscellaneous species *Rumex acetosa* was more prominent.

Again, the results afford a pretty direct test of the validity of the principle according to which all, and neither more nor less, of the constituents removed from the land in crops, should be returned to it in manure.

In LIEBIG's earlier writings he did not recognise the fact that a considerable proportion of the constituents removed from the land in crops is, in the actual practice of agriculture, periodically returned to it, and that, therefore, the loss to the soil is not measured by the amount of constituents in the crops grown, but more nearly by that in the produce sold off the farm. Further, his recommendations for the carrying out of his principle were confined to the application of the "mineral" or *ash-constituents;* he maintaining that the atmosphere would supply the necessary nitrogen. It is true that subsequently, in the course of controversy, he changed the meaning of his terms, and then included ammonia-salts in the category of mineral manures. It is seen, however, that even with a supply of the amount of nitrogen, as well as ash-constituents, contained in 1 ton of hay, not two-thirds of a ton of increase of produce was obtained.

With regard to the applicability of the principle under the actual conditions of practical agriculture, attention may be called to the following considerations:—

In the first place, there is no conceivable condition of chemical combination, and of distribution within the soil, in which the various constituents could be annually supplied so as to be all annually taken up by growing vegetation; and there is conclusive evidence that, in some cases, the unrecovered residue is, in greater or less part, lost by drainage; and that, so far as it is not so, it becomes so locked up, or distributed within the soil, that it is—at any rate, very slowly, and in some cases, perhaps, never fully—recovered in subsequent crops.

In the second place, the principle ignores the difference in the character and capabilities of different soils. Take, by way of illustration, two extremely opposite cases: A light, porous, almost exclusively sandy soil, which itself yields up little or nothing to growing plants, but which may, nevertheless, produce good crops under high farming, will probably suffer great loss of manurial constituents by natural drainage; so that, if no more were to be supplied than were removed, there must obviously be a decline of fertility. Suppose, on the other hand, a rich and deep loam, which would, under good mechanical cultivation and drainage, supply annually a considerable amount of potass, for example, to say nothing of other constituents, for hundreds and perhaps for thousands of years;—surely, in such a case, it is not necessary to supply as much in manure as has been removed in the crops?

Further, experience teaches that, in the actual condition of our soils, and of agricultural practice, the exact composition of the crops we remove, or wish to grow, is no direct guide to the description, and the amount, of manurial constituents which will be the most effective. Thus, an average crop of wheat will remove even rather more phosphoric acid than an average crop of barley; but experience teaches that, in the case of land of the same description, and in the same condition, superphosphate of lime is, as a rule, used with very much more benefit to the spring-sown barley than to the autumn-sown wheat. The wheat, being put in four or five months earlier, has so much more time for root-distribution, and acquires a greater capability of food-collection. The barley, on the other hand, depends very much more upon the stores available within the surface soil. Again, superphosphate of lime is, in practice, of very special benefit to the so-called "root-crops," though the amount of phosphoric acid they take up compared with other crops would not indicate this. Then, turning from the mineral or ash-constituents to the nitrogen, an average crop of beans will contain from two to three, and one of clover-hay from three to four, or more times, as much nitrogen as one of wheat or barley; but land in such condition as to grow a full crop of the rich-in-nitrogen beans, or clover, without nitrogenous manure, would not grow a full crop of wheat or barley, containing so much less nitrogen, without liberal nitrogenous manuring.

It is, then, under the existing conditions of practical agriculture, certainly not necessary to supply to the land all the constituents that have been removed from it, or that would be contained in the crops it is wished to grow, and neither more nor less of them than would be so removed. On the contrary, we should supply all, or only some, and more or less, according to the circumstances.

17. *Farmyard Manure, alone, and with Ammonia-Salts in addition; Plots* 2 *and* 1.

Having now considered the effects of various important individual constituents of manures, and of various combinations of them, on the mixed herbage of permanent meadow land, we are in a position the better to interpret the results obtained on the application of that complex and heterogeneous mixture—farmyard manure.

For eight years—1856–1863 inclusive—plot 2 received annually farmyard manure at the rate of 14 tons per acre. Over the same period, plot 1 received the same quantities of farmyard manure, but with 200 lbs. ammonia-salts per acre per annum in addition. At the end of the eight years, the application of the farmyard manure was stopped on both plots; but the ammonia-salts were still annually applied to plot 1. The cessation of the application of the farmyard manure was decided upon—partly because so large a quantity annually applied was obviously not thoroughly taken up by the soil, and it was thought somewhat obstructed the vegetation; and partly because calculation indicated how small a proportion of the constituents applied was recovered in the increase of crop, and that there was, therefore, a considerable accumulated residue, the amount, and the duration, of the effects of which, it would be of interest to trace.

The following table shows the average produce of hay, and its contents of nitrogen and mineral matter, over the eight years of the application of the dung, over the first six, and the second six, succeeding years, and over the total period of 20 years, on both plots 2 and 1. For comparison, there is also given the average produce on the unmanured plot 3, over the same periods; and lastly, the increase on plots 2 and 1 respectively, over the produce without manure, and the increase on plot 1 with, over the yield on plot 2 without, the ammonia-salts.

TABLE XX.—Average, per acre per annum, by Farmyard Manure alone, and with Ammonia-Salts; and by the residue of the Dung, also without and with Ammonia-Salts; Plots 2 and 1.

	Average per acre per annum.					
	Plot 3. Unmanured continuously.	Plot 2. Farmyard manure alone 8 years, 1856-1863; unmanured 12 years, 1864-1875.	Plot 1. Farmyard manure and ammonia-salts 8 years, 1856-1863; ammonia-salts only 12 years, 1864-1875.	Plot 2. + or − unmanured.	Plot 1. + or − unmanured.	Plot 1. + or − plot 2.
HAY.						
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
First period, 8 years, 1856-1863	2665	4804	5538	+2139	+2873	+734
Second period, 6 years, 1864-1869	2699	4846	5366	+2147	+2667	+520
Third period, 6 years, 1870-1875	1692	2517	3331	+ 825	+1639	+814
Total period, 20 years, 1856-1875	2383	4130	4824	+1747	+2441	+694
Second period, per cent. + or − first period	+ 1·3	+ 0·9	− 3·1			
Third period, per cent. + or − first period .	−36·5	−47·6	−39·9			
NITROGEN.						
First period, 8 years, 1856-1863	37·2	58·2	68·3	+21·0	+31·1	+10·1
Second period, 6 years, 1864-1869	37·3	53·3	63·9	+16·0	+26·6	+10·6
Third period, 6 years, 1870-1875	23·0	27·5	41·0	+ 4·5	+18·0	+13·5
Total period, 20 years, 1856-1875	33·0	47·5	58·8	+14·5	+25·8	+11·3
Second period, per cent. + or − first period	+ 0·3	− 8·4	− 6·4			
Third period, per cent. + or − first period .	−38·2	−52·7	−40·0			
MINERAL MATTER (ASH).						
First period, 8 years, 1856-1863	160·5	329·1	370·5	+168·6	+210·0	+41·4
Second period, 6 years, 1864-1869	152·2	275·0	291·8	+122·8	+139·6	+16·8
Third period, 6 years, 1870-1875	91·5	136·4	164·7	+ 44·9	+ 73·2	+28·3
Total period, 20 years, 1856-1875	137·3	255·0	285·1	+117·7	+147·3	+30·1
Second period, per cent. + or − first period	− 5·2	−16·4	−21·2			
Third period, per cent. + or − first period .	−43·0	−58·6	−55·5			

It will be convenient first to consider the effects of the farmyard manure used alone, over the eight years of its application, and those of its residue afterwards; and then the difference of effect produced by the use of 200 lbs. of ammonia-salts per acre per annum, both during the eight years of the application of the farmyard manure, and during the subsequent 12 years of the action of the residue.

Over the eight years of the application of the farmyard manure, there was an average produce of 4804 lbs. of hay, equal to an average annual increase over that without manure of 2139 lbs. It is remarkable that, over the first six years after the cessation of the application of the manure, the average produce was almost exactly the same as, and even rather more than, during the eight years of the application, namely, 4846 lbs; and the average annual increase was also almost identical, being 2147 lbs. This result was, indeed, partly due to one or two of the six years being seasons of very high productiveness; but still it is very striking. During the next six years there was one season of unusual drought, and others of much less than average productiveness; and, under these circumstances, together with the fact that the then remaining residue would doubtless be in a less readily available condition, there was little more than half as much average produce, and less than two-fifths as much average increase, as over the first six years of the action of the manurial residue. Still, there was an increase of between 6 and 7 cwts. of hay, due to the residue, in 1875, the twelfth year after the cessation of the application of the manure, and of more in 1876 and 1877.

The table further shows that very much more nitrogen was removed in the crops during the first, than during the second six years, of the action of the residue. But the amount was not, as was that of the hay, as much per acre per annum over the first six years as it had been over the eight of the application; and over the second six years there was little more than half as much as over the first six, and the average annual increase was only about 4½ lbs., against 16 lbs. over the first six years.

Of mineral matter, again, there was twice as much removed in the first as in the second six years of the action of the residue; and there was of it a greater falling off in the first six years compared with the years of the application, than of either the hay or the nitrogen.

The effect of the farmyard manure was to reduce the number of species developed, to bring into greater prominence the gramineous and the miscellaneous herbage, but to reduce the leguminous. The gramineous herbage became less mixed, a few individual species contributing a larger proportion of the hay. Of these, *Poa trivialis*, *Bromus mollis*, and *Avena flavescens*, were the most prominent; whilst, without manure, neither of these was in any marked quantity, but *Festuca ovina*, *Agrostis vulgaris*, *Avena pubescens*, and *Holcus lanatus* were the leading grasses, and *Lolium perenne*, *Anthoxanthum odoratum*, and some others, were fairly represented. After the cessation of the application of farmyard manure, however, the same grasses as without manure gradually became the more prominent on plot 2, whilst the *Poa trivialis* and *Bromus mollis* were, in 1877, represented in very insignificant amounts. Upon the whole, indeed, the general character of the herbage on the unmanured, and the farmyard-manure plot, especially in the later years, has not been very strikingly different; and the difference may, for our present purpose, be summed up in the statement that there was, on the manured plot, a higher percentage, and a considerably larger quantity per acre, of gramineous herbage; a considerably lower percentage, and a smaller amount per acre, of leguminous herbage; and lastly, a rather lower percentage, but a somewhat larger actual amount, of miscellaneous herbage.

With the greater proportion of gramineous and less of leguminous herbage in the produce by the farmyard-manure, there was a considerably lower, and a decreasing, percentage of nitrogen in it; but there was a higher percentage of total mineral matter. Of lime, magnesia, soda, and sulphuric acid, there was more, and of potass, phosphoric acid, chlorine, and silica, very much more, removed per acre from the farmyard-manured than from the continuously unmanured plot. The percentages of lime, magnesia, soda, and sulphuric acid, were, however, all lower in the dry substance of the produce by the farmyard manure; but those of potass, phosphoric acid, chlorine, and silica, were not only in very much larger actual amount, but in higher percentage in the produce of the manured plot; and (excepting chlorine) this was so even in the later years. It is clear, therefore, that there was a greater or less available residue of all the mineral constituents, and more especially of potass, phosphoric acid, and silica, many years after the cessation of the application of the manure.

It is obviously a matter of very great interest to consider at what rate, and in what proportion, the several constituents of the farmyard manure which were not recovered during the years of the application, have been so since, and what prospect there is of their final total recovery? These are questions which will be considered more in detail in the chemical section of our report, but a few general observations on the subject seem to be called for in this place, to enable us to form any just conception of the character of the agricultural results.

We have sufficient analytical data on which to found a pretty correct estimate of the amounts of nitrogen, and of the several mineral constituents, removed in the crops during the separate periods; but we have not any such sure basis for the estimation of the amount of the same constituents contained in the 112 tons of farmyard manure applied per acre to the plot during the eight years. We are obliged, therefore, to adopt the best estimate we can of the constituents supplied in the manure, founded on much data, both synthetic and analytic, relating to the subject, and a careful consideration of them. The result must obviously be only approximative, but it is doubtless sufficiently so to give considerable value and importance to its general indications.

Of nitrogen, it is estimated that the farmyard manure will, on the average, contain 0·64 per cent., and therefore that 200·7 lbs. were applied per acre per annum; or, in all, 1606 lbs. in the eight years. The total produce removed, during the eight years of the application, about 29 per cent. as much nitrogen as is thus estimated to be supplied in the manure. During the next six years about 20 per cent., and during the last six of the 20 years about 10 per cent. more, were removed, making in all about 59 per cent.

as much removed in the total produce of the 20 years, as had been supplied during the eight years. But obviously we must not assume that the whole of the nitrogen of the crops was derived from the manure. It would be nearer the truth to deduct the amount obtained in the unmanured produce, and to suppose only the so reckoned increase to be due to the manure. But, inasmuch as the manured produce contained considerably more gramineous herbage, which would probably depend largely on the artificial supply of nitrogen, and, further, as it contained upon the whole less total leguminous herbage, which would depend more on natural sources, it is probable that the manured produce might derive less nitrogen from such sources than the unmanured, especially in the presence of the presumably more readily available supply by the manure. Still, it is doubtless a nearer approximation to the truth to assume that only so much of the nitrogen of the manured produce as was in excess of that of the unmanured was derived from the nitrogen of the manure. Reckoned in this way, only 10·4 per cent. of the estimated supplied nitrogen was recovered as increased yield during the eight years of the application, only 6 per cent. more during the next six years, and only 1·7 per cent. during the second six years since the application; making, in all, a recovery as increase during the 20 years, of about 18·1 per cent. of that supplied in the first eight years.

Thus, on the supposition that the whole of the nitrogen of the produce was derived from the manure, there would still remain, at the end of the 20 years, about 41 per cent. of the 1606 lbs., or about 658 lbs., not accounted for. But on the supposition that only the increase above that in the unmanured produce was derived from the manure, which is doubtless at any rate much nearer the truth, there would remain unaccounted for, at the end of the 20 years, 81·9 per cent. of the nitrogen supplied—that is, about 1315 lbs.; and yet, during the last six years of the 20 less than 2 per cent. of the original amount supplied was recovered as increase. The prospect of recovering the whole, or even a considerable proportion, would thus seem, to say the least, extremely remote.

The question arises—whether this large amount of unrecovered supplied nitrogen does remain in the soil, and in such a condition of combination, and distribution, as to be available to succeeding crops? or whether some of it be not lost by drainage, or in other ways, and the remainder so locked up, or distributed, as to be so slowly recoverable, if ever, that it can be reckoned of scarcely appreciable practical value?

Samples of the soils of all the experimental grass-plots were taken in February and March, 1876—that is, after the experiments had been in progress for 20 years, and before the next growing season had commenced. Table XXI. shows, for the unmanured plot (3), and for the plot (2) manured for eight years with farmyard manure, and then unmanured for 12 years—

(1) The calculated average amounts of soil, free from stones, and free from moisture expelled at 100° C., in pounds per acre, for each layer of nine inches down to a depth of 54 inches;

(2) The percentage of nitrogen in the dry mould of each nine-inch layer, as determined by the soda-lime method;

(3) The calculated nitrogen per acre in pounds;

(4) The difference in amount between the two plots.

The recorded amounts of dry soil per acre at each depth are the averages calculated from direct experimental data relating to the point, obtained in the case of three samples taken at each depth, from each of 22 plots. The determinations, and the estimates of the nitrogen per acre, in the respective soils, do not include the amount in roots separated from the soil-samples. Whilst, however, from the first nine inches of the unmanured soil 0·32 per cent. of roots was separated, in the corresponding layer of the farmyard-manured soil only 0·24 per cent. was found. There would, therefore, be no further accumulation in the manured soil due to this source. In the lower depths, again, the amounts of separated roots were very inconsiderable.

TABLE XXI.—Estimated amount of dry soil per acre at each depth; Nitrogen per cent. in the dry soils, and estimated Nitrogen per acre, at each depth.

	Average dry soil per acre, exclusive of stones.	Nitrogen.				
		Per cent. in dry soil.		Per acre.		
		Plot 3. Unmanured continuously.	Plot 2. Farmyard manure 8 years; unmanured 12 years.	Plot 3. Unmanured continuously.	Plot 2. Farmyard manure 8 years; unmanured 12 years.	Plot 2. + or − plot 3.
	lbs.	Per cent.	Per cent.	lbs.	lbs.	lbs.
First depth, 1–9 inches . .	2,183,375	0·2565	0·2800	5600	6113	+513
Second depth, 10–18 inches .	2.835,339	0·0724	0·0849	2053	2407	+354
Third depth, 19–27 inches .	2,964,176	0·0458	0·0473	1358	1402	+ 44
Fourth depth, 28–36 inches .	3,049,436	0·0425	0·0402	1296	1226	− 70
Fifth depth, 37-45 inches. .	3.104 583	0·0381	0·0337	1183	1046	−137
Sixth depth, 46–54 inches .	3,080,909	0·0376	0·0319	1158	983	−175
Total. 27 inches . . .	7,982,890			9011	9922	+911
Total, 54 inches . . .	17,217,818			12,648	13,177	+529

Adopting the foregoing results as to the accumulation of the nitrogen of the manure within the soil on the basis of the determinations to the total depth of 54 inches, the next table shows—the estimated amounts supplied in the farmyard manure, recovered in the increase of crop, and not so recovered; the residue found in the soil to the depth in question; and the amount not so recovered, either in the increase or in the soil.

Table XXII.—Estimated Nitrogen supplied in the Manure, recovered in the increase of the hay-crop, determined as residue in the soil, and not recovered in either the increase or the soil, to the depth of 54 inches.

	Nitrogen of manure.	
	Per acre.	Per cent.
	lbs.	
Supplied in farmyard manure in eight years	1606	
Recovered in increase in 20 years (over plot 3)	291	18·1
Not recovered in increase	1315	81·9
Residue, determined by soda lime, in soil 54 inches deep	529	32·9
Not recovered in increase or in soil	786	49·0

In interpreting these figures we have to bear in mind the uncertainty in the estimate of the amount of nitrogen supplied in the manure; the difficulty in determining how much of the nitrogen of the produce was derived from that supplied; the possible natural difference, apart from the influence of manure, in the soils and subsoils of the respective plots; and also the unavoidable range of error in the sampling of, and determinations of the nitrogen in, the soils, and the calculation of such data into quantities per acre. Indeed, it appears at first sight anomalous, that the manured soil should show actually less nitrogen in the fourth, fifth, and sixth depths, than the unmanured; and, although the result may be due to differences in the subsoils themselves, irrespectively of the difference of recent treatment, it is at the same time not inconceivable that, with the accumulation of manurial matter in the upper layers of the manured soil, and the consequent increased luxuriance of the vegetation, more moisture, and with it more soluble nitrogenous compounds, might be drawn upwards from the lower layers than without manure, and in that case the mode of estimate adopted above would, so far, be the most correct. But if we take into account the figures relating to the first, second, and third, depths only—that is, only so far as the manured soil shows more nitrogen than the unmanured—the results as to accumulation and loss of nitrogen will stand as follows :—

TABLE XXIII.—Estimated Nitrogen supplied in the Manure, recovered in the increase of the hay-crop, determined as residue in the soil, and not recovered in either increase or soil, to the depth of 27 inches.

	Nitrogen of manure.	
	Per acre.	Per cent.
	lbs.	
Supplied in farmyard manure in eight years	1606	
Recovered in increase in 20 years (over plot 3)	291	18·1
Not recovered in increase	1315	81·9
Residue, determined by soda lime, in soil 27 inches deep . . .	911	56·7
Not recovered in increase or in soil	404	25·2

But even allowing due weight to the difficulties and uncertainties above referred to, and whichever basis of calculation be adopted, there can be little doubt—that only a comparatively small proportion of the nitrogen supplied in the farmyard manure was recovered in the increase of crop; that there was a considerable accumulation of it within the soil; and that there was also a very considerable amount so far unaccounted for.

Such a result would seem to require some confirmation. This is not wanting. Thus, among the series of field experiments at Rothamsted, wheat has been grown year after year on the same land from 1843–44 up to the present time, and barley in like manner from 1852 up to the present time. In the case of each crop, one plot has received 14 tons of farmyard manure per acre per annum, and another a mixed mineral manure, without nitrogen, every year. The following Table (XXIV.) shows, for 20 years, the average annual amounts of nitrogen—estimated to be supplied in the farmyard manure, obtained in the produce by the mineral manure without nitrogen, in the produce by the farmyard manure, in the increase by the farmyard over the mineral manure, and the percentage recovered, and not recovered, in the increase.

TABLE XXIV.—Nitrogen supplied in Farmyard Manure, recovered, and not recovered, in the increase of produce, of Wheat and of Barley.

	Nitrogen per acre, per annum.				For 100 nitrogen in farmyard manure.	
	Supplied in farmyard manure.	In produce by mineral manure.	In produce by farmyard manure.	In increase by farmyard manure.	Recovered in increase.	Not recovered in increase.
	lbs.	lbs.	lbs.	lbs.	Per cent.	Per cent.
Wheat, 20 years, 1852-71 . .	200·7	20·1	49·3	29·2	14·6	85·4
Barley, 20 years, 1852-71 . .	200·7	23·9	45·3	21·4	10·7	89·3

In comparing these results with one another, and with those obtained on the mixed herbage of permanent meadow land, it must be borne in mind that, in the case of the wheat, the farmyard manure had been applied, and the crop grown, for eight years preceding the period for which the calculations are made, and there was doubtless, therefore, already a considerable accumulated residue within the soil; and that, for both the wheat and the barley, the manure was applied in each of the 20 years, instead of only for eight years as for the grass. Further, in the case of the wheat and the barley the increase is reckoned over the produce by purely mineral manure, whereas in that of the grass it is taken over that without manure, since the effect of the mineral manure on the mixed herbage is to develop so much more of the highly nitrogenous leguminous plants.

The general result is that, according to the estimates, a higher proportion of the supplied nitrogen was annually recovered over the 20 years in the autumn-manured and autumn-sown (and longer so grown) wheat, than in the spring-manured and spring-sown barley; and that, about the same proportion was recovered in the barley, as over the eight years in the grass. But it will be obvious that the estimate of the nitrogen in the increase is likely to be nearer the truth in the case of the two cereal crops than in that of the mixed herbage, the character of which, and consequently the capability of collection from normal sources, is so changed by manure. The comparison of the results with the cereal crops may indeed be taken to indicate that the proportion of the nitrogen of the farmyard-manured hay crops estimated to be derived from the manure is too low; and it would seem to be a further reason for this supposition, that, with the much greater variety of root-distribution, and of root-capacity, and a much longer season of greater or less activity of growth, in the case of the mixed herbage, there would also be a greater power of gathering up the supplies by manure. However this may be, the evidence as it stands points to the conclusion that, neither with the wheat, the barley, nor the mixed herbage, was there more than from 10 to 15 per cent. of the nitrogen supplied in the farmyard manure recovered in the increase of crop during the years of the application.

In the case of the barley experiments, after the farmyard manure had been applied for 20 years in succession, the plot was divided; to one-half, the dung was still annually applied, but the other was now left without any further manure. The following table shows the result as to the recovery of the nitrogen of the manure in the two cases. In the upper division, are given the particulars for the 20 years (as already quoted), for the next six years, and for the 26 years, where the application was continued; and in the lower division, for the 20 years of the application, for the six years after the discontinuance, and for the 26 years, 20 with, and six without, the manure.

TABLE XXV.—Nitrogen supplied in Farmyard Manure, recovered, and not recovered, in increased produce of Barley, during the years, and after the cessation, of the application.

	Nitrogen per acre per annum.				For 100 nitrogen in farmyard manure.	
	Supplied in farmyard manure.	In produce by mineral manure.	In produce by farmyard manure.	In increase by farmyard manure.	Recovered in increase.	Not recovered in increase.
FARMYARD Manure every Year; 26 Years.						
	lbs.	lbs.	lbs.	lbs.	Per cent.	Per cent.
20 years, 1852–1871	200·7	23·9	45·3	21·4	10·7	89·3
6 years, 1872–1877	200·7	15·5	47·1	31·6	15·7	84·3
Total period, 26 years . .	200·7	22·0	45·7	23·7	11·8	88·2
FARMYARD Manure, 20 Years; Without Manure, 6 Years.						
20 years, 1852–1871	200·7	23·9	45·3	21·4	10·7	89·3
6 years, 1872–1877	..	15·5	35·3	19·8	2·9	..
Total period, 26 years . .	..	22·0	43·0	21·0	13·6	86·4

Thus, over the 20 years of the application (on the whole plot), 10·7 per cent. of the supplied nitrogen is so estimated to be recovered in the increase of crop; over the next six years of the continued application, on half the plot, 15·7 per cent., and over the total period of 26 years 11·8 per cent. On the other half, there was the 10·7 per cent. of the 20 years' supply recovered during the 20 years, 2·9 per cent. more of it during the next six years, and, in all, 13·6 per cent. of the 20 years' supply recovered in the 26 years. That is to say, of the 4014 lbs. of nitrogen estimated to be supplied in the 20 years, 86·4 per cent., or 3468 lbs., remained unaccounted for in the increase of crop at the end of the 26 years.

In reference to these results and conclusions, it is to be observed that the great reduction in the yield of nitrogen by the mineral manure in the later years is an indication of a gradual reduction of the more readily available nitrogen within the soil, and it is hardly to be supposed that, with the large supplies in the farmyard manure, the less readily available stores of the soil itself would be drawn upon to so great an extent as under the influence of the purely mineral manure. It is obvious that, so far as this was so, too much of the yield of nitrogen on the farmyard manure plot has been reckoned as derived from the soil, and too little as increase due to the manure—at any rate in the earlier years of the comparison; and this observation would apply to the estimates relating to wheat as well as to barley.

We have not taken samples of the soils of the barley plots, and have not therefore the means of estimating how much of the unrecovered nitrogen of the manure still remains in the soil, possibly to be gradually yielded up to succeeding crops. But, supposing 89·3 per cent. remained unaccounted for at the end of the 20 years of the application, and only 2·9 per cent. of the original supply were recovered in the first six years after the cessation of the application, or say 0·5 per cent. per annum, it would obviously take from 170 to 180 years to recover the whole of it, if at the same rate as during the first six years. There can be little doubt, however, that part of the unrecovered amount has been lost by drainage, or otherwise; and that whatever residue remains, a gradually decreasing proportion of it will be annually recovered.

But samples of the soils of the experimental wheat plots have been taken, and the nitrogen determinable in them by the soda-lime method estimated, at various times. The results so obtained enable us to form some judgment as to whether or not the whole of the nitrogen of the farmyard manure which is not recovered in the increase of crop remains available within the soil.

In the autumn of 1865, after wheat had been grown 22 years in succession, samples of the soils were taken from many of the plots, in each case from the first nine, the second nine, and the third nine inches, of depth, or to a total depth of 27 inches. The results showed rather more than one-and-two-thirds as much nitrogen so determinable in the first nine inches of the farmyard-manured, as in the corresponding layer of the unmanured plot, and about one-and-a-half time as much as, to a corresponding depth, of any of the plots receiving artificial nitrogenous manures. The second and third nine inches also showed rather higher percentages than those of the unmanured, or of most of the artificially-manured plots. More recent determinations—that is, after the farmyard manure had been applied some years longer—showed more than twice as much nitrogen in the first nine inches as without manure. The following table gives the results for the two wheat plots, in the same form as at page 372 for the grass plots, founded on the determinations made on the samples of soil taken in 1865, after 22 years' application of farmyard manure, and 22 wheat crops had been removed. The estimates of dry soil per acre are, for each depth, the average results obtained relating to eight samples from each of 11 plots.

TABLE XXVI.—Experimental Wheat Field. Estimated amount of dry soil per acre at each depth; Nitrogen per cent. in the dry soils, and Estimated Nitrogen per acre, at each depth.

	Average dry soil per acre, exclusive of stones.	Nitrogen.				
		Per cent. in dry soil.		Per acre.		
		Plot 3. Unmanured.	Plot 2. Farmyard-manured.	Plot 3. Unmanured.	Plot 2. Farmyard-manured.	Manured + or − Unmanured.
	lbs.	Per cent.	Per cent.	lbs.	lbs.	lbs.
First depth, 1-9 inches . .	2,287,155	·1090	·1882	2493	4304	+1811
Second depth, 10-18 inches .	2,712,508	·0738	·0810	2002	2197	+ 195
Third depth, 19-27 inches .	2,848,973	·0561	·0619	1598	1764	+ 166
Total, 27 inches . . .	7,848,636			6093	8265	+2172

The next table shows the estimates of the nitrogen—supplied in the manure, recovered in the increase of crop, determined in the soil, and not recovered in either the increase or the soil.

TABLE XXVII.—Experiments on Wheat. Estimated Nitrogen supplied in the Manure, recovered in the increase of crop, determined as residue in the soil, and not recovered in either the increase or the soil, to the depth of 27 inches.

	Nitrogen of manure.	
	Per acre.	Per cent.
	lbs.	
Supplied in farmyard manure in 22 years	4415	
Recovered as increase of crops	470	10·7
Not recovered in increase	3945	89·3
Residue determined by soda-lime in soil 27 inches deep	2172	49·2
Not recovered in increase or in soil	1773	40·1

Here we have, over the first 22 years of the wheat experiments, only 10·7 per cent. of the supplied nitrogen estimated to be recovered as increased yield, against 14·6 per cent. over the last 20, of the first 28 years (*see* p. 374). There is about one-and-a-half time as large a proportion determined as residue in the wheat soil, and therefore a less proportion estimated as loss, than in the case of the hay-plot reckoned to the depth of 54 inches. On the other hand, if the latter be taken to only the same depth as the wheat plots—that is, to 27 inches instead of 54—the proportion of the supplied nitrogen estimated to remain as residue in the soil is greater, and that estimated as loss less, in the case of the permanent meadow than in that of the wheat plots. Thus, in the case of the wheat experiments, the loss of nitrogen, by drainage or otherwise, is estimated to be about 40 per cent. of that supplied; in that of the hay-plot reckoned to the same, or 27 inches of depth, 25·2 per cent., or to the depth of 54 inches 49 per cent.

It must be borne in mind that the whole of the estimates on this point, whether relating to wheat, to barley, or to grass, are based on the assumption that the 14 tons of farmyard manure annually applied contained 0·64 per cent., or 200·7 lbs., of nitrogen; and if it did not contain so much, the percentage recovered was of course greater, and that not recovered less, than the foregoing calculations indicate. But it is at any rate clear that only a comparatively small proportion of the nitrogen so supplied is recovered in the increase of crop. The residue actually determined in the soil is very large; and it is possible that the whole of the nitrogen existing as nitric acid, especially in the subsoil, is not accounted for by the soda-lime determinations. It is very remarkable, however, that notwithstanding this great ascertained accumulation, and the annually renewed supply by manure, larger quantities of corn, or of straw, or of both, and also of hay, are every year obtained by the use (in conjunction with mineral manures) of much less than half as much nitrogen applied as ammonia-salts or as nitrate of soda. The wheat plots so manured, and so yielding, at the same time show less than two-thirds as high a percentage of nitrogen in the first nine inches of depth.

But, according to the estimates, besides the actually determined large, but comparatively ineffective, residue within the soil, there was also in each case a very large amount unaccounted for, either in the increase of crop or in the soil, to the depths examined. The quantity to be estimated as lost would, of course, be less if the estimate of the amount supplied be too high; again, by so much as may be retained as nitric acid, and not accounted for by the soda-lime method; and, again, by so much as may remain, either as nitric acid or in other forms, below the depth experimented on, but nevertheless not beyond the reach of root-collection, or of capillary action bringing up the stores from the lower to the upper layers.

Direct experiments have shown that the soil in the experimental wheat field which is manured annually with farmyard manure retains near the surface, owing to its greatly increased porosity, very much more of the rainfall than the soil of the plots not so manured. It is, accordingly, found that the drain from the farmyard-manured plot runs much less frequently than do those from the unmanured or the artificially manured plots. There will, obviously, be less loss of water by drainage. Direct experiments further show, however, that a given volume of the drainage water which does run from the farmyard-manured plot contains from two to three or more times as much nitrogen as nitrates and nitrites as that from the unmanured plot, or from the plots with mineral without artificial nitrogenous manure. We have here, therefore, a determined source of loss of the supplied nitrogen. But such calculations as the data admit of lead to the conclusion that the whole of the estimated loss cannot be accounted for in this way. The probability is that there is a considerable additional loss by decomposition of the nitrogenous organic matter within the soil, and evolution as free nitrogen.

There is, then, cumulative evidence to show—that the nitrogen supplied as farmyard manure was recovered in very small proportion during the years of its application; that in after years it was recovered in constantly decreasing proportion; that there nevertheless remained a considerable, but very slowly available, residue; that there was a considerable loss of it by drainage; and, finally, that there is probably a further loss by decomposition, and evolution into the atmosphere.

However significant these illustrations may be, it must be borne in mind that in ordinary agriculture much less farmyard manure would be applied than in these special experiments, and the losses by drainage would, from that cause alone, be proportionally less. Much, obviously, would also depend upon the character of the soil and the subsoil. Again, in an ordinary rotation of various crops, more of the supplied nitrogen would probably be gathered up before it finally passed beyond the reach of vegetation, than in the case of a single cereal crop grown year after year on the same land. For somewhat similar reasons it might indeed have been expected that, with the very varied herbage of grass-land, and possession of the soil by the roots of many descriptions of plant, the year round, a better result would have been obtained with the mixed herbage as compared with the cereal crops, than the evidence, as it stands, would show. That it was not so, may perhaps be taken to indicate that, in estimating the proportion of the nitrogen of the produce due to that supplied in the manure, it should not be assumed that as much was derived from natural sources as in the case of the unmanured produce, but more should be reckoned as derived from the manure.

So much for the nitrogen. It will be of interest also to consider whether or not the most important mineral constituents of the manure are as slowly available to the crop.

Of the amount of lime estimated to be supplied in the manure in the eight years, only about 12½ per cent. was obtained in the total produce of those years, about 9 per cent. in the next six, and little more than 4 per cent. in the last six years, making in all only about 25½ per cent. in the 20 years. Deducting the yield in the unmanured produce, however, there was an increase obtained representing only 3½ per cent. of the amount estimated to be supplied during the eight years, little more than 2 per cent. in the next six years, a small fraction of 1 per cent. in the second six, and not quite 6 per cent. in the 20 years.

Of the magnesia, a much larger proportion would appear to be taken up; that in the total produce (of the 20 years) amounting to nearly 70 per cent., and the estimated increased yield to about 21 per cent. of that supplied.

Of the potass estimated to be supplied in the eight years, the produce of the period contained 44 per cent. as much, that of the next six years 22½ per cent. more, and that of the last six years 10½ per cent. more; in all, 77 per cent. But the increased yield of potass represented only 30½ per cent. during the eight years, scarcely 13 per cent. during the next six, and only 4½ per cent. during the last six years; in all, only about 48 per cent. of that supplied. This result, however, upon whichever basis taken, shows a great effect from the potass of the dung, though a much diminishing one in the later years. It is known that potass is, at any rate in moderately clayey soils, very little subject to loss by drainage; but it would appear that the unrecovered residue becomes so locked up (or distributed) as to be but slowly available to succeeding crops.

In the presence of an abundant supply of potass, there was even less soda taken up in the manured than in the unmanured crop, during the years of the application; though, during the subsequent years, there was some, but comparatively little, increase in the amount compared with that in the unmanured.

Of phosphoric acid, a larger proportion of that supplied, though not so much as of the potass, would appear to be taken up. In the 20 years, about 57 per cent. of the quantity estimated to be supplied was contained in the total produce, whilst the increased yield represented 33 per cent. The residue of the phosphoric acid, like that of the potass, is very little subject to loss by drainage.

Of sulphuric acid, there is a much less increased amount found in the ash of the crop, in proportion to that estimated to be supplied, than of phosphoric acid.

Of chlorine, the increased amount found in the produce is, during the years of the application of the manure, greater in proportion to the estimated supply than that of any other constituent; but the proportion diminishes more rapidly in the subsequent years than that of any other constituent. Both chlorine and sulphuric acid are very subject to loss by drainage.

Lastly, of silica, the produce of the 20 years contained about 41½ per cent. as much as there was estimated to be supplied of soluble silica in the dung, and the increased yield of it represented about 22 per cent.

Thus, of the three more important constituents of manure, nitrogen, potass, and phosphoric acid, when these are supplied in farmyard manure, the nitrogen is recovered in the least proportion in the increase of the crop for which it is applied; it leaves a large determinable residue within the soil, which, however, is very slowly available to succeeding crops; and, finally, it is subject to serious loss by drainage, and probably by evolution into the atmosphere also. The potass, so supplied, is recovered in increase in much greater proportion during the years of the application; in much greater, though still rapidly decreasing, proportion, in subsequent years, and is very little subject to loss by drainage. The phosphoric acid, again, is recovered in much greater proportion than the nitrogen, but not in so large a proportion as the potass; it too, like the potass, is but little subject to loss by drainage.

The much less immediate effect of a given amount of nitrogen when supplied in farmyard manure than when in ammonia-salts or nitrate of soda, the consequent necessity to supply so much more in that form to obtain a given result, and the very slow action of the remaining residue, are important elements in the scientific explanation of the practically recognised much lower money value of a given amount of nitrogen so supplied.

It remains to point out the difference of effect when, besides the farmyard manure, 200 lbs. of ammonia-salts were also annually applied per acre, both over the eight years of the application of the dung, and over the next 12 years of the action of the residue. As already explained, plot 1 was devoted to this experiment.

Under the influence of the addition of the ammonia-salts, the growing herbage acquired a darker green colour; gramineous species became more, and both leguminous and miscellaneous species less prominent, than either on the unmanured plot or that with the farmyard manure alone. Compared with the latter, in the early years, *Poa trivialis*, and *Bromus mollis*, were even more prominent, as also was *Dactylis glomerata;*

and these three grasses made up a large proportion of the total produce. During the later years, as on the plot without ammonia, but in a greater degree, *Agrostis vulgaris*, and *Holcus lanatus*, became very prominent, as also did *Anthoxanthum odoratum*, and *Festuca ovina;* whilst, on both plots, the *Poa trivialis*, the *Bromus mollis*, and the *Dactylis glomerata*, went down very much. Other grasses were represented in less number, and generally in less proportion, on the plot with ammonia.

Of leguminous plants, *Lathyrus pratensis* is the most prominent on both plots; but, whilst without the ammonia the quantity at the last separation was between 5 and 6 per cent., with the ammonia there has been but a fraction of 1 per cent. of it; and, again, without ammonia there was nearly 1 per cent. of *Trifolium pratense*, but with it there was none at all.

Of miscellaneous species, many more were fairly prominent without than with the ammonia; though with it, *Rumex acetosa* was not only much more prominent than without it, but yielded a very large proportion of the total miscellaneous herbage. Finally in reference to the botanical character of the produce—a much greater number of species, gramineous, leguminous, and miscellaneous, contribute to the produce without, than to that with, the ammonia-salts.

In Table XX., p. 368, the produce of hay, and its contents of nitrogen and mineral matter, are given, for each period, with the farmyard manure and ammonia-salts, side by side with those on plot 2 with the farmyard manure alone.

Over the eight years of the application of the dung, there was an average of 734 lbs., over the first six years after the cessation, 520 lbs., over the next six years, 814 lbs., and over the whole period of 20 years, 694 lbs., more hay by the dung and ammonia-salts, than by the dung alone. Although the increase of produce, due to the ammonia-salts, was thus pretty constant, the actual amount of produce per acre was nearly 40 per cent. less over the last six years than over the first eight. Where the farmyard manure was used alone, however, the decline was greater still, being nearly 48 per cent.

The 200 lbs. of ammonia-salts annually applied are estimated to supply about 41 lbs. of nitrogen, equal to about 50 lbs. ammonia. The increased yield of nitrogen in the crop was, however, only 10·1 lbs. per acre per annum over the first eight years, 10·6 lbs. over the next six, 13·5 lbs. over the last six, and 11·3 lbs. over the 20 years. In other words, there was, so reckoned, an average of only about 27½ per cent. of the nitrogen so supplied recovered as increase in the crop.

Of mineral matter, there was also an increased amount taken up under the influence of the ammonia-salts. But, as without manure, and as with farmyard manure alone, the actual amount annually taken up was very much less over each succeeding period; indeed, on both plots, 2 and 1, it was considerably less than half as much over the last six years as over the eight years of the application of the dung. Still, the ammonia-salts undoubtedly had the effect of increasing the amount of mineral matter taken up.

Over each period there was more lime, magnesia, phosphoric acid, and sulphuric acid, and very much more chlorine, taken up with, than without, the ammonia-salts. Of sulphuric acid and chlorine there was, of course, a considerable quantity annually applied in the mixture of sulphate and muriate of ammonia used. Of potass, there was an increased amount taken up during the first eight years, actually rather less during the next six years, and very little more during the last six—in all, over the 20 years, not 3 lbs. per acre per annum more taken up with, than without, the ammonia-salts. There was, however, over each period, very much more soda taken up with the ammonia-salts, and over the whole period there was nearly three times as great an increase of soda as there was of potass in the produce. Of silica, there was rather more taken up with ammonia than without it over the eight years of the application of the dung, but rather less subsequently, and over the 20 years all but identical amounts on both plots.

It is obvious that the increased amounts of lime and magnesia, of potass and soda, and of phosphoric acid, taken up under the influence of the ammonia-salts, must have had their source in the previous supplies within the soil, or in the residue from the farmyard manure; and, so far, the action of the ammonia-salts has been more rapidly to utilize, and therefore the more to exhaust, these otherwise dormant stores. But, from the very slight increase in the amount of potass taken up, and the very much greater increase in that of soda, it may be concluded that there was a relative deficiency of available potass within the soil, notwithstanding the comparatively large amount probably supplied in the dung; in fact, the increased yield of potass over that without manure amounted, on both plots, to only about half that estimated to be supplied. It would appear that the residue of the potass of the dung, which we know to be but little subject to drainage, was nevertheless but slowly available to the succeeding crops. Of phosphoric acid, again, there was very much less increased amount taken up under the influence of the ammonia-salts than there was of either sulphuric acid or chlorine, both of which were so liberally supplied. It was probable that, of both the potass and the phosphoric acid supplied in the dung, part remains unliberated from its original condition of combination in the manure, and part becomes so locked up (or distributed) within the soil, as to be only very slowly available.

Of nitrogen, there was a somewhat higher percentage in the dry substance of the produce with, than in that without, the application of ammonia-salts, but still less than in that without manure, which contained so much more leguminous and miscellaneous herbage. Of total mineral matter, there was a lower percentage with than without the ammonia-salts. Of magnesia and of sulphuric acid, but especially of soda and of chlorine, the percentage was higher in the dry substance of the hay grown with the ammonia-salts; but of lime, and of phosphoric acid, the percentage was less, and of potass and silica (especially in the later years) it was very much less, under the influence of the ammonia-salts.

Upon the whole, the evidence goes to show that the effect of the ammonia-salts was

—to reduce the complexity of the herbage, to render it more gramineous, to increase the amount of produce, and, with this, to draw more upon the mineral stores within the soil. It would, at the same time, appear, that where the ammonia-salts were applied, in amount supplying only 41 lbs. of nitrogen per acre per annum, the available supply of nitrogen was relatively higher than the available supply of some of the more important of the mineral constituents, more especially of potass and phosphoric acid, notwithstanding (to say nothing of the stores of the soil itself) the large residue of these estimated to have accumulated, and to remain, within the soil.

It will be remembered it was estimated that, over the eight years of the application of the dung alone, only about 10½ per cent. of the supplied nitrogen was recovered as increase in the crop during that period, and that little more than 18 per cent. of the eight years' supply was recovered in the 20 years. In a similar way, it was estimated that, over the eight years of the application of both the farmyard manure and the ammonia-salts, about 13 per cent. of the total nitrogen supplied was recovered, and over the 20 years—eight with dung and ammonia-salts, and 12 with ammonia-salts alone—about 21½ per cent.; or, if the increased yield of nitrogen with the ammonia-salts over that without them be assumed to represent the amount derived from them, it would result that about 27½ per cent. of the nitrogen of the ammonia-salts was recovered, whilst only about 18 per cent. of that of the farmyard manure was recovered. It should be observed, however, that much more than 27½ per cent. of the nitrogen of ammonia-salts, or of nitrate of soda, is estimated to be recovered when these were employed in conjunction with an annual supply of soluble artificial mineral manures.

There is thus, here, further evidence of the much less effect of a given amount of nitrogen supplied in farmyard manure than as ammonia-salts. It is also clear that, although the application of the ammonia-salts was the means of turning to account some of the accumulated residue of the mineral constituents supplied in the dung, the limit of the immediately available supply was very soon reached, the remainder becoming less and less rapidly recoverable. It was, in fact, retained in a condition so slowly available as to be of but little effect in increasing immediate crops, and therefore of but little practical value, except as a storehouse against exhaustion.

THE SECOND CROPS.

So far, the produce of the first crops only, of each year, has been taken into account. As already explained (pp. 7 and 8), the second crops were, as a rule, fed off by sheep, having no other food, that they might contribute nothing in their manure that they had not derived from the land; whilst, having no other food, they sometimes even lost weight, especially in bad weather. They would, therefore, generally retain but little, and sometimes none at all, of either the nitrogen or the mineral constituents of the grass they consumed; and they might even sometimes themselves lose something. On this point it may be observed that, even if progressing favourably, they would probably retain not more than from 5 to 10 per cent. of the nitrogen of such food.

The object was, indeed, to secure that the treatment of the second crops should interfere as little as possible with the balance of constituents of the respective plots, as affected, on the one hand by the application of manure of known composition, and on the other by the removal of the first crops only, the quantity and composition of which were determined. In 15 out of the 20 years the second crops were so fed off by sheep; but, as the animals frequently suffered so much from change from better food, bad weather, or both, the plan was not always adopted in the later years of the 20, and has now been finally abandoned altogether. Thus, in the eleventh, fifteenth, eighteenth, and nineteenth seasons (1866, 1870, 1873, and 1874), the produce of aftergrass was cut, carefully spread on its own plot, and left to decay; whilst, in 1875, after the twentieth first crop had been removed, the second was cut, made into hay, removed, and weighed. In the four years in which the aftergrass was cut and spread on the land, we need not, perhaps, assume any loss of either nitrogen or mineral matter by the treatment. In the 15 seasons in which the second crops were fed off by sheep, there may have been some slight removal of those constituents; but, on the other hand, the nearly total amount returned to the land would be in a much more active manurial condition than in the cases when the produce was cut and spread on the land. Upon the whole, therefore, it is doubtless nearer the truth to assume the balance of constituents on the different plots to depend on the amounts supplied in manure, and those taken off in the first crops only, than to attempt any numerical estimate of the quantities lost by the feeding of the sheep on the plots. At the same time, it may be considered that the estimates of constituents removed in the first crops probably somewhat understate, especially in some seasons, the actual loss to the land.

Although it may thus be assumed that the chemical condition of the respective plots, so far as the actual amount of manurial constituents is concerned, would be little affected by the treatment of the second crops, it cannot be doubted that the character of the complex herbage would be influenced thereby, and differently according to the character of the manures, and of the seasons. As a means of forming some judgment on the point, botanical notes were generally made on the growing second crops; and, in order to arrive at some estimate of the actual and relative amounts of aftergrass, whenever the second crops were fed off by sheep, a given number of animals was allotted to each plot, according to its produce, penned on a portion of it, and the fold extended day by day as the grass was eaten down. In this way data were obtained for calculating how many sheep would be maintained per acre, for one week, on each plot; and, assuming that they would, on an average, consume grass equal to 16 lbs. of hay per head per week, the quantity of aftergrass, reckoned as hay, was estimated. The results so obtained are, of course, only approximations to the truth; but such they are; and they may be taken as at any rate giving some idea of the actual and relative amounts of growth on the different plots, and in the different seasons. Appendix-Table II. (p. 408), gives the so-estimated amounts of second crop, on each plot, in each season. But the general character and bearing of the results will be sufficiently brought to view in the following abstract table (XXVIII.), in which are given those for a selection of the plots only, representing very characteristically different conditions of manuring. The plots for which the results are given are—

Plot 3. Unmanured, every year.
Plot 7. Mixed mineral manure (including potass), without nitrogenous manure, every year.
Plot 9. Mixed mineral manure (including potass), and 400 lbs. ammonia-salts, every year.
Plot 11. Mixed mineral manure (including potass), and 800 lbs. ammonia-salts.
Plot 14. Mixed mineral manure (including potass), and nitrate of soda, containing the same amount of nitrogen as the ammonia-salts on plot 9.

The particulars given are—the actual amounts of produce of the first crops, the estimated amounts of the second crops, and the proportion of the second to the first as 100; and, at the bottom of the table are given the averages—of the first crops, of the estimated second crops, and the percentage of the second to the first, for the first eight years of the 20, in every one of which the second crops were fed off and their quantities estimated, and for eight subsequent (though not consecutive) years, in seven of which the second crops were fed and estimated, and in the eighth (the last of the 20) in which the second crops were cut, removed, and weighed. There are also given, at the foot of the table, the particulars of the produce, for the same plots, for the three years subsequent to the first 20.

TABLE XXVIII.—Actual amounts of the first crops of Hay, estimated amounts of the second crops, and proportion of the second to the first as 100, on selected plots.

Years.	Hay per acre, per annum.														
	Plot 3. Unmanured, continuously.			Plot 7. Mixed mineral manure (with potass), every year.			Plot 9. As Plot 7, and 400 lbs. ammonia-salts.			Plot 11. As Plot 7, and 800 lbs.* ammonia-salts.			Plot 14. As Plot 7, and 550 lbs. nitrate soda.		
	Actual first crops.	Estimated second crops.	Second to first as 100.	Actual first crops.	Estimated second crops.	Second to first as 100.	Actual first crops.	Estimated second crops.	Second to first as 100.	Actual first crops.	Estimated second crops.	Second to first as 100.	Actual first crops.	Estimated second crops.	Second to first as 100.
20 YEARS, 1856–1875 INCLUSIVE.															
	lbs.	lbs.	Per cent.	lbs.	lbs.	Per cent.	lbs.	lbs.	Per cent.	lbs.	lbs.	Per cent.	lbs.	lbs.	Per cent.
1856	2515	1188	47·2	3439	1280	37·3	6363	1507	23·7	6970	2013	28·9	..	..	..
1857	2856	950	32·3	3666	1485	40·5	6422	1645	25·6	6940	2195	31·6	..	..	..
1858	2472	1098	44·4	4082	1373	33·6	7172	1645	22·9	7508	2195	29·2	5646	1645	29·1
1859	2540	914	36·0	3416	1234	36·1	6198	1234	19·9	7150	1646	23·0	6072	960	15·8
1860	2760	2112	76·5	3928	2378	60·5	5624	2434	43·3	5744	2434	42·4	5586	2010	36·0
1861	2844	1426	50·1	4488	1782	39·7	6316	1902	30·1	6710	2021	30·1	5892	1728	29·3
1862	3052	1718	56·3	4424	1901	43·0	6402	1883	29·4	7114	2725	38·3	5718	2304	40·3
1863	2284	822	36·0	3870	1206	31·2	6026	1082	18·0	8364	1462	17·5	6528	603	9·2
1864	2688	549	20·4	4130	693	16·8	5628	869	15·4	7426	869	11·7	6816	1029	15·1
1865	1296	1462	112·8	2540	1600	63·0	3866	1234	31·9	6024	2742	45·5	5292	1603	30·3
1866	2660	..	..	4180	..	..	4956	..	..	8092	..	..	6576	..	..
1867	3332	1098	33·0	4458	1645	36·9	5382	1645	30·6	5808	2194	37·8	7183	768	10·7
1868	1960	460	23·5	4264	1024	24·0	6622	1024	15·5	7616	1280	16·8	7728	1029	13·3
1869	4256	1098	25·8	6124	1462	23·9	7700	1280	16·6	8610	1472	17·1	8526	1115	13·1
1870	644	..	..	1968	..	..	3306	..	..	5150	..	..	6300	..	..
1871	2844	1646	57·9	4414	1886	42·7	6576	1920	29·2	6856	2400	35·0	6930	3291	47·5
1872	1644	2011	122·3	4236	1920	45·3	5658	1874	33·1	7143	2149	30·1	6215	1371	22·0
1873	1872	..	..	3892	..	..	4984	..	..	5782	..	..	5796	..	..
1874	1412	..	..	3088	..	..	3290	..	..	3540	..	..	5484	..	..
1875†	2236	1384	61·9	4560	2684	58·8	5828	2729	46·8	5976	5132	85·9	7026	1974	28·1
AVERAGES.															
8 years, 1856–1863	2665	1279	48·0	3913	1580	40·4	6315	1667	26·4	7063	2086	29·5	5907‡	1542‡	26·1‡
8 years, 1864,'65,'67,'68,'69,'71,'72, and '75	2532	1213	47·9	4341	1614	37·2	5908	1572	26·6	6932	2280	32·9	6965	1522	21·9
16 years, 1856 =	2599	1246	47·9	4127	1597	38·7	6111	1619	26·5	6997	2183	31·2	6512§	1531§	23·5§
SUBSEQUENT YEARS.															
1876	1334	..	..	3886	..	..	5604	..	..	6858	..	..	7206	..	..
1877†	2364	1960	82·9	5100	2674	52·4	6054	2448	40·4	7648	4632	60·6	6258	2124	33·9
1878†	1848	1496	80·9	3926	2532	64·5	6290	2707	43·0	6260	4426	70·7	5382	1692	31·4

* 400 lbs. only in 1859, 1860, and 1861.

‡ Averages of 6 years only, 1858–1863.

† Second crops cut, removed, and weighed.

§ Averages of 14 years only, 1858–1863, and 1864, '65, '67, '68, '69, '71, '72, and '75.

We need not forestall, or incur the necessity of repetition, by discussing here the climatic characters of the different seasons. It will be sufficient to call attention to the very great variation in the estimated second, as in the actual first crops, from year to year, under the influence of the same manurial conditions. But reference to the columns showing the proportion of the second crops to the first each year will show that it sometimes varied very considerably. It is obvious that, a deficiency of rain, or a relatively too high, or too low, or too great a range of, temperature, during the spring and early summer, would give a deficient first crop, and leave the land correspondingly less exhausted. If then, the summer and autumn were favourable for growth, the second crop would have the additional advantage of unexhausted manurial condition of land. If, on the other hand, the earlier period were favourable for luxuriant first crop, the land would be comparatively exhausted for the second, even with suitable climatal conditions for its growth. And if, to add to exhaustion by the first crop, the succeeding climatal conditions are unfavourable for second growth, we shall have a still lower amount, and proportion, of second crop. Again, if both first and second crops are heavy, or the second only is heavy, in any particular season, this will have some effect, not only on the botany, and the general character, of the vegetation, but more or less on the condition of the land also, for the growth of the first crop the next year.

The results recorded in the table illustrate the great variation in the conditions referred to, from season to season. In regard to the amounts of second crop indicated by the estimates, it may be observed that, judging from the actual weights of the second crops obtained in 1875, 1877, and 1878, it may be concluded that the previously estimated amounts were more probably too low than too high; though it is true that these later seasons were more favourable for second growth than the majority of the earlier ones, for which estimates only were made. In fact, it was partly on account of the luxuriance of the second crops of these later years, that it was decided to cut and remove them from the land. It is probable, however, that a larger amount of the dry substance of the deficiently matured second, than of the better matured first crops of grass, would be required as food, and that therefore the estimate of 16 lbs. of such hay consumed per head per week may be too low.

Turning to the summaries at the bottom of the table, it is observed that, although without manure the amounts of produce, of both first and second crops, are small, the proportion of second crop to first is greater than under either of the selected manurial conditions; that is, it is greater where the total removal from the land is comparatively small, and where, especially, the variety of the herbage is the greatest, and where, consequently, the possession by the roots of the upper layers of the soil, and the capabilities of food-collection generally, will be the most varied.

Next in proportion of second crops to first comes the mineral-manured plot (7). Here, again, the crops, though much larger than without manure, are not really large;

but, as without manure, the herbage is complex, and the command by the roots, especially of the upper layers of the soil, will be very varied.

With the same mineral manure as on plot 7, and 400 lbs. ammonia-salts per acre per annum in addition (plot 9), the first crops average about one-and-a-half time as much as with the mineral manure alone, but the estimated average of the second crops is very nearly the same in the two cases. Accordingly, under the influence of the addition of the ammonia-salts, the proportion of the second crop to the first is much less than with the mineral manure alone, and in a still greater degree less than without manure. Thus, with the much more luxuriant growth of first crops under the influence of the ammonia-salts, and the much more simple, and almost exclusively gramineous herbage, the actual quantity of the second crop is small, and its proportion to the first little more than half as much as without manure, and only about two-thirds as much as with the mineral manure alone.

With, besides the mineral manure, nitrate of soda containing approximately the same amount of nitrogen as the 400 lbs. of ammonia-salts, the first crops averaged more still; they, also, consisted almost exclusively of free-growing (though chiefly other) grasses; and they comprised but few species. With these characters, the second crops averaged even rather less than with the ammonia-salts, and bore a smaller proportion to the first.

In the cases of plot 9 with the mixed mineral manure and ammonia-salts, and of plot 14 with the mixed mineral manure and nitrate of soda, the amount of nitrogen applied was sufficient to give nearly the maximum growth of first crops, but not so excessive as to give a percentage of nitrogen in the produce much higher than is normal in the almost exclusively gramineous hay; indicating, therefore, that the unrecovered amount of supplied nitrogen did not remain within the soil in a readily available condition; and hence, probably, in part, the comparatively little increase of growth of second crop. In the case of plot 11, however, with double the amount of ammonia-salts of plot 9, we have upon the whole still larger first crops, and almost exclusively gramineous herbage, which contained, however, a very abnormally high percentage of nitrogen; and, with the obvious excess available, there is here more second crop, and a higher proportion of second crop to first, than with the smaller amount of nitrogenous manure. There is, moreover, a tendency to a greater amount, and proportion, of second crop in the later years.

The general result is that, when (with mineral manure) active nitrogenous manures are used, but not in excessive amount, the increase of the first crop will, in favourable seasons, be such as to leave comparatively little available nitrogenous residue for the second crop; whilst, the produce under such circumstances being characteristically gramineous, and comprising comparatively few species, the condition of the herbage is not very favourable for subsequent growth. It will be seen further on, however, that the percentage of both mineral matter and nitrogen is generally much higher in the dry substance of the second and less matured produce, than in that of the first and more matured. The removal of the second crops, indeed, is a considerably greater drain upon the resources of the soil than might be judged from the comparatively small amounts of the produce. It is evident, too, that the actual and relative amounts of second crop depend not only on the balance of available constituents remaining within the soil, and on the climatal conditions, but also on the variety, and the unexhausted condition, of the plants themselves which are comprised in the mixed herbage.

INFLUENCE OF SEASON ON THE PRODUCE OF HAY.

In the foregoing discussion of the produce of both the first and the second crops of hay, the influence of variation in the climatal characters of the different seasons has been disregarded as far as practicable, with the object of bringing to view as prominently as possible the characteristic effects of the different manures as shown by the results obtained over series of various seasons. The object now will be to endeavour to trace the connexion between certain measurable characters of season on the one hand, and the luxuriance or sluggishness of growth of the mixed herbage on the other, with comparatively little reference to the effects of the different manures.

Common observation recognises a general connexion between the characters of the weather as to moisture, heat, and light, and the luxuriance or the scantiness of vegetation. When, however, we come to compare the amounts of growth in different seasons with the usual meteorological records of the period, we at once discern how complicated is the connexion, and how inadequate are such records for a full explanation of the differences of result obtained in different seasons.

As we have said elsewhere: "It is obvious that different seasons will differ almost infinitely at each succeeding period of their advance, and that, with each variation, the character of development of the plant will also vary, tending to luxuriance, or to maturation, that is, to quantity, or to quality, as the case may be. Hence, only a very detailed consideration of climatic statistics, taken together with careful periodic observations in the field, can afford a really clear perception of the connexion between the ever-fluctuating characters of season and the equally fluctuating characters of growth and produce. It is, in fact, the distribution of the various elements making up the season, their mutual adaptations, and their adaptation to the stage of growth of the plant, which throughout influence the tendency to produce quantity or quality. It not unfrequently happens, too, that some passing conditions, not indicated by a summary of the meteorological registry, may affect the crop very strikingly; and thus the cause will be overlooked, unless careful observations be also made, and the stage of progress, and tendencies of growth, of the crop itself at the time, be likewise taken into account." Again: "Those characters of season which are very unfavourable for land in poor condition, may be favourable to land in high condition, and *vice versâ*."

If such be the difficulty and the intricacy of the subject when the growth of a crop consisting of a single species only is concerned, how much more difficult and intricate must it be when we have to deal with the relations of the various climatal conditions to the development of a great variety of species growing together, as in the case of the mixed herbage, and numbering, as they do, under 20, or 50 or more, according to the varied manurial conditions provided?

Still, it will be of interest to show the fluctuations of produce from season to season, side by side with those in the characters of the seasons themselves, so far as such records as we have at command bearing upon the subject enable us to do so. It would, however, occupy far too much space to attempt to point out the connexion between the amount of produce and the climatal conditions of each individual season. It must suffice in this place to draw attention to the actual and the comparative characters of season under which some of the largest, and some of the smallest, amounts of produce have been grown. In Part II., relating to the Botanical Results, the connexion between climatal conditions and growth will be further brought to view, in discussing the characters of the seasons and the amounts and the characters of the growth, in the four years in which the botanical analysis of the mixed produce was undertaken.

Throughout the whole period of the experiments on mixed herbage, and for some years prior to their commencement, the rainfall has been registered at Rothamsted. At first the temperature also was observed, but owing to the difficulty of securing the completeness of the record throughout the year, the observations were abandoned. We are obliged to fall back, therefore, on the published records of the observations at Greenwich Observatory; which, though obviously inapplicable for the discussion of the subject in much detail, nevertheless sufficiently clearly indicate, for our present limited purpose, the distinctive and comparative characters of the different seasons, or parts of seasons, so far as temperature is concerned. With regard to light, there are not wanting so-called actinometric observations of various kinds; and there have also been of late years diurnal records of the number of hours of sunshine. There is, too, reason to hope that methods of systematic observation and record will before long be perfected and simplified; and that, at no very distant future, data relating to the quantity and quality of the sunlight at different seasons of the year, or in different years, may be available as another element in the study of the connexion between meteorological influences and vegetation. In the meantime, comparative temperature is the most reliable basis we possess for any deductions as to the comparative amounts of effective sunlight, as between one and the same period of different years; but it is obviously less directly applicable for the purpose of drawing conclusions as between different periods of the same season.

In Appendix-Tables III. to IX. (pp. 409 to 415), are given, for each year of the first 20, and also for each of the three subsequent years—

In Table III., the actual amounts of produce of hay per acre (first crops) on five selected plots, and the indices of those amounts.

In Table IV., the monthly rainfall in inches (at Rothamsted).

In Table V., the number of days in each month when the rainfall exceeded 0·01 inch.

In Table VI., the monthly mean maximum temperature (at Greenwich).

In Table VII., the monthly mean minimum temperature (at Greenwich).

In Table VIII., the monthly mean temperature (at Greenwich).

In Table IX., the monthly mean range of temperature (at Greenwich).

And, in the respective tables, there are given, for each season, the total rainfall, the total number of days on which 0·01 inch, or more, fell, and the means for each item of temperature—

1. For the total 12 months; July to June inclusive.
2. For four months—July, August, September, and October;
3. For four months—November, December, January, and February;
4. For four months—March, April, May, and June;
5. For three months—April, May, and June;
6. For two months—April and May.

It should be explained that, in Appendix-Table III., relating to produce, that of the same very characteristically differently manured plots is selected for illustration, as in the case of the consideration of the second crops; in the left-hand columns of the table the actual amounts of produce of each plot are given, and in the right-hand columns the indices of these amounts—that is to say, for each plot the highest amount of produce in any one year of the 20 is set down as 1, the second highest amount as 2, and so on.

An examination of the columns of indices of produce will show that, with each of the five very different conditions of manuring, 1869 gave the highest amount of produce. On the other hand, 1870 gave, with two out of the five conditions of manuring the lowest produce, with two others the lowest but one; but the remaining or nitrated plot was an exception, giving, in this year of drought, 1870, an abnormally high produce for the season—a fact which has been fully considered already (p. 334, *et seq.*). There can be no hesitation, therefore, in taking the season of 1869 as that of the highest productiveness of the 20, and that of 1870 (with the exception mentioned) as one of the lowest productiveness; the next in order in this respect being 1874. It will be interesting to compare the produce in these two most contrasted seasons, 1869 and 1870, and also the characters of the seasons themselves.

In Table XXIX. are given, for each of the selected plots, the average produce (of first crops) over the 20 years; the produce in 1869; the produce in 1870; the difference in amount over (+), or under (—), the average of the 20 years; and the deficiency in 1870 compared with 1869.

TABLE XXIX.—Produce of Hay per acre on selected plots. Average of the 20 years; produce of 1869, the year of highest productiveness; produce of 1870, the year of lowest productiveness; difference of each from the average; and difference of the one from the other.

	Plot 3. Unmanured, continuously.	Plot 7. Mixed mineral manure, alone.	Plot 9. Mixed mineral manure, and 400 lbs. ammonia-salts.	Plot 11. Mixed mineral manure, and 800 lbs.* ammonia-salts.	Plot 14. Mixed mineral manure, and 550 lbs. nitrate soda.	Means.
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Average 20 years, 1856–1875 .	2383	3958	5711	6726	6407†	5037
1869 .	4256	6124	7700	8610	8526	7043
1870 .	644	1968	3306	5150	6300	3474
+ or − average { 1869 .	+1873	+2166	+1989	+1884	+2119	+2006
+ or − average { 1870 .	−1739	−1990	−2405	−1576	− 107	−1563
1870–1869	−3612	−4156	−4394	−3460	−2226	−3569

The next Table (XXX.) shows some of the meteorological conditions under which the two very different crops were grown. The particulars are given—for each of the 12 months from July to June separately, and for series of months collectively, for the average of the 20 years, for 1868–1869, and for 1869–1870. The variations of each of the two seasons + or − the average, and the variations of 1869–1870, the year of lowest produce, + or − 1868–1869, the year of highest produce, are also given.

---

* 400 lbs. only in 1859, 1860, and 1861.

† Average of 18 years only, 1858–1875.

TABLE XXX.—Abstract of meteorological conditions of 1868–9, the year
comparison of each with the average of 20

	Monthly.							
	July.	August.	September.	October.	November.	December.	January.	February.
RAINFALL AT ROTHAMSTED.—INCHES.								
Average 20 years, 1855–6–1874-5	2·46	2·49	2·51	3·07	1·99	2·04	2·79	1·57
1868–1869	0·37	3·77	2·80	2·04	0·42	4·55	3·44	2·41
1869–1870	0·97	1·35	2·79	2·05	2·38	3·20	1·81	2·10
1868-9 + or − average of 20 years	−2·09	+1·28	+0·29	−1·03	−1·57	+2·51	+0·65	+0·84
1869–70 + or − average of 20 years	−1·49	−1·14	+0·28	−1·02	+0·39	+1·16	−0·98	+0·53
1869–70 + or − 1868–9	+0·60	−2·42	−0·01	+0·01	+1·96	−1·35	−1·63	−0·31
RAINFALL AT ROTHAMSTED.—NUMBER OF DAYS WHEN 0·01 INCH, OR MORE, FELL.								
Average 20 years, 1855–6–1874–5	12	13	13	16	14	14	16	12
1868–1869	4	15	9	11	8	22	14	16
1869–1870	6	11	12	17	12	18	17	21
1868-9 + or − average of 20 years	−8	+2	−4	−5	−6	+8	−2	+4
1869–70 + or − average of 20 years	−6	−2	−1	+1	−2	+4	+1	+9
1869–70 + or − 1868–9	+2	−4	+3	+6	+4	−4	+3	+5
MEAN MAXIMUM TEMPERATURE AT GREENWICH (FAHRENHEIT).								
Average 20 years, 1855–6–1874–5	75·2	73·5	67·9	59·0	48·3	44·4	43·8	46·1
1868–1869	82·0	75·1	71·7	56·9	46·9	50·5	46·0	51·8
1869–1870	77·0	72·3	68·6	57·5	49·1	42·0	42·6	41·4
1868-9 + or − average of 20 years	+6·8	+1·6	+3·8	−2·1	−1·4	+6·1	+2·2	+ 5·7
1869–70 + or − average of 20 years	+1·8	−1·2	+0·7	−1·5	+0·8	−2·4	−1·2	− 4·7
1869–70 + or − 1868–9	−5·0	−2·8	−3·1	+0·6	+2·2	−8·5	−3·4	−10·4
MEAN MINIMUM TEMPERATURE AT GREENWICH (FAHRENHEIT).								
Average 20 years, 1855–6–1874–5	53·2	53·0	49·4	44·1	36·7	35·9	34·0	34·7
1868–1869	55·7	55·0	51·1	40·2	36·1	41·1	36·5	39·7
1869–1870	54·5	52·4	52·4	42·0	37·4	33·4	34·0	31·9
1868–9 + or − average of 20 years	+2·5	+2·0	+1·7	−3·9	−0·6	+6·1	+2·5	+5·0
1869–70 + or − average of 20 years	+1·3	−0·6	+3·0	−2·1	+0·7	−1·6	0·0	−2·8
1869–70 + or − 1868–9	−1·2	−2·6	+1·3	+1·8	+1·3	−7·7	−2·5	−7·8
MEAN TEMPERATURE AT GREENWICH (FAHRENHEIT).								
Average 20 years, 1855-6-1874-5	62·6	61·7	57·4	50·6	42·2	39·8	38·8	39·8
1868–1869	67·5	63·6	60·5	47·9	41·5	46·0	41·1	45·3
1869–1870	64·5	60·8	59·0	48·9	43·0	37·9	38·3	36·2
1868-9 + or − average of 20 years	+4·9	+1·9	+3·1	−2·7	−0·7	+6·2	+2·3	+5·5
1869–70 + or − average of 20 years	+1·9	−0·9	+1·6	−1·7	+0·8	−1·9	−0·5	−3·6
1869–70 + or − 1868–9	−3·0	−2·8	−1·5	+1·0	+1·5	−8·1	−2·8	−9·1
MEAN RANGE OF TEMPERATURE AT GREENWICH (FAHRENHEIT).								
Average 20 years, 1855–6–1874–5	22·0	20·5	18·5	14·9	11·8	9·6	9·6	11·4
1868–1869	26·3	20·1	20·6	16·7	10·8	9·4	9·5	12·1
1869–1870	22·5	19·9	16·2	15·5	11·7	8·6	8·6	9·5
1868-9 + or − average of 20 years	+4·3	−0·4	+2·1	+1·8	−1·0	−0·2	−0·1	+0·7
1869–70 + or − average of 20 years	+0·5	−0·6	−2·3	+0·6	−0·1	−1·0	−1·0	−1·9
1869-70 + or − 1868–9	−3·8	−0·2	−4·4	−1·2	+0·9	−0·8	−0·9	−2·6

of highest productiveness; of 1869–70, the year of lowest productiveness; years; and comparison of the one with the other.

Monthly (continued).				12 months, July to June, inclusive.	4 months, July, August, September, October.	4 months, November, December, January, February.	4 months, March, April, May, June.	3 months, April, May, June.	2 months, April and May.
March.	April.	May.	June.						
Rainfall at Rothamsted.—Inches (continued).									
1·67	1·86	2·20	2·50	27·15	10·53	8·39	8·23	6·56	4·06
1·48	2·13	3·23	1·07	27·71	8·98	10·82	7·91	6·43	5·36
1·81	0·46	1·35	0·98	21·25	7·16	9·49	4·60	2·79	1·81
−0·19	+0·27	+1·03	−1·43	+0·56	−1·55	+2·43	−0·32	−0·13	+1·30
+0·14	−1·40	−0·85	−1·52	−5·90	−3·37	+1·10	−3·63	−3·77	−2·25
+0·33	−1·67	−1·88	−0·09	−6·46	−1·82	−1·33	−3·31	−3·64	−3·55
Rainfall at Rothamsted.—Number of Days when 0·01 inch, or more, fell (continued).									
13	11	12	12	158	54	56	48	35	24
16	12	16	8	151	39	60	52	36	28
15	8	10	5	152	46	68	38	23	18
+3	+1	+4	−4	−7	−15	+4	+4	+1	+5
+2	−3	−2	−7	−6	−8	+12	−10	−12	−5
−1	−4	−6	−3	+1	+7	+8	−14	−13	−10
Mean Maximum Temperature at Greenwich (Fahrenheit) (continued).									
49·9	58·6	64·4	71·3	58·5	68·9	45·7	61·1	64·8	61·5
44·8	61·6	60·7	67·4	59·6	71·4	48·8	58·6	63·2	61·2
46·9	62·0	66·9	74·8	58·4	68·9	43·8	62·7	67·9	64·5
−5·1	+3·0	−3·7	−3·9	+1·1	+2·5	+3·1	−2·5	−1·6	−0·3
−3·0	+3·4	+2·5	+3·5	−0·1	0·0	−1·9	+1·6	+3·1	+3·0
+2·1	+0·4	+6·2	+7·4	−1·2	−2·5	−5·0	+4·1	+4·7	+3·3
Mean Minimum Temperature at Greenwich (Fahrenheit) (continued).									
35·4	39·6	43·9	50·2	42·4	50·0	35·1	42·3	44·6	41·8
32·3	41·8	43·7	46·0	43·3	50·5	38·4	41·0	43·8	42·8
34·0	38·4	42·0	50·7	41·9	50·3	34·2	41·3	43·7	40·2
−3·1	+2·2	−0·2	−4·2	+0·9	+0·5	+3·3	−1·3	−0·8	+1·0
−1·4	−1·2	−1·9	+0·5	−0·5	+0·3	−0·9	−1·0	−0·9	+1·6
+1·7	−3·4	−1·7	+4·7	−1·4	−0·2	−4·2	+0·3	−0·1	−2·6
Mean Temperature at Greenwich (Fahrenheit) (continued).									
41·7	47·7	52·8	58·8	49·5	58·1	40·2	50·3	53·1	50·3
37·5	50·3	50·5	55·3	50·6	59·9	43·5	48·4	52·0	50·4
39·6	48·9	53·4	60·9	49·3	58·3	38·9	50·7	54·4	51·2
−4·2	+2·6	−2·3	−3·5	+1·1	+1·8	+3·3	−1·9	−1·1	+0·1
−2·1	+1·2	+0·6	+2·1	−0·2	+0·2	−1·3	+0·4	+1·3	+0·9
+2·1	−1·4	+2·9	+5·6	−1·3	−1·6	−4·6	+2·3	+2·4	+0·8
Mean Range of Temperature at Greenwich (Fahrenheit) (continued).									
14·6	19·1	20·5	21·1	16·1	19·0	10·6	18·8	20·2	19·8
12·5	19·8	17·0	21·4	16·4	20·9	10·5	17·7	19·4	18·4
12·9	23·6	24·9	24·1	16·5	18·5	9·6	21·4	24·2	24·3
−2·1	+0·7	−3·5	+0·3	+0·3	+1·9	−0·1	−1·1	−0·8	−1·4
−1·7	+4·5	+4·4	+3·0	+0·4	−0·5	−1·0	+2·6	+4·0	+4·5
+0·4	+3·8	+7·9	+2·7	+0·1	−2·4	−0·9	+3·7	+4·8	+5·9

The bottom line of Table XXIX. shows that (excluding the nitrate plot) there was an average yield per acre of nearly 4000 lbs. more hay in 1869 than in 1870; and without manure, and with purely mineral manure, the excess amounted to more than the average produce of those plots.

In the preceding year, 1868, there had been, under most conditions of manuring, and especially with high manuring, more than average first crops, but considerably less than average second crops. The very limited second growth is accounted for by the fact that there had been a great deficiency of rain in May, June, and July, with a considerable excess of temperature in each of those three months; whilst the rainfall of August and September, though in excess of the average, was still insufficient to restore the balance or to compensate for the previous drought, and there were again, in those two months, higher than average temperatures. Indeed, September 1868 ended an extraordinarily warm period of nearly nine months' duration. October and November were, throughout, with very few exceptions, colder than usual, both day and night, whilst in October there was a deficiency of rain, and in November a very great deficiency. So far, then, the period preceding the enormous growth of 1869 was unfavourable to luxuriant condition of the herbage.

From this time, however, throughout the three winter months of December, January, and February, there was a considerable excess of rain, with temperatures ranging considerably higher than the average. To go a little more into detail, December was almost throughout very much warmer than the average, with a very great excess of rain, some violent gales of wind, very variable, but upon the whole very low barometric pressures, and high degree of humidity of the atmosphere. The average temperature of December had, indeed, been exceeded only twice during the preceding 98 years, namely, in 1806 and 1852. Again, with the exception of a week after the middle of January, 1869, the very warm period continued until the end of February, completing three winter months of average temperature about six degrees higher than the average of 98 years; and, after the enormous excess of rain in December, there was again a moderate excess in both January and February. March, on the contrary, was several degrees colder than the average, with rather less than the average amount of rain. Early in April warm weather again set in, and lasted till nearly the end of the month, the temperature during this period being several degrees higher than the average, whilst the rainfall was only about the average. May and June were, with few exceptions, of short duration, very much colder than the average. Towards the end of May the cold was very extreme for the season, and the greater part of June was very unusually cold, both day and night; and there was in May a considerable excess, but in June a greater deficiency, of rain.

To sum up: after five months of unusually high temperatures, and unusual drought during the first three of them, October and November (1868) were again dry but cold. The three winter months were very warm, and all more or less, but December especially, very wet. The result was an unusual winter growth of grass. The dry and cold weather of March, however, checked vegetation; but with its early start, and marked progress in the winter, it recovered rapidly under the influence of the very warm and sufficiently wet weather of April. The two remaining months of the grass season were, however, unusually cold, May being at the same time very wet, but June dry, a condition which was compensated by the previous abundance of moisture; whilst, although the ruling temperatures were low for those months, they were yet actually sufficient for active vegetation. The result was luxuriant, and succulent, though not maturing, growth. Indeed, according to notes taken on the ground, the plots generally manifested great luxuriance; but the most prominent plants, whether gramineous, leguminous, or miscellaneous, were much more backward than usual at the time of cutting.

It would thus appear that the excessive produce of grass in 1869 was by no means due alone, or even perhaps chiefly, to the climatic conditions during the limited period of really active accumulation and aboveground growth, which, in fact, were not specially favourable; but the result was due to the only moderately favourable conditions during that period, succeeding upon very favourable, instead of as usual detrimental ones, throughout the three winter months, thus bringing the herbage unusually forward, and rendering it the more capable of turning to the best account such climatic elements of growth as followed.

It may here be remarked that this result is somewhat analogous to that observed in the case of our experiments on the continuous growth of wheat; the seasons of extraordinary productiveness of that crop having been marked, rather by moderately favourable conditions throughout, than by specially favourable ones during the period of most active above-ground growth.

Let us now contrast with the foregoing conditions of extremely luxuriant growth, those under which the smallest produce of the 20 years was obtained.

After the enormous first crops of 1869, less than average second crops were grown. Not only would there be comparative exhaustion of manurial constituents, but, succeeding upon the dry weather of June, and the cold weather of both May and June, there was a considerable deficiency of rain in July and August, but little more than the average in September, and again a deficiency in October; and, with the continued defect of rain in July and August, July was warmer, but August for the most part unseasonably colder, than usual; whilst September, with its fair amount of rain, was generally warmer, and October, with its defect, at times much colder than the average.

The autumn conditions were, therefore, upon the whole adverse to growth. Over the five months—November, 1869, to March, 1870, inclusive—the rain gauge indicated more than the average total fall, though there was a considerable deficiency in January. There were heavy and continuous falls of rain in November and December, with great fluctuations of temperature, some very warm, and some very cold weather, and numerous gales. The first three months of 1870, again, were characterised by frequent alternations of warm and very cold weather, the colder periods prevailing, and being sometimes very severe; snow was frequent, but the total fall was deficient in January, and but little above the average in February and March. Vegetation generally was very backward, and grass land was very brown and bare. April, May, and June, the three months of most active accumulation and growth, were largely deficient in rain; and with the exception of about a fortnight at the end of April and the beginning of May, when the weather was cold and cloudy, the whole period was unusually warm and sunny; the three months together being not only much warmer than the average, but very unusually deficient in rain. The day temperatures especially were high, though the night temperatures were in April and May low; and throughout the three months the degree of humidity of the atmosphere was considerably below the average.

Thus, after an autumn very deficient in rain, and fluctuating as to temperature, a winter and early spring very stormy, very fluctuating as to temperature, in fact very inclement, and vegetation consequently very backward to start with, the three months of most active above-ground growth were very unusually dry, very unusually hot in the days, and frequently colder than the average in the nights. It was under these conditions that the smallest crop of the 20 years was obtained.

Upon the whole, then, the registered meteorological conditions of the season of least productiveness more obviously account for the deficient crop, than do those of the previous season account for its excessive yield. In the case of the very defective crop of 1870, the conditions previous to the period of active growth were strikingly unfavourable for the herbage; and the period of active growth was itself strikingly adverse, both in its extreme dryness, and in its coincident high day and low night temperatures. In the case of the excessive, but succulent and immature growth of 1869, the climatic conditions previous to the period of most active vegetation were obviously very favourable; but those of the period of active above-ground growth itself were such as would only conduce to great luxuriance provided there were an already forward condition of the herbage.

It may be well to notice briefly, the characters of the seasons, and of the produce, of 1868 and 1874; the former perhaps the second in order of productiveness, and the latter the second in order of unproductiveness. The following table (XXXI.) summarises, in the same form as before, the results relating to the produce of these two seasons; and Table XXXII. summarises the actual and comparative characters of the seasons themselves, also in the same form as before.

TABLE XXXI.—Produce of Hay per acre on selected plots. Average of the 20 years; produce of 1868, the year second in order of productiveness; produce of 1874, the year second in order of unproductiveness; difference of each from the average; and difference of the one from the other.

	Plot 3. Unmanured, continuously.	Plot 7. Mixed mineral manure, alone,	Plot 9. Mixed mineral manure, aud 400 lbs. ammonia-salts.	Plot 11. Mixed mineral manure, and 800 lbs.* ammonia-salts.	Plot 14. Mixed mineral manure, and 550 lbs. nitrate soda.	Means.
	lbs.	lbs.	lbs.	lbs.	lbs.	
Average 20 years, 1856-1875 .	2383	3958	5711	6726	6407†	5037
1868 .	1960	4264	6622	7616	7728	5638
1874 .	1412	3088	3290	3540	5484	3363
+ or − average { 1868 .	− 423	+ 306	+ 911	+ 890	−1321	+ 601
+ or − average { 1874 .	− 971	− 870	−2421	−3186	− 923	−1674
1874-1868	− 548	−1176	−3332	−4076	−2244	−2275

---

* 400 lbs. only in 1859, 1860, and 1861.

† Average of 18 years only, 1858–1875.

TABLE XXXII.—Abstract of meteorological conditions of 1867–8, the year second comparison of each with the average of 20

	Monthly.							
	July.	August.	September.	October.	November.	December.	January.	February.
RAINFALL AT ROTHAMSTED.—INCHES.								
Average 20 years, 1855-6–1874-5	2·46	2·49	2·51	3·07	1·99	2·04	2·79	1·57
1867–1868	4·10	2·15	2·06	1·86	0·32	2·04	3·93	1·49
1873–1874	2·54	2·69	2·38	2·83	1·99	0·71	1·93	1·73
1867–8 + or – average of 20 years	+1·64	–0·34	–0·45	–1·21	–1·67	0·00	+1·14	–0·08
1873–4 + or – average of 20 years	+0·08	+0·20	–0·13	–0·24	0·00	–1·33	–0·86	+0·16
1873–4 + or – 1867–8	–1·56	+0·54	+0·32	+0·97	+1·67	–1·33	–2·00	+0·24
RAINFALL AT ROTHAMSTED.—NUMBER OF DAYS WHEN 0·01 INCH, OR MORE, FELL.								
Average 20 years, 1855-6–1874-5	12	13	13	16	14	14	16	12
1867–1868	17	9	7	17	9	10	13	11
1873–1874	10	19	12	22	19	12	18	16
1867–8 + or – average of 20 years	+5	– 4	–6	+1	– 5	–4	–3	–1
1873–4 + or – average of 20 years	–2	+ 6	–1	+6	+ 5	–2	+2	+4
1873–4 + or – 1867–8	–7	+10	+5	+5	+10	+2	+5	+5
MEAN MAXIMUM TEMPERATURE AT GREENWICH (FAHRENHEIT).								
Average 20 years, 1855–6–1874–5	75·2	73·5	67·9	59·0	48·3	44·4	43·8	46·1
1867–1868	71·1	73·3	68·0	57·2	47·8	42·2	41·4	49·9
1873–1874	76·6	74·7	65·2	57·0	50·1	45·6	47·3	45·0
1867–8 + or – average of 20 years	–4·1	–0·2	+0·1	–1·8	–0·5	–2·2	–2·4	+3·8
1873–4 + or – average of 20 years	+1·4	+1·2	–2·7	–2·0	+1·8	+1·2	+3·5	–1·1
1873–4 + or – 1867–8	+5·5	+1·4	–2·8	–0·2	+2·3	+3·4	+5·9	–4·9
MEAN MINIMUM TEMPERATURE AT GREENWICH (FAHRENHEIT).								
Average 20 years, 1855-6–1874–5	53·2	53·0	49·4	44·1	36·7	35·0	34·0	34·7
1867–1868	50·8	53·4	50·3	42·0	35·3	32·1	32·8	36·8
1873–1874	53·9	54·4	46·1	40·6	38·8	35·2	36·2	33·5
1867–8 + or – average of 20 years	–2·4	+0·4	+0·9	–2·1	–1·4	–2·9	–1·2	+2·1
1873–4 + or – average of 20 years	+0·7	+1·4	–3·3	–3·5	+2·1	+0·2	+2·2	–1·2
1873–4 + or – 1867–8	+3·1	+1·0	–4·2	–1·4	+3·5	+3·1	+3·4	–3·3
MEAN TEMPERATURE AT GREENWICH (FAHRENHEIT).								
Average 20 years, 1855–6–1874–5	62·6	61·7	57·4	50·6	42·2	39·8	38·8	39·8
1867–1868	59·4	62·0	57·6	48·7	41·1	37·5	37·2	43·0
1873–1874	63·4	62·7	54·7	47·8	44·2	40·6	41·7	38·7
1867–8 + or – average of 20 years	–3·2	+0·3	+0·2	–1·9	–1·1	–2·3	–1·6	+3·2
1873–4 + or – average of 20 years	+0·8	+1·0	–2·7	–2·8	+2·0	+0·8	+2·9	–1·1
1873–4 + or – 1867–8	+4·0	+0·7	–2·9	–0·9	+3·1	+3·1	+4·5	–4·3
MEAN RANGE OF TEMPERATURE AT GREENWICH (FAHRENHEIT).								
Average 20 years, 1855–6–1874–5	22·0	20·5	18·5	14·9	11·8	9·6	9·6	11·4
1867–1868	20·3	19·9	17·7	15·2	12·5	10·1	8·6	13·1
1873–1874	22·7	20·3	19·1	16·4	11·3	10·4	11·1	11·5
1867–8 + or – average of 20 years	–1·7	–0·6	–0·8	+0·3	+0·7	+0·5	–1·0	+1·7
1873–4 + or – average of 20 years	+0·7	–0·2	+0·6	+1·5	–0·5	+0·8	+1·5	+0·1
1873–4 + or – 1867–8	+2·4	+0·4	+1·4	+1·2	–1·2	+0·3	–2·5	–1·6

in order of productiveness; of 1873–4, the year second in order of unproductiveness; years; and comparison of the one with the other.

Monthly—continued.				12 months, July to June, inclusive.	4 months, July, August, September, October.	4 months, November, December, January, February.	4 months, March, April, May, June.	3 months, April, May, June.	2 months, April and May.
March.	April.	May.	June.						
RAINFALL AT ROTHAMSTED.—INCHES (continued).									
1·67	1·86	2·20	2·50	27·15	10·53	8·39	8·23	6·56	4·06
1·92	2·19	0·73	0·37	23·16	10·17	7·78	5·21	3·29	2·92
0·65	2·14	1·19	1·59	22·37	10·44	6·36	5·57	4·92	3·33
+0·25	+0·33	−1·47	−2·13	−3·99	−0·36	−0·61	−3·02	−3·27	−1·14
−1·02	+0·28	−1·01	−0·91	−4·78	−0·09	−2·03	−2·66	−1·64	−0·73
−1·27	−0·05	+0·46	+1·22	−0·79	+0·27	−1·42	+0·36	+1·63	+0·41
RAINFALL AT ROTHAMSTED.—NUMBER OF DAYS WHEN 0·01 INCH, OR MORE, FELL (continued).									
13	11	12	12	158	54	56	48	35	24
13	10	6	4	126	50	43	33	20	16
14	10	11	13	176	63	65	48	34	21
0	−1	−6	−8	−32	− 4	−13	−15	−15	−7
+1	−1	−1	+1	+18	+ 9	+ 9	0	− 1	−2
+1	0	+5	+9	+50	+13	+22	+15	+14	+5
MEAN MAXIMUM TEMPERATURE AT GREENWICH (FAHRENHEIT) (continued).									
49·9	58·6	64·4	71·3	58·5	68·9	45·7	61·1	64·8	61·5
52·9	58·8	70·5	76·3	59·1	67·4	45·4	64·6	68·5	64·7
52·8	61·5	63·2	71·1	59·2	68·4	47·0	62·2	65·3	62·4
+3·0	+0·2	+6·1	+5·0	+0·6	−1·5	−0·3	+3·5	+3·7	+3·2
+2·9	+2·9	−1·2	−0·2	+0·7	−0·5	+1·3	+1·1	+0·5	+0·9
−0·1	+2·7	−7·3	−5·2	+0·1	+1·0	+1·6	−2·4	−3·2	−2·3
MEAN MINIMUM TEMPERATURE AT GREENWICH (FAHRENHEIT) (continued).									
35·4	39·6	43·9	50·2	42·4	50·0	35·1	42·3	44·6	41·8
36·8	40·1	46·1	50·8	42·3	49·1	34·3	43·5	45·7	43·1
36·6	41·3	40·7	48·3	42·1	48·8	35·9	41·7	43·4	41·0
+1·4	+0·5	+2·2	+0·6	−0·1	−0·9	−0·8	+1·2	+1·1	+1·3
+1·2	+1·7	−3·2	−1·9	−0·3	−1·2	+0·8	−0·6	−1·2	−0·8
−0·2	+1·2	−5·4	−2·5	−0·2	−0·3	+1·6	−1·8	−2·3	−2·1
MEAN TEMPERATURE AT GREENWICH (FAHRENHEIT) (continued).									
41·7	47·7	52·8	58·8	49·5	58·1	40·2	50·3	53·1	50·3
44·0	48·1	57·3	62·0	49·8	56·9	39·7	52·9	55·8	52·7
43·7	50·0	50·5	58·0	49·7	57·2	41·3	50·6	52·8	50·3
+2·3	+0·4	+4·5	+3·2	+0·3	−1·2	−0·5	+2·6	+2·7	+2·4
+2·0	+2·3	−2·3	−0·8	+0·2	−0·9	+1·1	+0·3	−0·3	0·0
−0·3	+1·9	−6·8	−4·0	−0·1	+0·3	+1·6	−2·3	−3·0	−2·4
MEAN RANGE OF TEMPERATURE AT GREENWICH (FAHRENHEIT) (continued).									
14·6	19·1	20·5	21·1	16·1	19·0	10·6	18·8	20·2	19·8
16·1	18·7	24·4	25·5	16·8	18·3	11·1	21·2	22·9	21·5
16·2	20·2	22·5	22·8	17·0	19·6	11·1	20·4	21·8	21·3
+1·5	−0·4	+3·9	+4·4	+0·7	−0·7	+0·5	+2·4	+2·7	+1·7
+1·6	+1·1	+2·0	+1·7	+0·9	+0·6	+0·5	+1·6	+1·6	+1·5
+0·1	+1·5	−1·9	−2·7	+0·2	+1·3	0·0	−0·8	−1·1	−0·2

A comparison of Table XXXI. with Table XXIX. will show that the excess of produce over the average was very much less in 1868 than in 1869, and that it was the most considerable under the influence of highly nitrogenous in conjunction with mineral manures. Indeed, with mineral manures alone (plot 7) the increase was very small, and without manure there was even a considerable deficiency in this season, which nevertheless is taken as, upon the whole, illustrative of high productiveness. There are, in fact, other seasons giving more produce on one or other of the plots; but, excepting 1869, there is no other that gives so much on each of the three selected highly-manured plots. The season was, therefore, one of great luxuriance with high manuring, but of deficient growth with low manuring.

The season of 1874 gave more uniformly bad results than did that of 1868 give uniformly good ones; the produce being the twentieth, or lowest, in order of amount on two of the highly-manured plots, seventeenth on the third, eighteenth on the mineral manured, and seventeenth on the unmanured plot.

The difference between the produce of the two seasons was the most marked where ammonia-salts were used; there being about 1½ ton less hay in 1874 than in 1868 where the smaller amount of ammonia-salts was applied (plot 9), and nearly 2 tons less with the larger amount (plot 11); whilst, with the nitrate of soda, the produce in the reputed good season was only about 1 ton more than in the reputed bad one. This was due, however, not to any deficiency of growth with the nitrate in the good season, but to much better growth with it than with the ammonia-salts in the bad season—a result explained by the fact of the less dependence of the herbage growing under the influence of the nitrate on atmospheric supplies of moisture during the period of active vegetation.

In 1867, the season previous to the productive one of 1868, there had been fairly average both first and second crops. October and November were considerably deficient in rain, and both were colder than usual. December was characterised by great and rapid variations of temperature and barometric pressure, some extremely heavy gales, sometimes frost, snow, and sleet, at others very warm weather, with (in the aggregate) a fair amount of total fall. Nearly the first half of January (1868) was also very cold, but from that time to the end of the summer the weather was unusually warm. February, March, April, May, and June, were, indeed, all considerably warmer than the average. The average temperature of April had, however, frequently, and that of May and June occasionally, been exceeded in the corresponding months of other years; but the average temperature of the three months together had only once been exceeded in any corresponding three months in 98 years—namely, in 1865—when, though April was hotter, May and June were not quite so hot; and the average temperature of the whole period from the middle of January to the end of June was only exceeded in 1822. Concurrently with this long-continued warm weather, there was a considerable excess of rain in January, with in the early part of the month several gales of wind; there was scarcely the average fall in February, a slight excess in March and April, a great deficiency in May, and a still greater in June. The degree of humidity of the atmosphere was also below the average in each of the six months, January to June inclusive.

With the excess of rain in January, the fair amounts in February, March, and April, and the unusual warmth of those early spring months, vegetation became active very early, and was very forward before the dry, and very hot, weather of May and June set in. The result was, as Table XXXI. shows, that with liberal manuring there were very heavy crops, which fully matured, and were cut very early; whilst, without manure, or with only mineral manure, the herbage suffered greatly from the heat and drought of May and June.

Thus, the season second in order of productiveness was characterised by unusually high temperatures throughout the whole period of growth, with a sufficiency of rain up to the end of April—conditions which brought the herbage very early forward, and rendered it comparatively independent of the extreme heat and drought of the months of May and June, which would otherwise have been fatal at that period, and which were, in fact, very injurious where the conditions of manuring had not been such as to bring the vegetation sufficiently forward previously.

We have, it may be observed, the curious result of the lowest produce of the 20 years in the year of 1870, which was the one of the most extreme heat and drought of the series, and of the highest produce but one in 1868, which was only second to 1870 in heat and drought of the growing period. But there was this difference between the two seasons: the winter and early spring of 1870 had been very adverse, the herbage was in a very backward state, and the heat and drought commenced a month earlier; whereas, from the commencement of 1868, for a period of nearly four months, the conditions both as to heat and moisture were favourable, and it was not until May—that is, a month later than in 1870, and when with high manuring the herbage was already very luxuriant—that the heat and drought succeeded, then serving to elaborate and mature, rather than materially to check, vegetation in such condition of luxuriance and forwardness.

Let us now call attention to the characters of the season of 1874, the second of the 20 in order of unproductiveness. The preceding season (1873) had been one of less than average productiveness of first crops, but of fairly luxuriant second growth, which was cut twice—namely, about the middle of August, and after the middle of October—and the produce was, in each case, spread on the respective plots to decay.

In every month, from September, 1873, to June, 1874, inclusive, excepting in February and April (when the excess was only slight), there was more or less defect of rain compared with the average of the 20 years, and in December, January, March, May, and June, the deficiency was very considerable. With this marked deficiency of rain almost throughout both the earlier and the later periods of the grass season, the months of November, December, and January, and again of March and April, were upon the whole warmer than usual, both day and night; whilst February, May, and

June, were unseasonably cold. This was more especially the case with May, and especially so far as its night temperatures were concerned. With regard to the seven months—December, 1873, to June, 1874, inclusive—Mr. GLAISHER states that there is no instance on record of so small a fall of rain during those months collectively. And, although December, January, March, and April were, on the average, notably warmer than usual, February was on the average colder; the period from the 4th to the 12th of the month was extremely cold, as also was that from the 4th to the 13th of March, when snow was general. But it was the first three weeks of May which showed the most unseasonable reductions of temperature, the night frosts during that period being exceptionally severe; and from the middle to the end of June also the weather was very cold; whilst, throughout both May and June there was a great deficiency of rain.

Thus, although the winter and early spring of 1873–4 were upon the whole considerably warmer than usual, there were periods of considerable severity as to temperature, and the whole period was very deficient in rain; so that, instead of the warmth of the usually cold months availing to bring vegetation forward, it remained very backward, when it was overtaken by the unfavourably cold days, and the very unusually severe frosty nights, of the greater part of May; whilst the short period of warmer weather which then set in was unaccompanied by sufficient moisture, and the herbage was already too much damaged to recover. According to notes taken on the ground at the time, the foliage of the grasses became spotted, and the early flowering stems were bleached, and in many cases killed. Although a number of the grasses suffered considerably, the greatest damage was done on the plots highly manured with ammonia-salts, where *Dactylis glomerata,* which was both abundant and forward, suffered very much. The leguminous, and several of the miscellaneous species, also suffered very considerably.

The four seasons which have been selected for the purpose of attempting to trace the connexion between the amount of growth of the mixed herbage of grass land, and such meteorological conditions as we have sufficient record of, most strikingly illustrate the intricacy and difficulty of the subject, as referred to in our introductory remarks. Very dissimilar climatal conditions characterised the two seasons of highest productiveness, and again very dissimilar ones those of lowest productiveness. The character of the produce was also very different in the two cases of the largest crops, and again very different in the two of the smallest crops.

In both cases of high productiveness, the period prior to that of most active above-ground growth had brought the herbage into an unusual state of forwardness; when, in the one, abundance of rain, with, upon the whole, low temperatures, gave great luxuriance, but comparatively leafy, succulent, and immature produce; whilst, in the other, the luxuriant early growth was followed by both unusual drought and unusual heat, yielding quantity by virtue of high development and maturation, as distinguished from succulence and immaturity. As in both the cases of high productiveness the period antecedent to that of most active growth had been favourable, so in both those of very defective growth they had been very unfavourable. The winter and early spring of 1870 had not, upon the whole, been deficient in rain, but the period had been extremely variable as to temperature, frequently very inclement, and, upon the whole, colder than usual. The herbage was, from these causes, very backward at the commencement of the active growing period. April, May, and June followed, with a great deficiency of rain, very high day, and low night, temperatures, yielding very stunted and prematurely ripened produce. The winter and early spring of 1874 had, on the contrary, been very unusually deficient in rain, whilst the temperatures had ruled higher than the average, both day and night, until the end of April. Then followed, instead of drought and heat, drought and unusual cold, both day and night, especially the latter, and the already backward herbage was very materially damaged, yielding not only checked and stunted, but really damaged crops.

In the discussion of the characteristic effects of the different manures, it has been pointed out, in general terms, that both the botanical and the chemical composition of the mixed herbage varied very greatly according to the description of manure applied; and the foregoing typical illustrations of the effects of the varying climatic conditions of different seasons clearly indicate how different, both botanically and chemically, will be the character of the produce dependent on the character of the seasons. In fact, a given quantity of gross produce of the mixed herbage may be one thing in one season, and quite another in another season, both as to the proportion of the different species composing it, and their condition of development and maturity. In this place, however, we have to deal with the question of the quantity rather than of the quality of the produce, and from this point of view the few illustrations of the influence of season which have been given will suffice. But, in the next section of our report—"Part II.—The Botanical Results"—the characters of the four seasons in which the botanical analyses of the produce were undertaken will be considered; and then, not only will the effects of variation of climatal conditions on the quantity of produce yielded be further elucidated, but an endeavour will be made to trace a connexion between the distinctive habits of growth of plants of different orders, genera, or species, and the way in which, accordingly, their predominance, and the amount, and the character, of their development, are influenced by the characters of the seasons.

APPENDIX-TABLE I.—Showing (taken together with the footnotes) the description and quantities from each plot, in each year,

Plots.	Manures per acre per annum (unless otherwise stated in the footnotes).	Produce of hay per acre, first crops only, first 20 years, 1856-1875.													Plots.
		Each year.													
		1856.	1857.	1858.	1859.	1860.	1861.	1862.	1863.	1864.	1865.	1866.	1867.	1868.	
		lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	
1	1856-63, 8 years, 14 tons farmyard manure and 200 lbs. ammonia-salts[*] 1864 and since, 200 lbs. ammonia-salts alone	5009	6008	5320	5356	5704	5320	5556	6028	6216	3616	4968	5880	4676	1
2	1856-63, 8 years, 14 tons farmyard manure 1864 and since, unmanured	4030	5328	4164	4584	5208	5052	5060	5004	5392	2848	4816	5716	4116	2
3	Unmanured continuously	2515	2856	2472	2540	2760	2844	3052	2284	2688	1296	2660	3332	1960	3
4-1	1856-58, 3 years, 2000 lbs. sawdust (average produce 2299 lbs.) 1859 and since, 3½ cwts. superphosphate of lime[†]	2313	2340	2244	2828	3176	3400	3252	2592	2632	1244	2884	3540	2200	4-1
4-2	1856-58, 3 years, 2000 lbs. sawdust (average produce 2299 lbs.) 1859 and since, 3½ cwts. superphosphate of lime and 400 lbs. ammonia-salts	2313	2340	2244	4996	4788	4968	4756	4356	4312	2904	3864	4352	3276	4-2
5	1856 and since, 400 lbs. ammonia-salts	4028	3774	3982	3644	2940	3808	3854	3508	2940	1722	2920	3306	2688	5
6[‡]	1856-68, 13 years, 400 lbs ammonia-salts 1869-78, 300 lbs. sulph. potass, 100 lbs. sulph. soda, 100 lbs. sulph. magnesia, and 3½ cwts. superphosphate.	3954	3710	4166	3664	2982	3943	4452	3412	3108	1792	3066	3192	3080	6
7	1856-78, 300 lbs. sulph. potass, 100 lbs.[§] sulph. soda, 100 lbs. sulph. magnesia, and 3½ cwts. superphosphate.	3429	3666	4082	3416	3928	4488	4424	3870	4130	2540	4180	4458	4264	7
8[‡]	1856-61, 6 years, 300 lbs. sulph. potass, 200 lbs. sulph. soda, 100 lbs. sulph. magnesia, and 3½ cwts. superphosphate 1862 and since, 250 lbs.[‖] sulph. soda, 100 lbs sulph. magnesia, and 3½ cwts. superphosphate	3711	3994	4376	3396	4074	4608	4524	3710	3424	1894	3212	3824	3058	8
9	1856-78, 300 lbs. sulph. potass, 100 lbs.[§] sulph. soda, 100 lbs. sulph. magnesia, 3½ cwts. superphosphate, and 400 lbs. ammonia-salts	6363	6422	7172	6198	5624	6316	6402	6026	5628	3866	4956	5382	6622	9
10[‡]	1856-61, 6 years, 300 lbs. sulph. potass, 200 lbs. sulph. soda, 100 lbs. sulph. magnesia, 3½ cwts. superphosphate, 400 lbs. ammonia-salts 1862 and since, 250 lbs.[‖] sulph. soda, 100 lbs. sulph. magnesia, 3½ cwts. superphosphate, and 400 lbs. ammonia-salts	6369	6428	6892	5852	5362	6432	6180	6582	5356	3666	4964	4984	4992	10
11-1	1856-78, 300 lbs. sulph. potass, 100 lbs.[§] sulph. soda, 100 lbs. sulph. magnesia, 3½ cwts. superphosphate, and 800 lbs[¶] ammonia-salts	6970	6940	7508	7160	5744	6710	7108	8036	7152	5816	7812	5360	7140	11-1
11-2	1856-78, 300 lbs. sulph. potass, 100 lbs.[§] sulph. soda, 100 lbs. sulph. magnesia, 3½ cwts. superphosphate, 800 lbs.[¶] ammonia-salts, and 400 lbs. silicate of soda[**]	6970	6940	7508	7150	5744	6710	7120	8692	7700	6232	8372	6256	8092	11-2
12	Unmanured continuously	2351	2592	3360	2576	2884	3304	3424	2844	2808	1932	3012	3048	2676	12
13	1856-78, 300 lbs. sulph. potass, 100 lbs.[§] sulph. soda, 100 lbs. sulph. magnesia, 3½ cwts. superphosphate, 400 lbs. ammonia-salts, and 2000 lbs. cut wheat straw.	5412	6050	6752	6370	5698	6462	6308	7298	6594	4908	6868	6064	6832	13
14	1858-78, 550 lbs. nitrate of soda,[††] 300 lbs. sulph. potass, 100 lbs.[‡‡] sulph. soda, 100 lbs. sulph. magnesia, and 3½ cwts. superphosphate	...	...	5646	6072	5586	5892	5718	6528	6816	5292	6576	7188	7728	14
15	1858-75, 18 years, 550 lbs. nitrate of soda 1876-78, 300 lbs. sulph. potass, 100 lbs. sulph. soda, 100 lbs. sulph. magnesia, and 3½ cwts. superphosphate.	...	...	3564	4116	4410	4452	4086	4116	4410	3150	4578	5286	3570	15
16	1858-78, 275 lbs. nitrate of soda, 300 lbs. sulph. potass, 100 lbs.[‡‡] sulph. soda, 100 lbs. sulph. magnesia, and 3½ cwts. superphosphate	...	...	4236	4956	4812	5514	5178	5694	5964	4110	5754	5940	5736	16
17	1858-78, 275 lbs. nitrate of soda	...	...	2952	3588	3948	4092	4446	4014	4452	3204	4254	5082	3234	17
18	1865-78, mixture supplying the quantity of potass, soda, lime, magnesia, phosphoric acid, silica, and nitrogen contained in 1 ton of hay	...	...	...	...	...	...	...	...	...	2352	4684	4332	3074	18
19	1872-78, 275 lbs. nitrate of soda, 290 lbs. sulphate of potass, and 3½ cwts. superphosphate (equal in nitrogen and potass to plot 20)	...	...	...	...	...	...	...	...	...	...	...	...	...	19
20	1872-78, 327 lbs. nitrate of potass and 3½ cwts. superphosphate (equal in nitrogen and potass to plot 19)	...	...	...	...	...	...	...	...	...	...	...	...	...	20

---

[*] "Ammonia-salts" in all cases, equal parts sulphate and muriate of ammonia of commerce.

[†] "Superphosphate of lime" in all cases made from 200 lbs. bone ash, 150 lbs. sulphuric acid, sp. gr. 1·7 (and water).

[‡] Plots 6, 8, and 10 had, besides the manures specified, 2000 lbs. sawdust per acre per annum for the first 7 years, 1856–62, but without effect.

[§] 200 lbs., 1856–63 inclusive.

[‖] 500 lbs., 1862 and 1863.

[¶] Only 400 lbs. in 1859–60–61.

[**] The application of silicates did not commence until 1862; 9 years (1862–1870), 200 lbs. silicate of lime, and 200 lbs. silicate of soda; 1871 and since, 400 lbs. silicate of soda.

[††] 550 lbs. nitrate of soda is reckoned to contain the same amount of nitrogen as 400 lbs. "ammonia-salts."

[‡‡] 200 lbs., 1858–63 inclusive.

manures applied per acre, on each plot, in each year, and the amount of hay per acre, removed years, 1856–1878, inclusive.

Produce of hay per acre, first crops only, first 20 years, 1856-1875 (continued).											Subsequent years.										Plots.
Each year (continued).							Averages.				1875.			1876.	1877.			1878.			
1869.	1870.	1871.	1872.	1873.	1874.	1875.	First period.	Second period.	Total period.	Duration of periods.	First crop.	Second crop.	Total.	One crop only.	First crop.	Second crop.	Total.	First crop.	Second crop.	Total.	
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.		lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	
6832	1828	4904	3531	3332	2628	3764	5538	4348	4824	8, 12, and 20 years .	3764	1960	5724	3320	4732	2248	6980	3448	1960	5408	1
6188	1556	3800	2831	2112	1816	2988	4804	3682	4131	8, 12, and 20 years .	2988	1320	4308	2288	3612	1824	5436	2340	1776	4116	2
4256	644	2844	1644	1372	1412	2236	2531	2236	2383	10, 10, and 20 years .	2236	1384	3620	1384	2364	1960	4324	1848	1496	3344	3
4508	812	2788	1772	1516	1472	2344	2732	2384	2527	7, 10, and 17 years .	2344	1728	4072	1860	3092	2100	5192	2184	1704	3888	4–1
5096	924	4284	3195	2912	2140	4100	4440	3414	3837	7, 10, and 17 years .	4100	1612	5712	3760	4696	1500	6196	3648	2408	6056	4–2
3990	628	3320	2514	1894	756	2696	3420	2471	2946	10, 10, and 20 years .	2696	2024	4720	1982	2952	2240	5192	1915	2104	4019	5
6328	1820	4206	2822	2918	2414	4008	3425	3502	3452	13, 7, and 20 years .	4008	1678	5686	3590	4222	2196	6418	4140	2114	6254	6
6124	1968	4414	4236	3892	3088	4560	3797	4118	3958	10, 10, and 20 years .	4560	2684	7244	3886	5100	2674	7774	3926	2532	6458	7
5194	1428	3366	2568	2058	1936	3182	4027	3098	3377	6, 14, and 20 years .	3182	1788	4970	2702	3624	1748	5372	2544	1932	4476	8
7700	3306	6576	5658	4894	3290	5828	6002	5421	5711	10, 10, and 20 years .	5828	2729	8557	5604	6054	2448	8502	6290	2707	8997	9
6412	2376	5218	4328	3700	2582	4806	6222	4725	5674	6, 14, and 20 years .	4806	2713	7519	4484	4880	2806	7686	4600	2456	7056	10
8428	4748	6348	7131	5236	2628	5224	6913	6006	6459	10, 10, and 20 years .	5224	5684	10908	6452	6796	5420	12216	5808	4604	10412	11–1
8792	5552	7364	7155	6328	4452	6728	7077	6909	6993	10, 10, and 20 years .	6728	4580	11308	7264	8500	3844	12344	6712	4248	10960	11–2
4352	1260	2960	2252	1804	1642	2632	2808	2564	2686	10, 10, and 20 years .	2632	1600	4232	1599	2165	2818	4983	1832	1784	3616	12
8694	5380	7066	7010	6384	5200	7286	6185	6678	6432	10, 10, and 20 years .	7286	3406	10692	7418	6276	3234	9510	6172	3302	9474	13
8526	6300	6930	6215	5796	5484	7026	5944	6777	6406	8, 10, and 18 years .	7026	1974	9000	7206	6258	2124	8382	5382	1692	7074	14
5976	1746	4326	3659	3780	2928	3276	4038	3913	3968	8, 10, and 18 years .	3276	1464	4740	3420	3784	2022	5806	2814	2394	5208	15
8316	3714	6384	4475	4662	3294	5040	5058	5332	5210	8, 10, and 18 years .	5040	1866	6906	4656	6096	2304	8400	4788	2292	7080	16
6090	2166	4308	3323	3192	2544	3360	3837	3755	3791	8, 10, and 18 years .	3360	1452	4812	2880	3738	1788	5526	3084	1590	4674	17
6230	1640	4250	3740	2932	2512	3900	3908	3301	3604	5½, 5½, and 11 years .	3900	1722	5622	3544	4500	2224	6724	3864	1936	5800	18
...	...	...	4488	4328	3840	4616	...	...	4318	4 years.	4616	2256	6872	4152	4728	2152	6880	4125	1928	6353	19
...	...	...	4312	4032	3280	4744	...	...	4092	4 years.	4744	2368	7112	4248	5144	1856	7000	4774	1576	6350	20

APPENDIX-TABLE II.—Estimated amounts per acre, of second crops, reckoned as Hay.

	Plot 1. Farm-yard manure first 8 years; ammonia-salts every year.	Plot 2. Farm-yard manure first 8 years; afterwards unmanured.	Plot 3. Unmanured continuously.	Plot 4–1. Superphosphate of lime alone; 1859 and since.	Plot 4–2. Superphosphate and ammonia-salts; 1859 and since.	Plot 5. Ammonia-salts alone.	Plot 6. Ammonia-salts first 13 years; afterwards mixed mineral manure.	Plot 7. Mixed mineral manure (with potass) every year.	Plot 8. Mixed mineral manure with potass 6 years; without afterwards.	Plot 9 as plot 7 and 400 lbs. ammonia-salts.	Plot 10 as plot 8 and 400 lbs. ammonia-salts.	Plot 11-1 as plot 7 and 800 lbs.* ammonia-salts.	Plot 11-2 as plot 11-1 with silicates in addition; 1862 and since.	Plot 12. Unmanured continuously.	Plot 13 as plot 9 and 2000 lbs. cut wheat straw.	Plot 14 as plot 7 and 550 lbs. nitrate soda.	Plot 15. 550 lbs. nitrate soda 18 years; mixed mineral manure afterwards.	Plot 16 as plot 7 and 275 lbs. nitrate soda.	Plot 17. 275 lbs. nitrate soda every year.	Plot 18. Mineral constituents and nitrogen of 1 ton hay.	Plot 19. Superphosphate, nitrate soda, and sulphate potass.	Plot 20. Superphosphate and nitrate potass.
20 YEARS, 1856–1875 INCLUSIVE.																						
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1856	1188	1188	1188	(1188)	(1188)	1280	1280	1280	1280	1507	1507	2013		1558	2330	..	..	..	..	..	..	..
1857	1427	1427	950	(950)	(950)	1187	1187	1485	1485	1645	1645	2195		1189	1782	..	..		..	..	..	..
1858	1210	1210	1098	(877)	(877)	1098	1098	1373	1373	1645	1645	2195		1043	1645	1645	1645	1645	1645	..	..	..
1859	1098	1098	914	822	987	906	906	1234	1234	1234	1234	1646		822	1234	960	960	822	822	..	..	..
1860	2432	2432	2112	2112	2323	2222	2222	2378	2378	2434	2434	2434		2146	2496	2010	1507	1576	1507	..	..	..
1861	1755	1755	1426	1536	1646	1664	1664	1782	1782	1902	1902	2021		1664	2021	1728	1426	1701	1426	..	..	..
1862	2395	2395	1718	1383	2157	1846	1846	1901	1901	1883	2085	2725	2725	1755	2725	2304	2304	2304	2304	..	..	..
1863	1317	1152	822	1152	987	1098	1098	1206	1206	1082	1006	1462	1462	1098	1462	603	549	603	603	..	..	..
1864	771	771	549	549	731	869	869	693	869	869	434	869	869	560	640	1029	771	1029	771	..	..	..
1865	2011	2011	1462	2011	1371	1871	1871	1600	1600	1234	1234	2742	2742	1462	2286	1603	1166	1603	1603	..	..	..
1866	Second	crops	cut and	spread	on the	respec	tive pl	ots.														
1867	1317	1317	1098	1098	1317	1206	1206	1645	1645	1645	1645	2194	2194	1098	1645	768	1371	1371	1371	1115	..	..
1868	640	480	460	768	768	640	640	1024	1024	1024	1024	1280	1280	640	1280	1029	411	1029	1029	1029	..	..
1869	960	914	1098	1280	1280	1262	1590	1462	1462	1280	1280	1472	1472	778	1420	1115	864	1152	1098	1280	..	..
1870	Second	crops	cut and	spread	on the	respec	tive pl	ots.														
1871	1859	1371	1646	1714	2742	1376	1792	1886	1646	1920	2286	2400	2400	2171	1772	3291	2058	3072	2987	2304	..	..
1872	1578	1462	2011	1576	2194	1166	1646	1920	1782	1874	1766	2149	2149	1954	2286	1871	1646	1782	1920	1800	2194	1578
1873	Second	and th	ird cro	ps cut	and sp	read on	the res	pective	plots.													
1874	Second	and th	ird cro	ps cut	and sp	read on	the res	pective	plots.													
1875†. . . .	1960	1820	1384	1728	1612	2024	1678	2684	1788	2729	2718	5684	4580	1600	3406	1974	1464	1866	1452	1722	2256	2368
AVERAGES.																						
8 years 1856–63.	1603	1582	1279	1501‡	1620‡	1413	1413	1580	1580	1667	1682	2086	2086	1409	1984	1542‖	1899‖	1442‖	1385‖	..	..	..
8 years, 1864–5–7–8–9–71–2–5.	1387	1206	1213	1341	1502	1239	1349	1614	1477	1572	1548	2349	2211	1283	1842	1522	1219	1613	1529	..	..	..
16 years, 1856–65–7–8–9–71–2–5.	1495	1394	1246	1402§	1547§	1326	1381	1597	1528	1619	1615	2218	2149	1346	1913	1531¶	1296¶	1540¶	1467¶	..	..	..
SUBSEQUENT YEARS.																						
1876	Second	crops	cut and	spread	on the	respec	tive pl	ots.														
1877†. . . .	2248	1824	1960	2100	1500	2240	2196	2674	1748	2448	2806	5420	3544	2818	3234	2124	2022	2304	1788	2224	2152	1856
1878†. . . .	1960	1776	1496	1704	2408	2104	2114	2532	1932	2707	2456	4604	4248	1784	3302	1692	2394	2292	1590	1936	1928	1576

* 400 lbs. only in 1859, 1860, and 1861.

‡ Averages of 5 years only, 1859–63.

‖ Averages of 6 years only, 1858–63.

† Second crops cut, removed, and weighed.

§ Averages of 13 years only, 1859–65, and '67, '68, '69, '71, '72, and '75.

¶ Averages of 14 years only, 1858–65, and '67, '68, '69, '71, '72, and '75.

APPENDIX-TABLE III.—Actual amounts of Hay per acre (first crops), each year, on five selected plots; and the indices of those amounts.

Years.	Actual produce per acre (first crops); lbs.						Indices of actual amounts; highest any year = 1.						Years.
	Plot 3. Unmanured continuously.	Plot 7. Mixed mineral manure alone.	Plot 9. Mixed mineral manure and 400 lbs. ammonia-salts.	Plot 11. Mixed mineral manure and 800 lbs.* ammonia-salts.	Plot 14. Mixed mineral manure and 550 lbs. nitrate soda.	Means of plots 7, 9, and 11.	Plot 3. Unmanured continuously.	Plot 7. Mixed mineral manure alone.	Plot 9. Mixed mineral manure, and 400 lbs. ammonia-salts.	Plot 11. Mixed mineral manure, and 800 lbs.* ammonia-salts.	Plot 14. Mixed mineral manure and 550 lbs. nitrate soda.	Of mean produce of plots 7, 9, and 11.	
20 YEARS, 1856–1875 INCLUSIVE.													
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.							
1856	2515	3429	6363	6970	..	5587	11	16	7	10	..	13	1856
1857	2856	3666	6422	6940	..	5676	4	15	5	11	..	11	1857
1858	2472	4082	7172	7508	5646	6254	12	11	2	5	15	2	1858
1859	2540	3416	6198	7150	6072	5588	10	17	9	7	11	12	1859
1860	2760	3928	5624	5744	5586	5099	7	12	14	18	16	16	1860
1861	2844	4488	6316	6710	5892	5838	5	3	8	13	12	7	1861
1862	3052	4424	6402	7114	5718	5980	3	5	6	9	14	5	1862
1863	2284	3870	6026	8364	6528	6087	13	14	10	2	8	4	1863
1864	2688	4130	5628	7426	6816	5728	8	10	13	6	6	9	1864
1865	1296	2540	3866	6024	5292	4143	19	19	18	14	18	18	1865
1866	2660	4180	4956	8092	6576	5743	9	9	16	3	7	8	1866
1867	3332	4458	5382	5808	7188	5216	2	4	15	16	3	15	1867
1868	1960	4264	6622	7616	7728	6167	15	7	3	4	2	3	1868
1869	4256	6124	7700	8610	8526	7478	1	1	1	1	1	1	1869
1870	644	1968	3306	5150	6300	3475	20	20	19	19	9	19	1870
1871	2844	4414	6576	6856	6930	5949	5	6	4	12	5	6	1871
1872	1644	4236	5658	7143	6215	5679	16	8	12	8	10	10	1872
1873	1372	3892	4894	5782	5796	4856	18	13	17	17	13	17	1873
1874	1412	3088	3290	3540	5484	3306	17	18	20	20	17	20	1874
1875	2236	4560	5828	5976	7026	5455	14	2	11	15	4	14	1875
Means	2383	3958	5711	6726	6407†	5465							Means
SUBSEQUENT YEARS.													
1876	1384	3886	5604	6858	7206	5449							1876
1877	2364	5100	6054	7648	6258	6267							1877
1878	1848	3926	6290	6260	5382	5492							1878

* 400 lbs. only in 1859, 1860, and 1861.

† Averages of 18 years only, 1858–1875.

APPENDIX-TABLE IV.—Rainfall at Rothamsted.—Inches.

Years.	Monthly.												Totals.						Years.
	July.	August.	Sept.	October.	Nov.	Dec.	Jan.	Feb.	March.	April.	May.	June.	12 months, July to June, inclusive.	4 months, July, August, September, October.	4 months, November, December, January, February.	4 months, March, April, May, June.	3 months, April, May, June.	2 months, April and May.	
20 Years, 1855–56 to 1874–75.																			
	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	
1855–6	6·96	2·63	1·54	5·50	2·47	1·72	2·78	1·35	1·00	2·61	4·71	1·91	35·18	16·63	8·32	10·23	9·23	7·32	1855–6
1856–7	1·48	2·65	2·19	2·87	1·42	2·24	3·71	0·57	1·48	2·16	1·11	2·21	24·09	9·19	7·94	6·96	5·48	3·27	1856–7
1857–8	1·61	3·08	4·17	5·91	2·25	0·58	0·97	1·43	0·80	2·58	2·55	0·96	26·89	14·77	5·23	6·89	6·09	5·13	1857–8
1858–9	3·19	1·56	1·54	1·60	0·88	1·99	1·34	2·01	1·73	2·70	2·13	3·41	24·08	7·89	6·22	9·97	8·24	4·83	1858–9
1859–60	3·02	2·78	3·44	2·86	2·28	2·79	3·42	1·22	2·08	1·94	4·30	6·26	36·34	12·10	9·71	14·53	12·50	6·24	1859–60
1860–1	2·03	4·22	2·77	1·77	2·45	1·43	0·81	2·41	2·29	1·28	1·04	2·98	25·48	10·79	7·10	7·59	5·30	2·32	1860–1
1861–2	3·19	0·89	1·63	1·46	3·99	1·58	1·77	0·60	3·06	2·84	2·91	3·41	27·33	7·17	7·94	12·22	9·16	5·75	1861–2
1862–3	1·80	2·50	2·29	4·05	1·34	1·74	4·04	0·74	0·91	0·96	1·01	4·60	25·98	10·64	7·86	7·48	6·57	1·97	1862–3
1863–4	0·70	2·87	2·91	2·35	2·22	1·64	1·28	0·77	2·47	1·25	1·88	1·79	22·13	8·83	5·91	7·39	4·92	3·13	1863–4
1864–5	0·89	0·78	3·14	1·29	2·47	0·55	4·01	1·84	1·42	0·47	3·05	0·68	20·59	6·10	8·87	5·62	4·20	3·52	1864–5
1865–6	2·93	5·17	0·17	7·36	2·66	1·46	3·97	3·24	1·65	1·95	1·24	4·51	36·31	15·63	11·33	9·35	7·70	3·19	1865–6
1866–7	3·01	3·44	4·10	1·82	2·16	2·70	2·56	1·94	2·17	2·82	3·35	1·06	31·13	12·37	9·36	9·40	7·23	6·17	1866–7
1867–8	4·10	2·15	2·06	1·86	0·32	2·04	3·93	1·49	1·92	2·19	0·73	0·37	23·16	10·17	7·78	5·21	3·29	2·92	1867–8
1868–9	0·37	3·77	2·80	2·04	0·42	4·55	3·44	2·41	1·48	2·13	3·23	1·07	27·71	8·98	10·82	7·91	6·43	5·36	1868–9
1869–70	0·97	1·35	2·79	2·05	2·38	3·20	1·81	2·10	1·80	0·46	1·35	0·98	21·24	7·16	9·49	4·59	2·79	1·81	1869–70
1870–1	1·12	1·59	2·30	4·13	1·40	2·65	1·45	1·63	1·50	2·88	0·96	3·86	25·47	9·14	7·13	9·20	7·70	3·84	1870–1
1871–2	4·00	0·77	4·07	1·79	0·66	1·42	4·68	1·47	2·15	1·63	2·89	3·09	28·62	10·63	8·23	9·76	7·61	4·52	1871–2
1872–3	2·89	2·29	1·36	4·67	3·87	4·04	4·02	1·34	2·05	0·63	1·66	1·75	30·57	11·21	13·27	6·09	4·04	2·29	1872–3
1873–4	2·54	2·69	2·38	2·83	1·99	0·71	1·93	1·73	0·65	2·14	1·19	1·59	22·37	10·44	6·36	5·57	4·92	3·33	1873–4
1874–5	2·81	1·75	3·62	3·23	2·35	1·80	3·99	1·18	0·87	1·56	2·74	3·53	29·43	11·41	9·32	8·70	7·83	4·30	1874–5
Means	2·46	2·49	2·51	3·07	1·99	2·04	2·79	1·57	1·67	1·86	2·20	2·50	27·21	10·53	8·39	8·23	6·56	4·06	**Means**
Subsequent Years.																			
1875–6	5·66	1·10	2·80	5·90	4·43	1·19	1·81	3·05	2·90	3·33	0·78	1·35	34·30	15·46	10·48	8·36	5·46	4·11	1875–6
1876–7	1·46	2·98	5·02	1·52	4·20	6·00	4·99	2·10	2·55	2·76	2·82	1·43	37·83	10·98	17·29	9·56	7·01	5·58	1876–7
1877–8	3·28	2·60	1·53	1·95	5·16	2·28	1·75	1·80	0·98	4·09	4·98	2·51	32·91	9·36	10·99	12·56	11·58	9·07	1877–8

APPENDIX-TABLE V.—Rainfall at Rothamsted.—Number of days when 0·01 inch, or more, fell.

Years.	Monthly.												Totals.						Years.
	July.	August.	Sept.	October.	Nov.	Dec.	Jan.	Feb.	March.	April.	May.	June.	12 months, July to June, inclusive.	4 months, July, August, September, October.	4 months, November, December, January, February.	4 months, March, April, May, June.	3 months, April, May, June.	2 months, April and May.	
20 YEARS, 1855–56 TO 1874–75.																			
1855–6	14	9	9	20	15	12	18	10	5	13	20	6	151	52	55	44	39	33	1855–6
1856–7	14	12	10	14	11	17	21	10	12	15	10	11	157	50	59	48	36	25	1856–7
1857–8	9	10	18	12	14	9	7	6	5	11	16	7	124	49	36	39	34	27	1857–8
1858–9	16	11	12	11	14	19	13	14	9	15	9	13	156	50	60	46	37	24	1858–9
1859–60	8	12	13	14	10	16	20	8	13	12	13	29	168	47	54	67	54	25	1859–60
1860–1	13	21	15	13	13	11	8	20	21	5	7	15	162	62	52	48	27	12	1860–1
1861–2	21	11	14	13	20	14	16	7	18	14	18	16	182	59	57	66	48	32	1861–2
1862–3	13	11	11	23	18	16	17	9	12	9	9	16	164	58	60	46	34	18	1862–3
1863–4	6	17	16	20	16	13	12	14	14	6	8	15	157	59	55	43	29	14	1863–4
1864–5	6	8	16	7	15	3	18	10	11	6	17	5	122	37	46	39	28	23	1864–5
1865–6	17	14	6	18	17	7	20	18	10	11	9	14	161	55	62	44	34	20	1865–6
1866–7	12	18	24	11	12	15	12	11	12	18	15	10	170	65	50	55	43	33	1866–7
1867–8	17	9	7	17	9	10	13	11	18	10	6	4	126	50	43	33	20	16	1867–8
1868–9	4	15	9	11	8	22	14	16	16	12	16	8	151	39	60	52	36	28	1868–9
1869–70	6	11	12	17	12	18	17	21	15	8	10	5	152	46	68	38	23	18	1869–70
1870–1	12	8	10	23	14	8	12	12	12	14	8	16	149	53	46	50	38	22	1870–1
1871–2	18	5	13	10	12	13	21	16	10	13	18	15	164	46	62	56	46	31	1871–2
1872–3	16	14	14	21	21	23	16	7	18	9	13	10	182	65	67	50	32	22	1872–3
1873–4	10	19	12	22	19	12	18	16	14	10	11	13	176	63	65	48	34	21	1873–4
1874–5	13	16	17	20	14	13	24	10	10	9	10	15	171	66	61	44	34	19	1874–5
Means	12	13	13	16	14	14	16	12	13	11	12	12	157	54	56	48	35	24	Means
SUBSEQUENT YEARS.																			
1875–6	15	9	16	19	19	15	12	22	17	19	9	11	183	59	68	56	39	28	1875–6
1876–7	11	14	25	15	20	27	26	18	23	18	18	7	217	65	91	61	38	31	1876–7
1877–8	17	17	15	17	23	19	17	15	9	16	23	13	201	66	74	61	52	39	1877–8

APPENDIX-TABLE VI.—Mean maximum temperature, at Greenwich (Fahrenheit).

Years.	Monthly.												Averages.						Years.
	July.	August.	Sept.	October.	Nov.	Dec.	Jan.	Feb.	March.	April.	May.	June.	12 months, July to June, inclusive.	4 months, July, August, September, October.	4 months, November, December, January, February.	4 months, March, April, May, June.	3 months, April, May, June.	2 months, April and May.	
20 Years, 1855–56 to 1874–75.																			
1855–6	73·3	72·9	68·5	58·7	46·5	40·2	43·7	47·3	45·7	57·4	59·1	70·8	57·0	68·4	44·4	58·3	62·4	58·3	1855–6
1856–7	73·3	75·9	65·5	59·8	47·1	45·1	41·8	47·2	50·5	55·8	67·1	75·9	58·8	68·6	45·3	62·3	66·3	61·5	1856–7
1857–8	77·8	78·1	70·2	61·6	51·9	50·3	43·8	41·8	50·7	57·6	63·7	79·5	61·0	72·1	47·9	63·0	66·9	60·7	1857–8
1858–9	73·8	75·6	70·9	59·9	46·1	45·1	45·5	50·4	54·2	56·9	64·9	73·9	59·3	70·1	46·8	62·5	65·2	60·9	1858–9
1859–60	81·8	76·1	67·1	59·0	49·4	41·5	45·0	42·5	49·2	53·7	65·5	65·0	58·0	71·0	44·6	58·4	61·4	59·6	1859–60
1860–1	69·2	67·2	63·4	58·6	46·7	40·5	39·6	48·2	52·7	55·0	63·5	70·8	56·3	64·6	43·8	60·5	63·1	59·3	1860–1
1861–2	72·3	75·6	68·3	64·1	47·3	45·9	43·9	46·5	50·0	57·5	66·4	67·1	58·7	70·1	45·9	60·3	63·7	62·0	1861–2
1862–3	70·8	71·0	67·6	60·5	45·8	48·0	46·9	49·5	53·7	61·2	64·4	70·1	59·1	67·5	47·6	62·4	65·2	62·8	1862–3
1863–4	74·3	73·8	63·5	58·8	51·1	48·6	41·4	41·5	49·8	58·3	64·8	69·5	58·0	67·6	46·1	60·9	64·2	61·6	1863–4
1864–5	75·3	72·8	67·3	58·2	48·5	42·5	40·9	42·2	44·0	66·3	67·9	73·6	58·3	68·4	43·5	63·0	69·3	67·1	1864–5
1865–6	75·7	70·9	76·4	60·0	50·8	46·7	47·8	47·1	48·4	58·2	61·4	73·2	59·7	70·8	48·1	60·3	64·3	59·8	1865–6
1866–7	72·6	69·4	65·1	58·2	50·5	47·6	39·5	50·7	44·5	58·7	64·7	70·2	57·6	66·3	47·1	59·5	64·5	61·7	1866–7
1867–8	71·1	73·3	68·0	57·2	47·8	42·2	41·4	49·9	52·9	58·8	70·5	76·3	59·1	67·4	45·4	64·6	68·5	64·7	1867–8
1868–9	82·0	75·1	71·7	56·9	46·9	50·5	46·0	51·8	44·8	61·6	60·7	67·4	59·6	71·4	48·8	58·6	63·2	61·2	1868–9
1869–70	77·0	72·3	68·6	57·5	49·1	42·0	42·6	41·4	46·9	62·0	66·9	74·8	58·4	68·9	43·8	62·7	67·9	64·5	1869–70
1870–1	78·1	72·6	66·8	58·2	47·8	38·0	37·4	43·3	55·0	57·8	64·4	66·3	57·6	68·9	42·9	60·9	62·8	61·1	1870–1
1871–2	72·6	78·1	67·5	58·6	43·2	42·2	46·3	51·7	53·5	59·3	62·1	71·3	58·9	69·2	45·9	61·6	64·2	60·7	1871–2
1872–3	78·2	72·9	68·2	56·7	50·8	47·0	46·8	39·2	51·3	57·4	62·3	70·2	58·4	69·0	46·0	60·3	63·3	59·9	1872–3
1873–4	76·6	74·7	65·2	57·0	50·1	45·6	47·3	45·0	52·8	61·5	63·2	71·1	59·2	68·4	47·0	62·2	65·3	62·4	1873–4
1874–5	79·0	72·1	63·4	59·5	48·4	37·9	47·8	40·4	47·1	57·3	64·7	69·4	57·7	69·8	43·6	59·6	63·8	61·0	1874–5
Means	75·2	73·5	67·9	59·0	48·3	44·4	43·8	46·1	49·9	58·6	64·4	71·3	58·9	68·9	45·7	61·1	64·8	61·5	Means
Subsequent Years.																			
1875–6	67·6	73·7	70·2	57·2	48·8	43·2	42·8	47·5	49·2	57·6	60·7	69·4	57·3	67·2	45·6	59·2	62·6	59·2	1875–6
1876–7	77·7	74·5	65·3	60·2	49·9	48·6	49·5	50·4	49·3	54·3	59·1	74·4	59·4	69·4	49·6	59·3	62·6	56·7	1876–7
1877–8	70·6	71·6	62·5	57·8	52·1	45·9	45·3	47·6	49·9	58·2	64·8	71·1	58·1	65·6	47·7	61·0	64·7	61·5	1877–8

APPENDIX-TABLE VII.—Mean minimum temperature, at Greenwich (Fahrenheit).

Years.	Monthly.												Averages.						Years.
	July.	August.	Sept.	October.	Nov.	Dec.	Jan.	Feb.	March.	April.	May.	June.	12 months, July to June, inclusive.	4 months, July, August, September, October.	4 months, November, December, January, February.	4 months, March, April, May, June.	3 months, April, May, June.	2 months, April and May.	
20 YEARS, 1855–56 TO 1874–75.																			
1855–6	54·1°	53·7°	47·7°	45·2°	36·8°	30·9°	35·2°	37·6°	33·3°	38·3°	42·6°	50·0°	42·1°	50·2°	35·1°	41·1°	43·6°	40·5°	1855–6
1856–7	52·4	54·7	47·6	46·1	35·4	35·3	32·1	32·5	35·9	38·8	44·0	50·5	42·1	50·2	38·8	42·3	44·4	41·4	1856–7
1857–8	54·3	56·4	52·1	47·1	40·6	39·6	31·7	29·8	33·6	38·0	42·7	53·9	43·3	52·5	35·4	42·1	44·9	40·4	1857–8
1858–9	51·8	52·1	52·6	43·9	33·6	36·6	35·5	36·3	40·5	39·1	43·9	53·0	43·2	50·1	35·5	44·1	45·3	41·5	1858–9
1859–60	57·2	54·3	49·0	45·0	35·5	31·8	34·8	30·1	35·0	35·6	44·6	48·5	41·8	51·4	33·1	40·9	42·9	40·1	1859–60
1860–1	50·1	51·8	45·8	44·5	35·3	31·9	28·7	36·9	37·1	36·0	43·0	51·3	41·0	48·1	33·2	41·9	43·4	39·5	1860–1
1861–2	53·4	53·8	48·2	47·7	34·1	36·0	34·3	36·7	38·4	41·7	47·9	49·3	43·5	50·8	35·3	44·3	46·3	44·8	1861–2
1862–3	50·8	51·4	50·1	45·6	34·3	38·6	36·6	35·7	35·6	40·1	42·7	50·1	42·6	49·5	36·3	42·1	44·3	41·4	1862–3
1863–4	49·4	53·7	45·8	46·1	40·3	36·8	31·7	31·2	34·3	40·0	44·9	49·1	41·9	48·8	35·0	42·1	44·7	42·5	1863–4
1864–5	51·2	48·5	49·1	44·1	35·5	33·7	31·8	32·2	31·1	41·5	46·3	49·9	41·2	48·2	33·3	42·2	45·9	43·9	1864–5
1865–6	54·3	51·5	53·6	43·7	38·7	38·1	36·7	34·7	34·5	40·8	40·8	52·0	43·3	50·3	37·1	42·0	44·5	40·8	1865–6
1866–7	52·5	52·3	50·6	45·7	38·0	37·4	28·5	39·5	33·0	42·3	44·7	49·1	42·8	50·3	35·9	42·3	45·4	43·5	1866–7
1867–8	50·8	53·4	50·3	42·0	35·3	32·1	32·8	36·8	36·8	40·1	46·1	50·8	42·3	49·1	34·3	43·5	45·7	43·1	1867–8
1868–9	55·7	55·0	51·1	40·2	36·1	41·1	36·5	39·7	32·3	41·8	43·7	46·0	43·3	50·5	38·4	41·0	43·8	42·8	1868–9
1869–70	54·5	52·4	52·4	42·0	37·4	33·4	34·0	31·9	34·0	38·4	42·0	50·7	41·9	50·3	34·2	41·3	43·7	40·2	1869–70
1870–1	56·0	53·1	46·4	42·8	35·4	28·9	29·3	37·5	36·7	41·2	42·1	47·9	41·4	49·6	32·8	42·0	43·7	41·7	1870–1
1871–2	54·0	53·8	50·3	41·9	32·7	34·2	37·0	39·2	37·7	40·1	42·5	50·0	42·8	50·0	35·8	42·6	44·2	41·3	1871–2
1872–3	54·8	52·5	49·1	41·1	40·8	38·7	38·0	30·8	35·2	37·9	42·5	51·0	42·7	49·4	37·1	41·7	43·8	40·2	1872–3
1873–4	53·9	54·4	46·1	40·6	38·8	35·2	36·2	33·5	36·6	41·3	40·7	48·3	42·1	48·8	35·9	41·7	43·4	41·0	1873–4
1874–5	53·6	51·5	50·3	45·7	39·6	28·6	39·6	31·8	36·0	39·4	50·0	51·8	43·2	50·3	34·9	44·3	47·1	44·7	1874–5
Means	53·2	53·0	49·4	44·1	36·7	35·0	34·0	34·7	35·4	39·6	43·9	50·2	42·4	50·0	35·1	42·3	44·6	41·8	Means
SUBSEQUENT YEARS.																			
1875–6	53·2	55·3	53·0	42·4	37·2	34·1	32·0	36·1	34·6	40·2	40·2	49·5	42·3	51·0	34·9	41·1	43·3	40·2	1875–6
1876–7	56·1	54·2	48·9	46·9	38·3	39·5	36·8	38·0	34·1	39·5	41·1	51·5	43·7	51·5	38·4	41·6	44·0	40·3	1876–7
1877–8	53·4	55·8	46·4	41·4	39·7	35·9	35·7	37·3	36·4	39·6	47·5	51·4	43·4	49·3	37·1	43·7	46·1	43·6	1877–8

APPENDIX-TABLE VI.—Mean maximum temperature, at Greenwich (Fahrenheit).

Years.	Monthly.												Averages.						Years.
	July.	August.	Sept.	October.	Nov.	Dec.	Jan.	Feb.	March.	April.	May.	June.	12 months, July to June, inclusive.	4 months, July, August, September, October.	4 months, November, December, January, February.	4 months, March, April, May, June.	3 months, April, May, June.	2 months, April and May.	
20 YEARS, 1855–56 TO 1874–75.																			
1855–6	73·3	72·9	68·5	58·7	46·5	40·2	43·7	47·3	45·7	57·4	59·1	70·8	57·0	68·4	44·4	58·3	62·4	58·3	1855–6
1856–7	73·3	75·9	65·5	59·8	47·1	45·1	41·8	47·2	50·5	55·8	67·1	75·9	58·8	68·6	45·3	62·3	66·3	61·5	1856–7
1857–8	77·8	78·1	70·2	61·6	51·9	50·3	48·8	41·8	50·7	57·6	63·7	79·5	61·0	72·1	47·9	63·0	66·9	60·7	1857–8
1858–9	73·8	75·6	70·9	59·9	46·1	45·1	45·5	50·4	54·2	56·9	64·9	73·9	59·3	70·1	46·8	62·5	65·2	60·9	1858–9
1859–60	81·8	76·1	67·1	59·0	49·4	41·5	45·0	42·5	49·2	53·7	65·5	65·0	58·0	71·0	44·6	58·4	61·4	59·6	1859–60
1860–1	69·2	67·2	63·4	58·6	46·7	40·5	39·6	48·2	52·7	55·0	63·5	70·8	56·3	64·6	43·8	60·5	63·1	59·3	1860–1
1861–2	72·3	75·6	68·3	64·1	47·3	45·9	43·9	46·5	50·0	57·5	66·4	67·1	58·7	70·1	45·9	60·3	63·7	62·0	1861–2
1862–3	70·8	71·0	67·6	60·5	45·8	48·0	46·9	49·5	53·7	61·2	64·4	70·1	59·1	67·5	47·6	62 4	65·2	62·8	1862–3
1863–4	74·3	73·8	63·5	58·8	51·1	48·6	41·4	41·5	49·8	58·3	64·8	69·5	58·0	67·6	46·1	60·9	64·2	61·6	1863–4
1864–5	75·3	72·8	67·3	58·2	48·5	42·5	40·9	42·2	44·0	66·3	67·9	73·6	58·3	68·4	43·5	63·0	69·3	67·1	1864–5
1865–6	75·7	70·9	76·4	60·0	50·8	46·7	47·8	47·1	48·4	58·2	61·4	73·2	59·7	70·8	48·1	60·3	64·3	59·8	1865–6
1866–7	72·6	69·4	65·1	58·2	50·5	47·6	39·5	50·7	44·5	58·7	64·7	70·2	57·6	66·3	47·1	59·5	64·5	61·7	1866–7
1867–8	71·1	73·3	68·0	57·2	47·8	42·2	41·4	49·9	52·9	58·8	70·5	76·3	59·1	67·4	45·4	64·6	68·5	64·7	1867–8
1868–9	82·0	75·1	71·7	56·9	46·9	50·5	46·0	51·8	44·8	61·6	60·7	67·4	59·6	71·4	48·8	58·6	63·2	61·2	1868–9
1869–70	77·0	72·3	68·6	57·5	49·1	42·0	42·6	41·4	46·9	62·0	66·9	74·8	58·4	68·9	43·8	62·7	67·9	64·5	1869–70
1870–1	78·1	72·6	66·8	58·2	47·8	38·0	37·4	48·3	55·0	57·8	64·4	66·3	57·6	68·9	42·9	60·9	62·8	61·1	1870–1
1871–2	72·6	78·1	67·5	58·6	43·2	42·2	46·3	51·7	53·5	59·3	62·1	71·3	58·9	69·2	45·9	61·6	64·2	60·7	1871–2
1872–3	78·2	72·9	68·2	56·7	50·8	47·0	46·8	39·2	51·3	57·4	62·3	70·2	58·4	69·0	46·0	60·3	63·3	59·9	1872–3
1873–4	76·6	74·7	65·2	57·0	50·1	45·6	47·3	45·0	52·8	61·5	63·2	71·1	59·2	68·4	47·0	62·2	65·3	62·4	1873–4
1874–5	79·0	72·1	63·4	59·5	48·4	37·9	47·8	40·4	47·1	57·3	64·7	69·4	57·7	69·8	43·6	59·6	63·8	61·0	1874–5
Means	75·2	73·5	67·9	59·0	48·3	44·4	43·8	46·1	49·9	58·6	64·4	71·3	58·9	68·9	45·7	61·1	64·8	61·5	Means
SUBSEQUENT YEARS.																			
1875–6	67·6	73·7	70·2	57·2	48·8	43·2	42·8	47·5	49·2	57·6	60·7	69·4	57·3	67·2	45·6	59·2	62·6	59·2	1875–6
1876–7	77·7	74·5	65·3	60·2	49·9	48·6	49·5	50·4	49·3	54·3	59·1	74·4	59·4	69·4	49·6	59·3	62·6	56·7	1876–7
1877–8	70·6	71·6	62·5	57·8	52·1	45·9	45·3	47·6	49·9	58·2	64·8	71·1	58·1	65·6	47·7	61·0	64·7	61·5	1877–8

APPENDIX-TABLE VII.—Mean minimum temperature, at Greenwich (Fahrenheit).

Years.	Monthly.												Averages.						Years.
	July.	August.	Sept.	October.	Nov.	Dec.	Jan.	Feb.	March.	April.	May.	June.	12 months, July to June, inclusive.	4 months, July, August, September, October.	4 months, November, December, January, February.	4 months, March, April, May, June.	3 months, April, May, June.	2 months, April and May.	
							20 YEARS, 1855–56 TO 1874–75.												
1855–6	54·1°	53·7°	47·7°	45·2°	36·8°	30·9°	35·2°	37·6°	33·3°	38·3°	42·6°	50·0°	42·1°	50·2°	35·1°	41·1°	43·6°	40·5°	1855–6
1856–7	52·4	54·7	47·6	46·1	35·4	35·3	32·1	32·5	35·9	38·8	44·0	50·5	42·1	50·2	33·8	42·3	44·4	41·4	1856–7
1857–8	54·3	56·4	52·1	47·1	40·6	39·6	31·7	29·8	33·6	38·0	42·7	53·9	43·3	52·5	35·4	42·1	44·9	40·4	1857–8
1858–9	51·8	52·1	52·6	43·9	33·6	36·6	35·5	36·3	40·5	39·1	43·9	53·0	43·2	50·1	35·5	44·1	45·3	41·5	1858–9
1859–60	57·2	54·3	49·0	45·0	35·5	31·8	34·8	30·1	35·0	35·6	44·6	48·5	41·8	51·4	33·1	40·9	42·9	40·1	1859–60
1860–1	50·1	51·8	45·8	44·5	35·3	31·9	28·7	36·9	37·1	36·0	43·0	51·3	41·0	48·1	33·2	41·9	43·4	39·5	1860–1
1861–2	53·4	53·8	48·2	47·7	34·1	36·0	34·3	36·7	38·4	41·7	47·9	49·3	43·5	50·8	35·3	44·3	46·3	44·8	1861–2
1862–3	50·8	51·4	50·1	45·6	34·3	38·6	36·6	35·7	35·6	40·1	42·7	50·1	42·6	49·5	36·3	42·1	44·3	41·4	1862–3
1863–4	49·4	53·7	45·8	46·1	40·3	36·8	31·7	31·2	34·3	40·0	44·9	49·1	41·9	48·8	35·0	42·1	44·7	42·5	1863–4
1864–5	51·2	48·5	49·1	44·1	35·5	33·7	31·8	32·2	31·1	41·5	46·3	49·9	41·2	48·2	33·3	42·2	45·9	43·9	1864–5
1865–6	54·3	51·5	53·6	43·7	38·7	38·1	36·7	34·7	34·5	40·8	40·8	52·0	43·3	50·3	37·1	42·0	44·5	40·8	1865–6
1866–7	52·5	52·3	50·6	45·7	38·0	37·4	28·5	39·5	33·0	42·3	44·7	49·1	42·8	50·3	35·9	42·3	45·4	43·5	1866–7
1867–8	50·8	53·4	50·3	42·0	35·3	32·1	32·8	36·8	36·8	40·1	46·1	50·8	42·3	49·1	34·3	43·5	45·7	43·1	1867–8
1868–9	55·7	55·0	51·1	40·2	36·1	41·1	36·5	39·7	32·3	41·8	43·7	46·0	43·3	50·5	38·4	41·0	43·8	42·8	1868–9
1869–70	54·5	52·4	52·4	42·0	37·4	33·4	34·0	31·9	34·0	38·4	42·0	50·7	41·9	50·3	34·2	41·3	43·7	40·2	1869–70
1870–1	56·0	53·1	46·4	42·8	35·4	28·9	29·3	37·5	36·7	41·2	42·1	47·9	41·4	49·6	32·8	42·0	43·7	41·7	1870–1
1871–2	54·0	53·8	50·3	41·9	32·7	34·2	37·0	39·2	37·7	40·1	42·5	50·0	42·8	50·0	35·8	42·6	44·2	41·3	1871–2
1872–3	54·8	52·5	49·1	41·1	40·8	38·7	38·0	30·8	35·2	37·9	42·5	51·0	42·7	49·4	37·1	41·7	43·8	40·2	1872–3
1873–4	53·9	54·4	46·1	40·6	38·8	35·2	36·2	33·5	36·6	41·3	40·7	48·3	42·1	48·8	35·9	41·7	43·4	41·0	1873–4
1874–5	53·6	51·5	50·3	45·7	39·6	28·6	39·6	31·8	36·0	39·4	50·0	51·8	43·2	50·3	34·9	44·3	47·1	44·7	1874–5
Means	53·2	53·0	49·4	44·1	36·7	35·0	34·0	34·7	35·4	39·6	43·9	50·2	42·4	50·0	35·1	42·3	44·6	41·8	Means
									SUBSEQUENT YEARS.										
1875–6	53·2	55·8	53·0	42·4	37·2	34·1	32·0	36·1	34·6	40·2	40·2	49·5	42·3	51·0	34·9	41·1	43·3	40·2	1875–6
1876–7	56·1	54·2	48·9	46·9	38·3	39·5	36·8	38·0	34·1	39·5	41·1	51·5	43·7	51·5	38·4	41·6	44·0	40·3	1876–7
1877–8	53·4	55·8	46·4	41·4	39·7	35·0	35·7	37·3	36·4	39·6	47·5	51·4	43·4	49·3	37·1	43·7	46·1	43·6	1877–8

APPENDIX-TABLE VIII.—Mean temperature, at Greenwich (Fahrenheit).

Years.	Monthly.												Averages.						Years.
	July.	August.	Sept.	October.	Nov.	Dec.	Jan.	Feb.	March.	April.	May.	June.	12 months, July to June, inclusive.	4 months, July, August, September, October.	4 months, November, December, January, February.	4 months, March, April, May, June.	3 months, April, May, June.	2 months, April and May.	
20 YEARS, 1855–56 TO 1874–75.																			
1855–6	62·1°	62·1°	57·1°	51·2°	41·3°	35·6°	39·4°	42·0°	38·7°	46·8°	49·5°	55·2°	48·4°	58·1°	39·6°	47·6°	50·5°	48·2°	1855–6
1856–7	61·1	63·6	55·2	51·7	40·7	40·2	36·6	39·2	41·8	45·7	54·0	61·8	49·3	57·9	39·2	50·8	53·8	49·9	1856–7
1857–8	64·5	65·8	59·7	52·9	45·8	45·1	37·5	34·6	41·4	46·2	51·7	64·9	50·7	60·7	40·8	51·1	54·3	49·0	1857–8
1858–9	60·6	62·0	60·3	50·8	39·6	41·0	40·4	43·1	46·4	46·6	53·1	61·4	50·4	58·4	41·0	51·9	53·7	49·9	1858–9
1859–60	68·1	63·5	56·7	50·9	41·9	36·8	39·7	35·7	41·1	42·9	53·8	54·8	48·8	59·8	38·5	48·2	50·5	48·4	1859–60
1860–1	57·6	57·7	53·4	50·6	40·8	36·3	33·9	42·1	43·8	44·3	51·9	59·1	47·6	54·8	38·3	49·8	51·8	48·1	1860–1
1861–2	60·9	63·2	57·1	54·9	40·8	41·0	39·0	41·1	43·1	48·4	55·4	56·3	50·1	59·0	40·5	50·8	53·3	51·9	1861–2
1862–3	59·1	59·5	57·7	51·8	39·8	43·6	41·8	42·1	43·9	49·1	52·0	58·1	49·9	57·0	41·8	50·8	53·1	50·5	1862–3
1863–4	60·8	61·9	53·7	51·6	45·7	43·2	36·5	36·0	41·3	48·2	53·8	57·4	49·2	57·0	40·4	50·2	53·1	51·0	1863–4
1864–5	61·8	59·6	56·9	50·5	42·0	38·5	36·3	36·6	36·6	52·3	56·1	60·2	49·0	57·2	38·4	51·3	56·2	54·2	1864–5
1865–6	63·8	59·9	63·9	50·9	44·8	42·4	42·6	40·5	40·5	47·9	50·1	60·9	50·7	59·6	42·6	49·9	53·0	49·0	1865–6
1866–7	61·0	59·4	56·4	51·3	44·3	42·9	34·2	44·7	37·7	49·0	53·4	58·1	49·4	57·0	41·5	49·6	53·5	51·2	1866–7
1867–8	59·4	62·0	57·6	48·7	41·1	37·5	37·2	43·0	44·0	48·1	57·3	62·9	49·8	56·9	39·7	52·9	55·8	52·7	1867–8
1868–9	67·5	63·6	60·5	47·9	41·5	46·0	41·1	45·3	37·5	50·3	50·5	55·3	50·6	59·9	43·5	48·4	52·0	50·4	1868–9
1869–70	64·5	60·8	59·0	48·9	43·0	37·9	38·3	36·2	39·6	48·9	53·4	60·9	49·4	58·3	38·9	50·7	54·4	51·2	1869–70
1870–1	65·4	61·1	55·7	49·8	41·5	33·6	33·2	42·4	44·9	47·7	51·9	54·8	48·5	58·0	37·7	49·8	51·5	49·8	1870–1
1871–2	61·7	64·8	57·4	49·4	37·6	38·3	41·3	44·8	44·6	48·3	50·9	59·2	49·9	58·3	40·5	50·8	52·8	49·6	1871–2
1872–3	65·0	61·0	57·4	47·8	45·3	42·9	42·1	34·6	41·9	45·9	50·6	58·9	49·5	57·8	41·2	49·3	51·8	48·3	1872–3
1873–4	63·4	62·7	54·7	47·8	44·2	40·6	41·7	38·7	43·7	50·0	50·5	58·0	49·7	57·2	41·3	50·6	52·8	50·3	1873–4
1874–5	64·4	60·1	57·9	51·7	42·0	33·2	43·8	35·5	40·7	46·8	55·3	59·0	49·2	58·5	38·6	50·5	53·7	51·1	1874–5
Means.	62·6	61·7	57·4	50·6	42·2	39·8	38·8	39·8	41·7	47·7	52·8	58·8	49·5	58·1	40·2	50·3	53·1	50·3	Means.
SUBSEQUENT YEARS.																			
1875–6	58·7	62·6	60·0	49·0	42·5	39·0	37·4	41·5	41·2	47·5	49·1	58·0	48·9	57·6	40·1	49·0	51·5	48·3	1875–6
1876–7	65·0	62·9	55·8	52·7	44·0	44·3	43·1	44·0	40·8	45·6	48·7	61·4	50·7	59·1	43·9	49·1	51·9	47·2	1876–7
1877–8	60·6	62·1	53·3	49·2	46·0	41·3	40·6	42·4	42·4	47·6	54·5	59·4	50·0	56·3	42·6	51·0	53·8	51·1	1877–8

APPENDIX-TABLE IX.—Mean range of temperature, at Greenwich (Fahrenheit).

Years.	Monthly.												Averages.						Years.
	July.	August.	Sept.	October.	Nov.	Dec.	Jan.	Feb.	March.	April.	May.	June.	12 months, July to June, inclusive.	4 months, July, August, September, October.	4 months, November, December, January, February.	4 months, March, April, May, June.	3 months, April, May, June.	2 months, April and May.	
20 Years, 1855–56 to 1874–75.																			
1855–6	19°·2	19°·2	20°·8	13°·5	9°·7	9°·3	8°·5	9°·7	12°·4	19°·1	16°·5	20°·8	14°·9	18°·2	9°·3	17°·2	18°·8	17°·8	1855–6
1856–7	20·9	21·2	17·9	13·7	11·7	9·8	9·7	14·7	14·6	17·0	23·1	24·4	16·6	18·4	11·5	19·8	21·5	20·1	1856–7
1857–8	23·5	21·7	18·1	14·5	11·3	10·7	11·9	12·0	17·1	19·6	21·0	25·6	17·3	19·5	11·5	20·8	22·1	20·3	1857–8
1858–9	22·0	23·5	18·3	16·0	12·5	8·5	10·0	14·1	13·7	17·8	21·0	20·9	16·5	20·0	11·3	18·3	19·9	19·4	1858–9
1859–60	24·6	21·8	18·1	14·0	13·9	9·7	10·2	12·4	14·2	18·1	20·9	16·5	16·2	19·6	11·6	17·4	18·5	19·5	1859–60
1860–1	19·1	15·4	17·6	14·1	11·4	8·6	10·9	11·3	15·6	19·0	20·5	19·5	15·3	16·6	10·5	18·6	19·7	19·8	1860–1
1861–2	18·9	21·8	20·1	16·4	13·2	9·9	9·6	9·8	11·6	15·8	18·5	17·8	15·3	19·3	10·6	15·9	17·4	17·1	1861–2
1862–3	20·0	19·6	17·5	14·9	11·5	9·4	10·3	13·8	18·1	21·1	21·7	20·0	16·5	18·0	11·3	20·2	20·9	21·4	1862–3
1863–4	24·9	20·1	17·7	12·7	10·8	11·8	9·7	10·3	15·5	18·3	19·9	20·4	16·0	18·9	10·7	18·5	19·5	19·1	1863–4
1864–5	24·1	24·3	18·2	14·1	13·0	8·8	9·1	10·0	12·9	24·8	21·6	23·7	17·1	20·2	10·2	20·8	23·4	23·2	1864–5
1865–6	21·4	19·4	22·8	16·3	12·1	8·6	11·1	12·4	13·9	17·4	20·6	21·2	16·4	20·0	11·0	18·3	19·7	19·0	1865–6
1866–7	20·1	17·1	14·5	12·5	12·5	10·2	11·0	11·2	11·5	16·4	20·0	21·1	14·8	16·0	11·2	17·3	19·2	18·2	1866–7
1867–8	20·3	19·9	17·7	15·2	12·5	10·1	8·6	13·1	16·1	18·7	24·4	25·5	16·8	18·3	11·1	21·2	22·9	21·5	1867–8
1868–9	26·3	20·1	20·6	16·7	10·8	9·4	9·5	12·1	12·5	19·8	17·0	21·4	16·4	20·9	10·5	17·7	19·4	18·4	1868–9
1869–70	22·5	19·9	16·2	15·5	11·7	8·6	8·6	9·5	12·9	23·6	24·9	24·1	16·5	18·5	9·6	21·4	24·2	24·2	1869–70
1870–1	22·1	19·5	20·4	15·4	12·4	9·1	8·1	10·8	18·2	16·6	22·3	18·4	16·1	19·4	10·1	18·9	19·1	19·4	1870–1
1871–2	18·6	24·3	17·2	16·7	10·4	8·0	9·2	12·5	15·7	19·2	19·7	21·3	16·1	19·2	10·0	19·0	20·1	19·4	1871–2
1872–3	23·4	20·4	19·1	15·6	10·0	8·3	8·8	8·4	16·1	19·5	19·8	19·2	15·7	19·6	8·9	18·6	19·5	19·7	1872 3
1873–4	22·7	20·3	19·1	16·4	11·3	10·4	11·1	11·5	16·2	20·2	22·5	22·8	17·0	19·6	11·1	20·4	21·8	21·3	1873–4
1874–5	25·4	20·6	18·1	14·4	12·8	12·0	8·2	8·6	11·1	17·9	14·7	17·6	15·1	19·6	10·4	15·3	16·7	16·3	1874–5
Means	22·0	20·5	18·5	14·9	11·8	9·6	9·6	11·4	14·6	19·1	20·5	21·1	16·2	19·0	10·6	18·8	20·2	19·8	Means
Subsequent Years.																			
1875–6	14·4	18·4	17·2	14·8	11·6	9·1	10·8	11·4	14·6	17·4	20·5	19·9	15·0	16·2	10·7	18·1	19·3	19·0	1875–6
1876–7	21·6	20·3	16·4	13·3	11·6	9·1	12·7	12·4	15·2	14·8	18·0	22·9	15·7	17·9	11·5	17·7	18·6	16·4	1876–7
1877–8	17·2	15·8	16·1	16·4	12·4	10·9	9·6	10·3	13·5	18·6	17·3	19·7	14·7	16·4	10·6	17·3	18·5	18·0	1877–8

CONTENTS.

Pages.
Introduction 289–293

PART I.—THE AGRICULTURAL RESULTS.

Statement of the Question, and General Description of the Experiments 293–300
Effects of different Manuring Agents on the Mixed Herbage (First Crops) :—
1. Without Manure 300–302
2. Ammonia-salts, alone 302–303
3. Nitrate of Soda, alone 303–307
4. Mixed Mineral Manure, alone (including Potass) 307–313
5. Superphosphate of Lime, alone 313–315
6. Mixed Mineral Manure alone, with, and without, Potass 316–320
7. Ammonia-salts (400 lbs. per acre), with Mixed Mineral Manure (including Potass) 320–324
8. Ammonia-salts (400 lbs. per acre), with Mixed Mineral Manure (including Potass), and Cut Wheat Straw.. 324–327
9. Ammonia-salts (800 lbs. per acre), and Mixed Mineral Manure (including Potass) 327–332
10. Nitrate of Soda (550 lbs. per acre), with Mixed Mineral Manure (including Potass) 332–339
11. Nitrate of Soda (275 lbs. per acre), with Mixed Mineral Manure (including Potass) 339–342
12. Ammonia-salts (400 lbs. per acre), and Superphosphate of Lime 342–345
13. Ammonia-salts (400 lbs. per acre), and Mixed Mineral Manure, with, and without, Potass 345–351
14. Mixed Mineral Manure alone, seven years; succeeding Ammonia-salts alone, 13 years 351–359
15. Equal Nitrogen, and equal Potass, in Nitrate of Soda and Sulphate of Potass, and in Nitrate of Potass; in each case with Superphosphate of Lime .. 359–362
16. Mixture supplying the Ash-constituents, and the Nitrogen, of 1 ton of Hay.. 362–367
17. Farmyard Manure, alone, and with Ammonia-salts in addition.. 367–384

The Second Crops 384–390

Influence of Season on the Produce of Hay 390–405

Appendix-Tables I.–IX. 406–415

INDEX TO PART I.

OF THE

PHILOSOPHICAL TRANSACTIONS

FOR THE YEAR 1880.

A.

Abel (F. A.) and Noble (Captain) (see Noble).
Ayrton (W. E.) and Perry (J.) The Contact Theory of Voltaic Action.—No. III., 15.

B.

Battery, voltaic, *potential of a*, deduced from the differences of potential of its constituent parts, 15.
Brain, comparative structure of, *in different animals*, 35 (see Lewis).

C.

Clark (L.), potential of his standard cell, 25.
Clarke (G. S.) and M'Leod (H.) (see M'Leod).

D.

De La Rue (W.) and Müller (H. W.). Experimental Researches on the Electric Discharge with the Chloride of Silver Battery, 65 (for index see p. 115).

E.

Electric discharge, 65 (see De La Rue).
Ellipsoids of revolution, conduction of heat in, 117.
English reproduction table (see Farr).

F.

Farr (W.). English Reproduction Table, 281.

G.

Gilbert (J. H.) and Lawes (J. B.) (see Lawes).
Gunpowder, products of explosion of fired, 203 (see Noble).

H.

Heat, conduction of, *in ellipsoids* of revolution, 117.
Hyposulphite of potassium, question of the formation of, *in explosion of gunpowder*, 209, 277.

L.

Lawes (J. B.) and Gilbert (J. H.). Agricultural, Botanical, and Chemical results of Experiments on the Mixed Herbage of Permanent Meadow, conducted for more than Twenty Years in succession on the Same Land.—Part I. Agricultural results, 289 (for the contents see p. 416).
Lewis (W. B.). Researches on the Comparative Structure of the Cortex Cerebri, 35.—Brain of the pig, 35; of the sheep, 47; of the cat, 51; relative area of five-laminated cortex in different animals, 55; divergence in types of ganglionic cell, 58; summary, 61.

M.

Manures, effects of (see Lawes).
M'Leod (H.) and Clarke (G. S.). On the Determination of the Rate of Vibration of Tuning-Forks, 1.
Medusæ, 161 (see Romanes).
Müller (H. W.) and De La Rue (W.) (see De La Rue).

N.

Niven (C.) On the Conduction of Heat in Ellipsoids of Revolution, 117.
Noble (W.) and Abel (F. A.). Researches on Explosives.—No. II. Fired Gunpowder, 203 (for contents see p. 203).

P.

Perry (J.) and Ayrton (W. E.) (see Ayrton).
Potential of voltaic battery, 15 (see Ayrton); *required for a discharge in gas*, 65 (see De La Rue).

R.

ROMANES (G. J.). Concluding Observations on the Locomotor System of Medusæ, 161.—Stimulation, 162; section, 190; general summary, 196; literature, 198.

ROSSE (Earl of). On some recent Improvements made in the Mountings of the Telescopes at Birr Castle, 153.

T.

Telescopes, mountings of large, 153.

Tuning-forks, determination of the rate of vibration of, 1.

V.

Voltaic action, contact theory of, 15.

LONDON:
PRINTED BY HARRISON AND SONS, ST. MARTIN'S LANE.

Mc Leod & Clarke.

Phil. Trans. 1880. Plate 1.

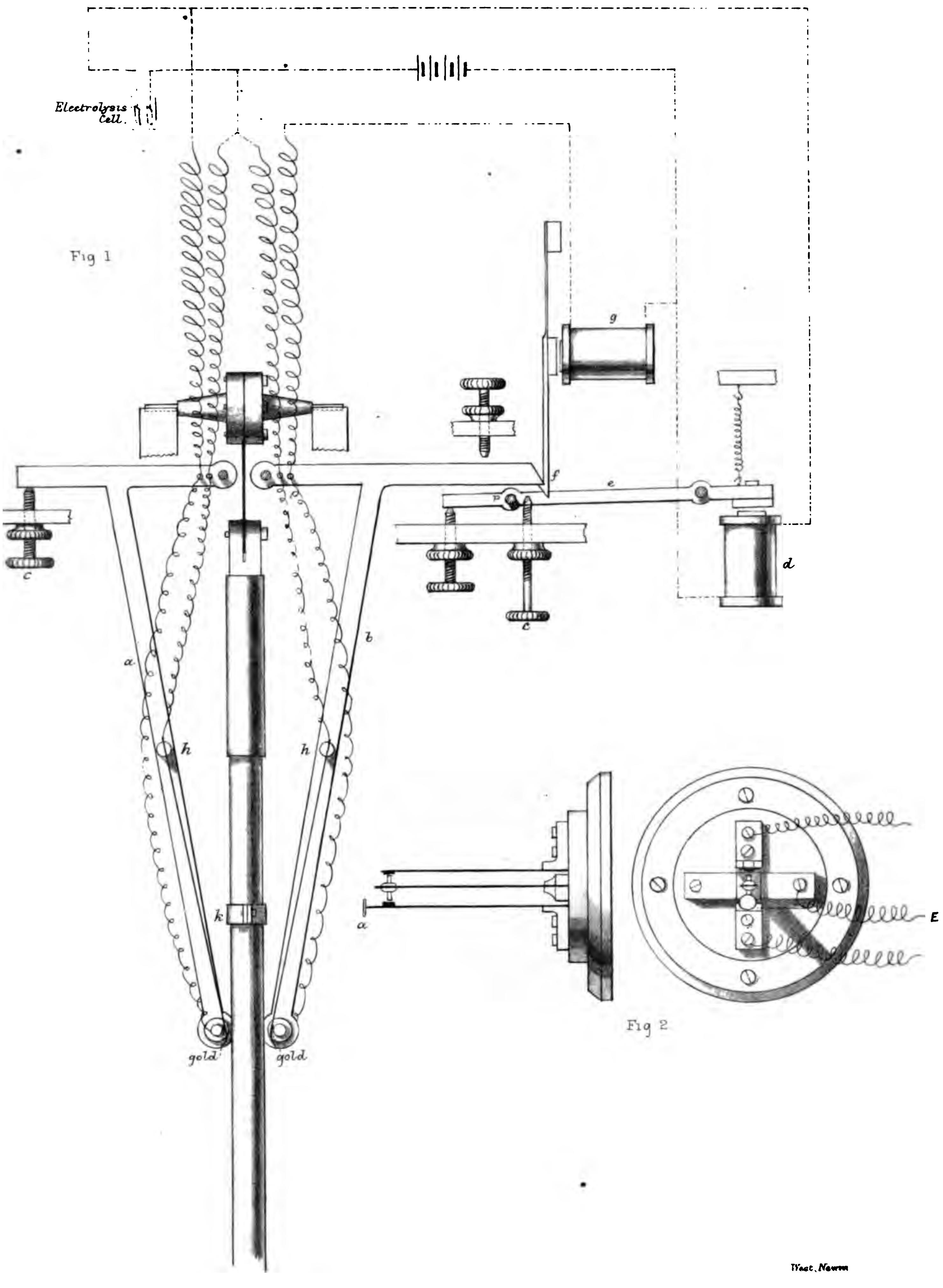
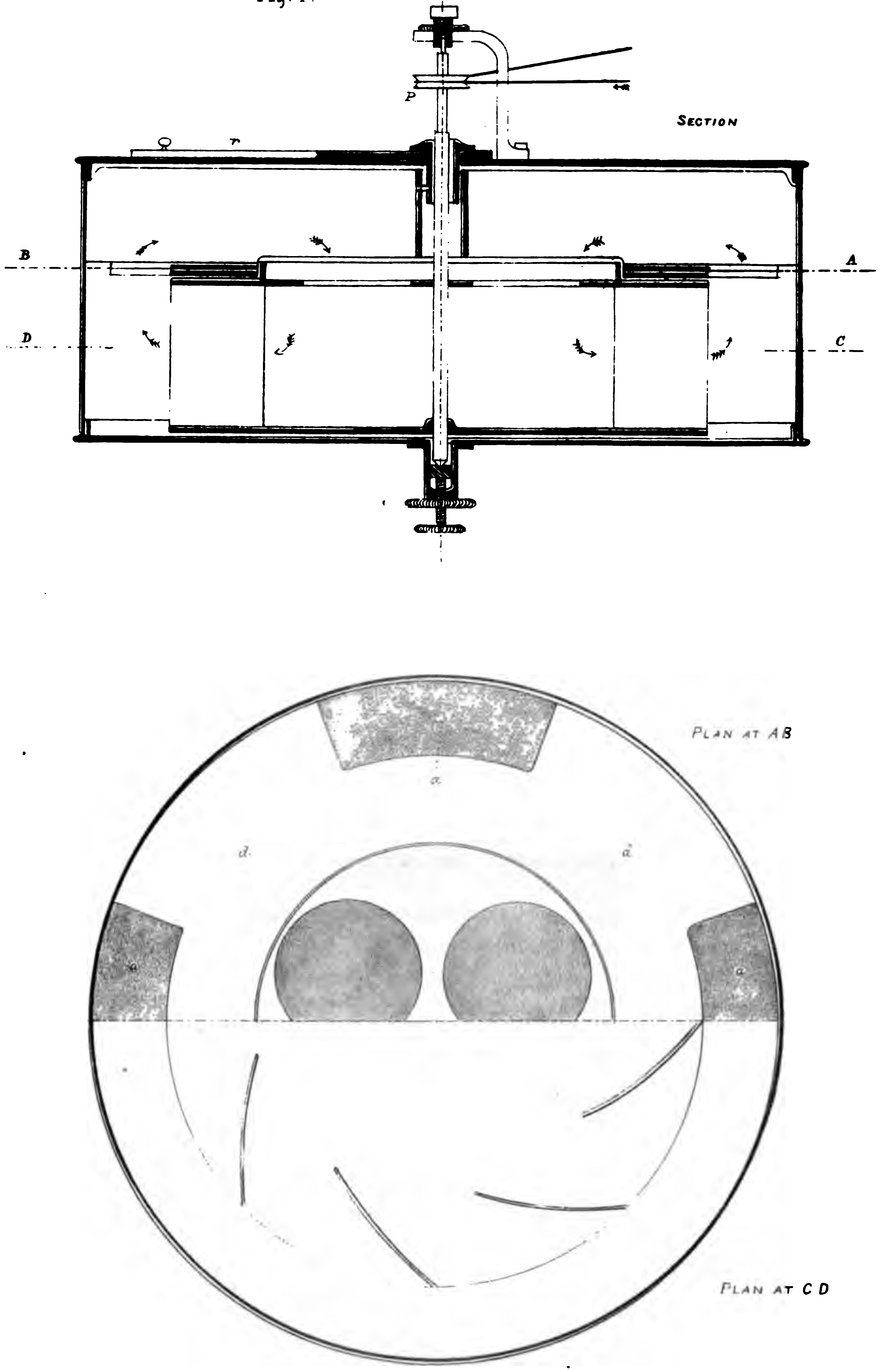


Fig 1

Fig 2

West, Newm.

Fig. 4.



West, Newman & Co. lith.

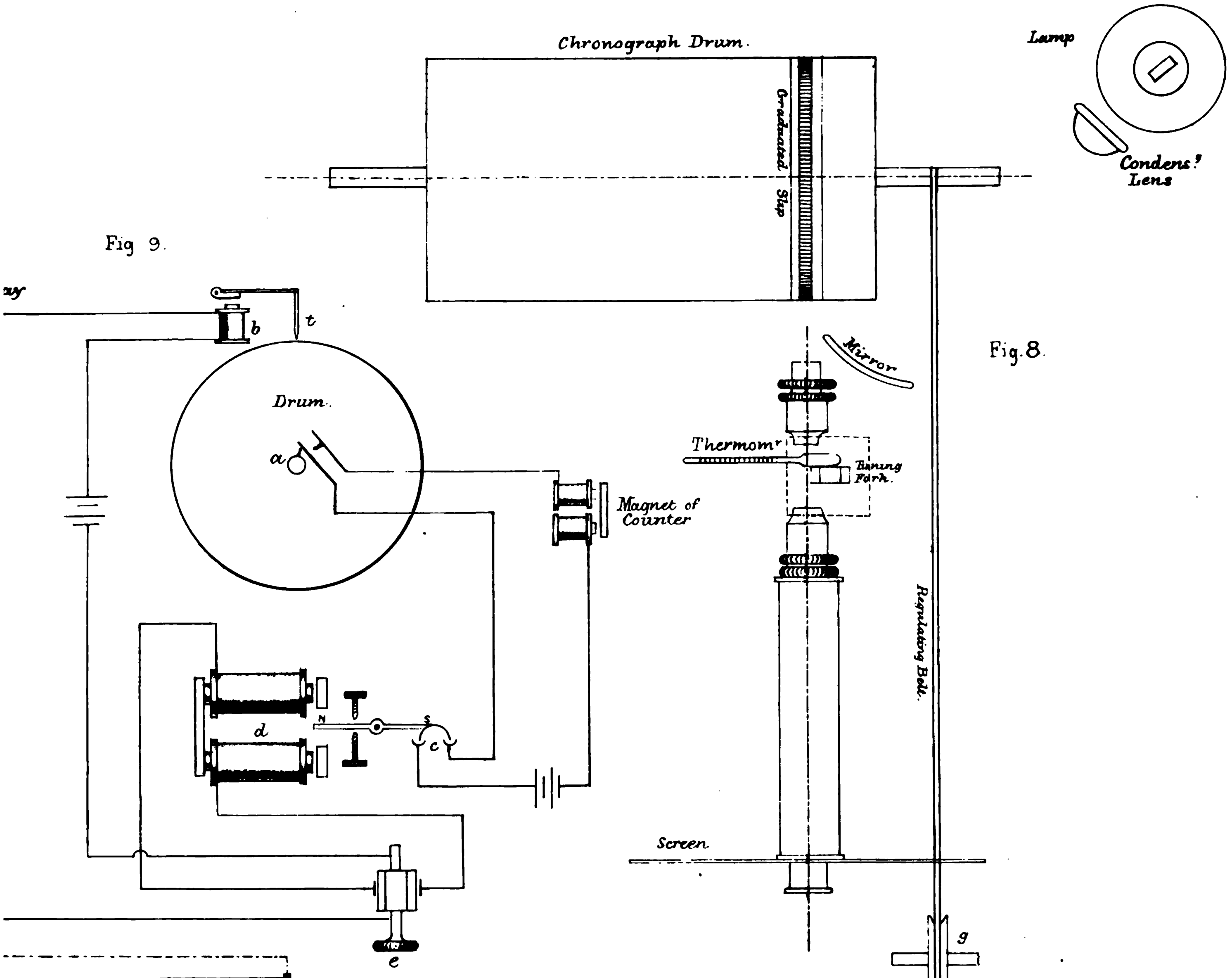


Fig 9.

Fig. 8.

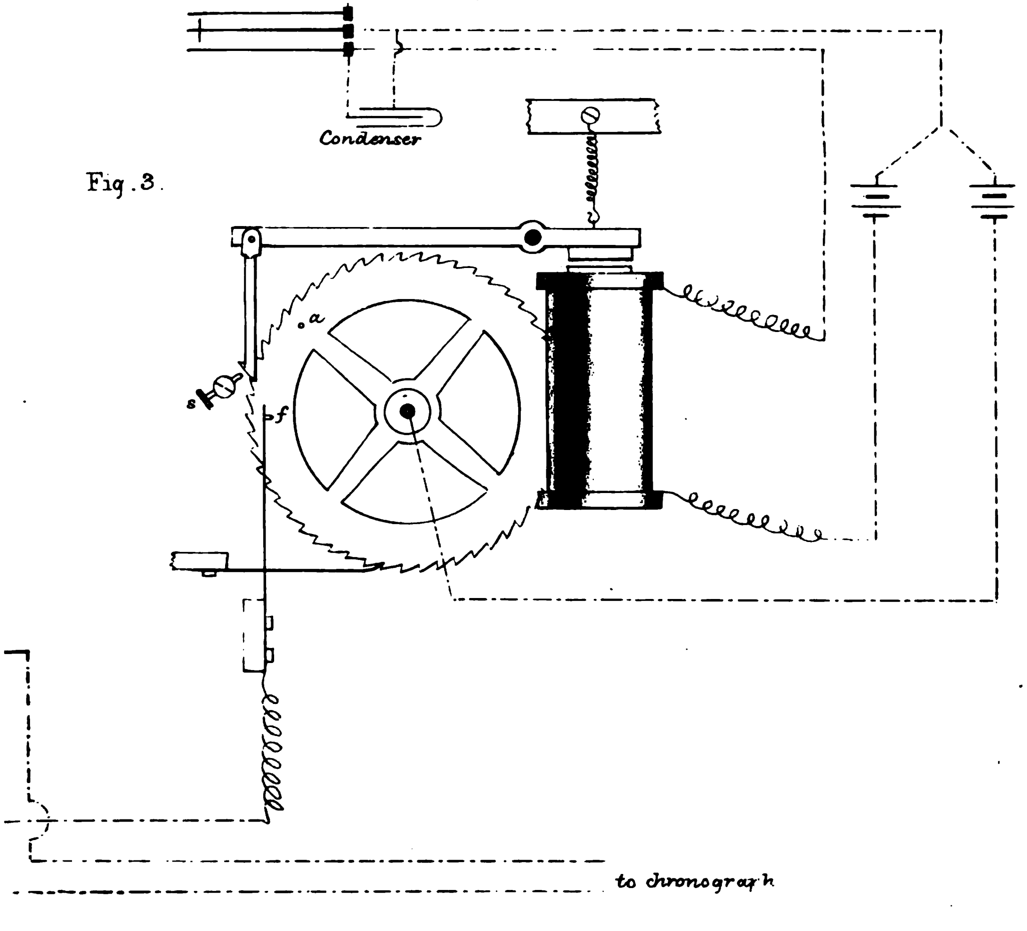


Fig. 3.

Fig. 6.

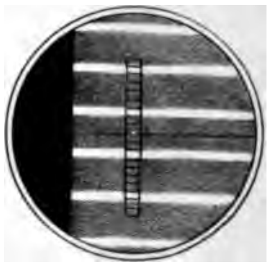
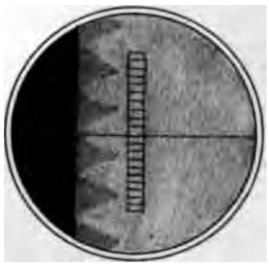
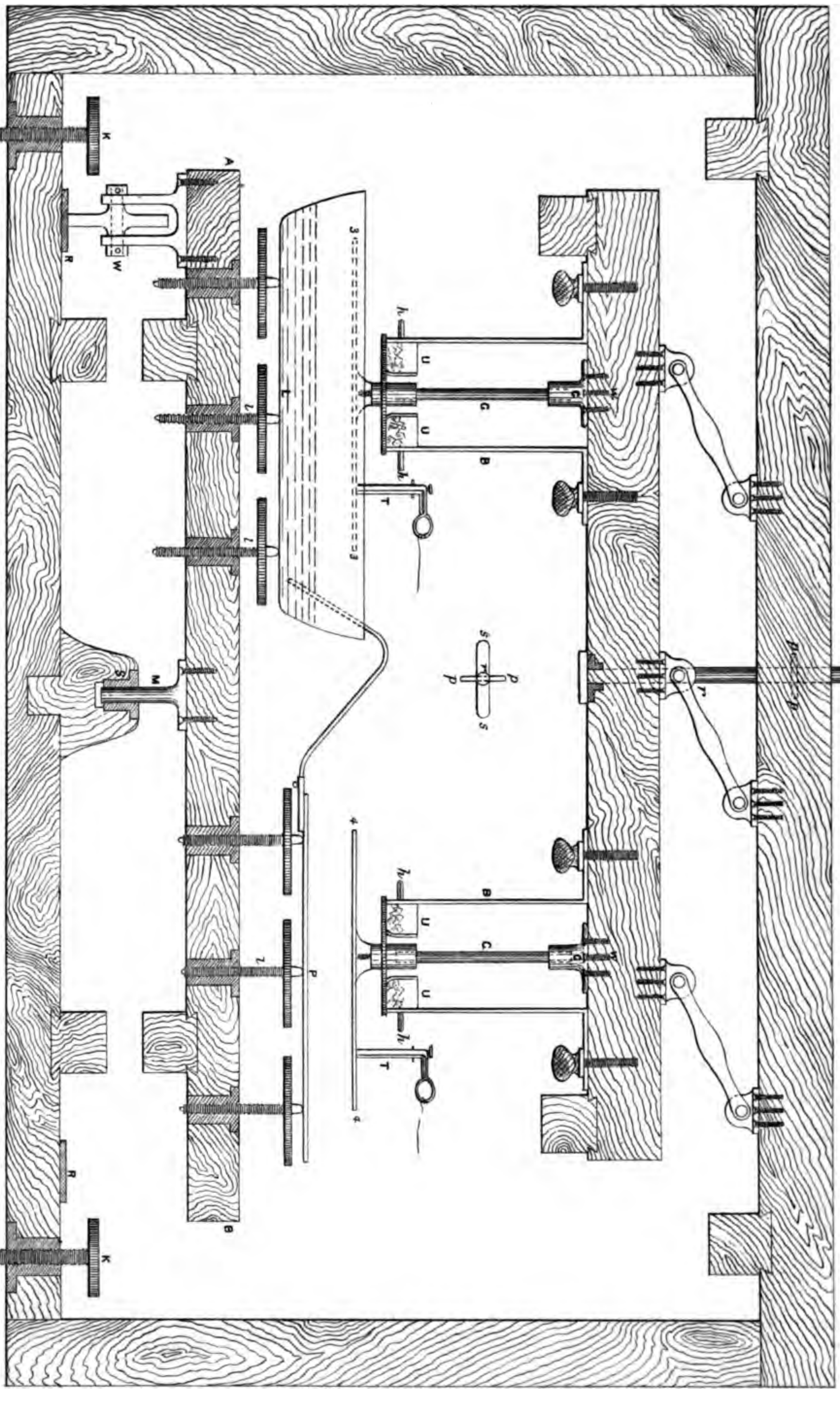


Fig. 7.

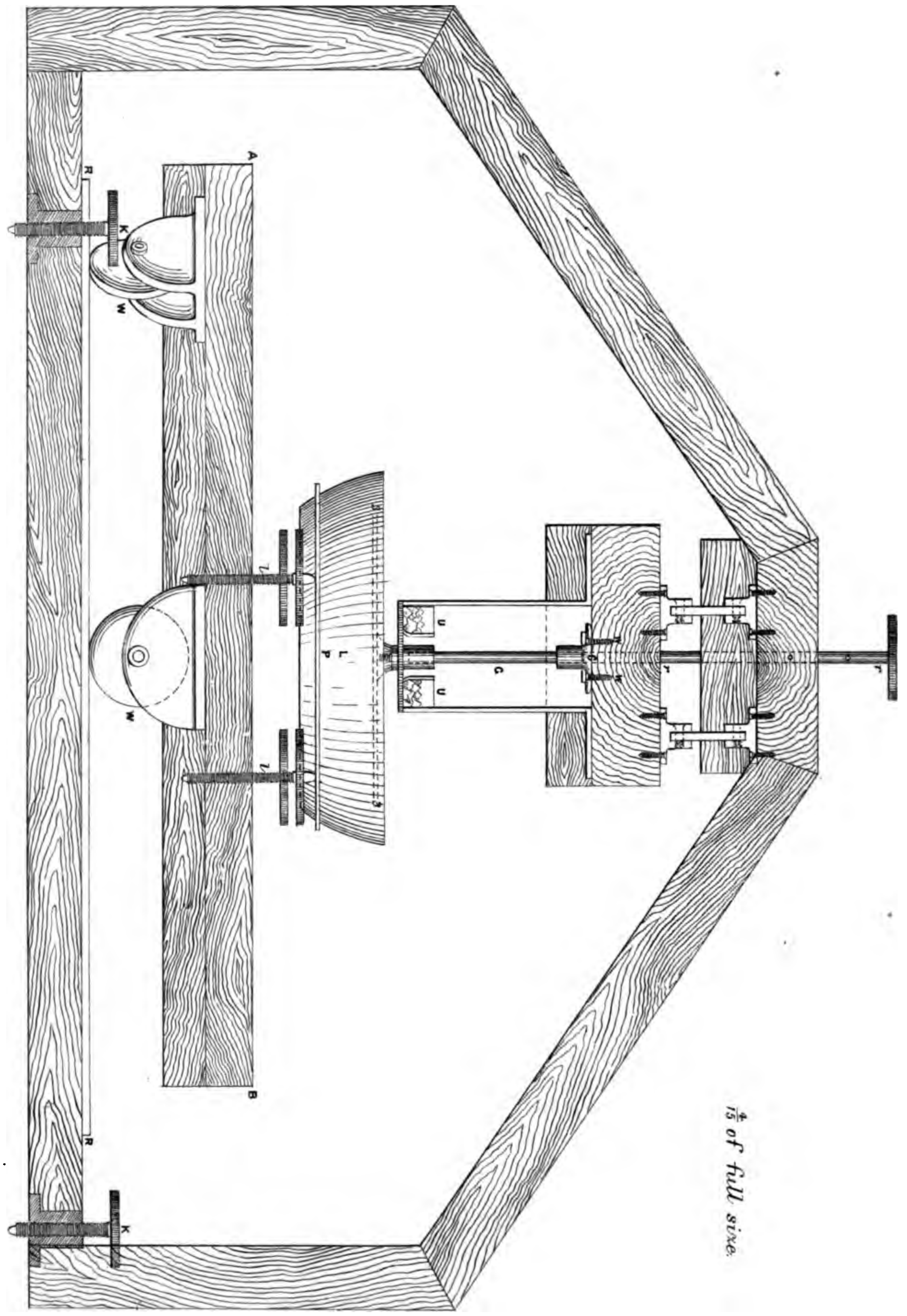


West Newman & Co lith.



$\frac{4}{13}$ *of full size.*

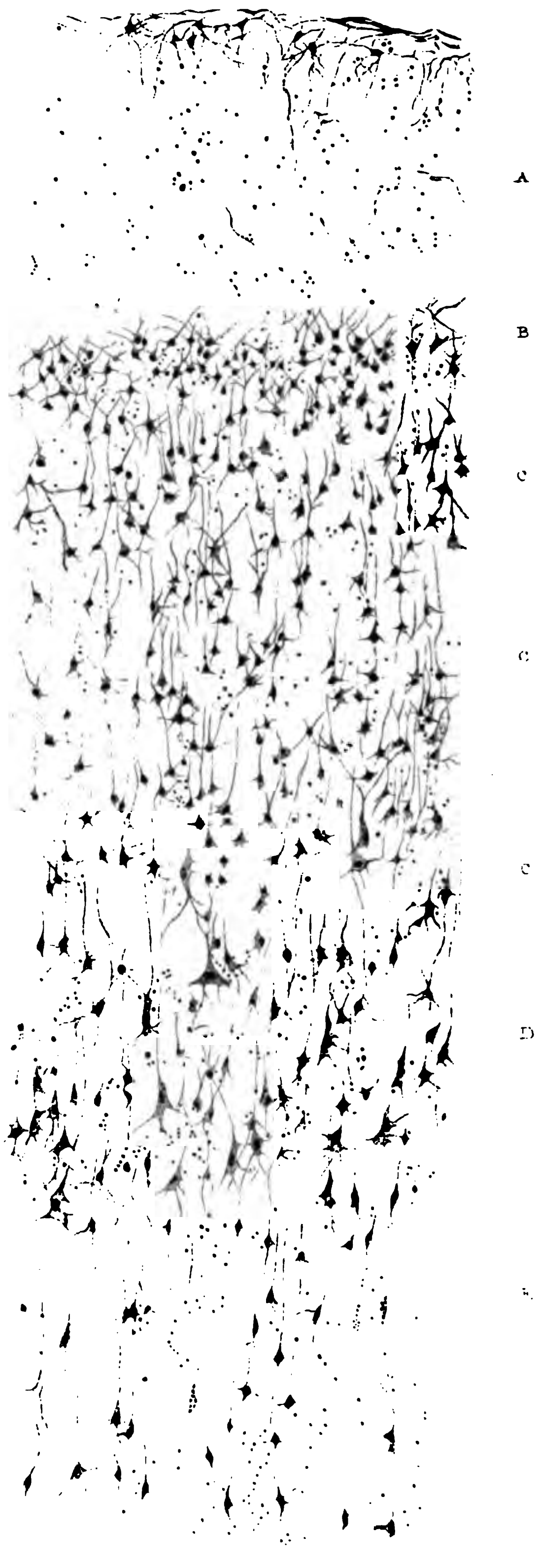
Harrison & Sons Lith. St Martins Lane W.C.



4/15 of full size.

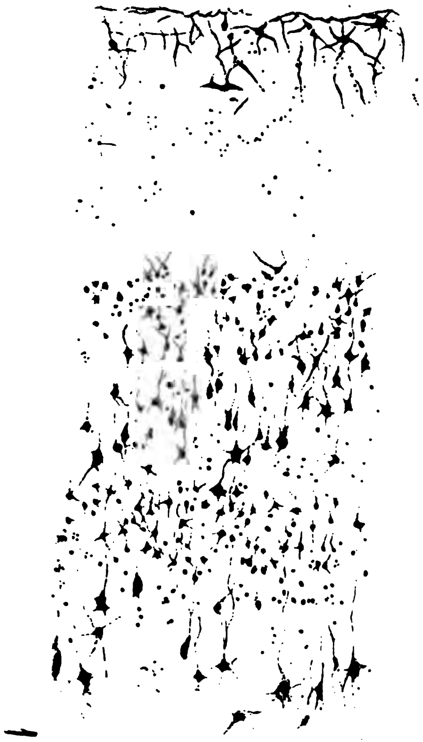
Harrison & Sons Lith. St Martins Lane W.C.

Section through Cortex Cerebri of Sheep.
From the Limbic Lobe.
x 78

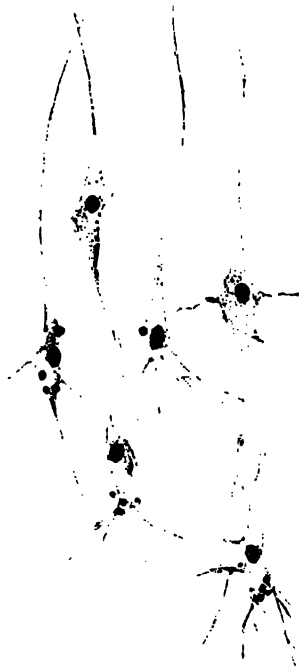


Section through Cortex Cerebri of Pig.
From the Limbic Lobe.
x 89

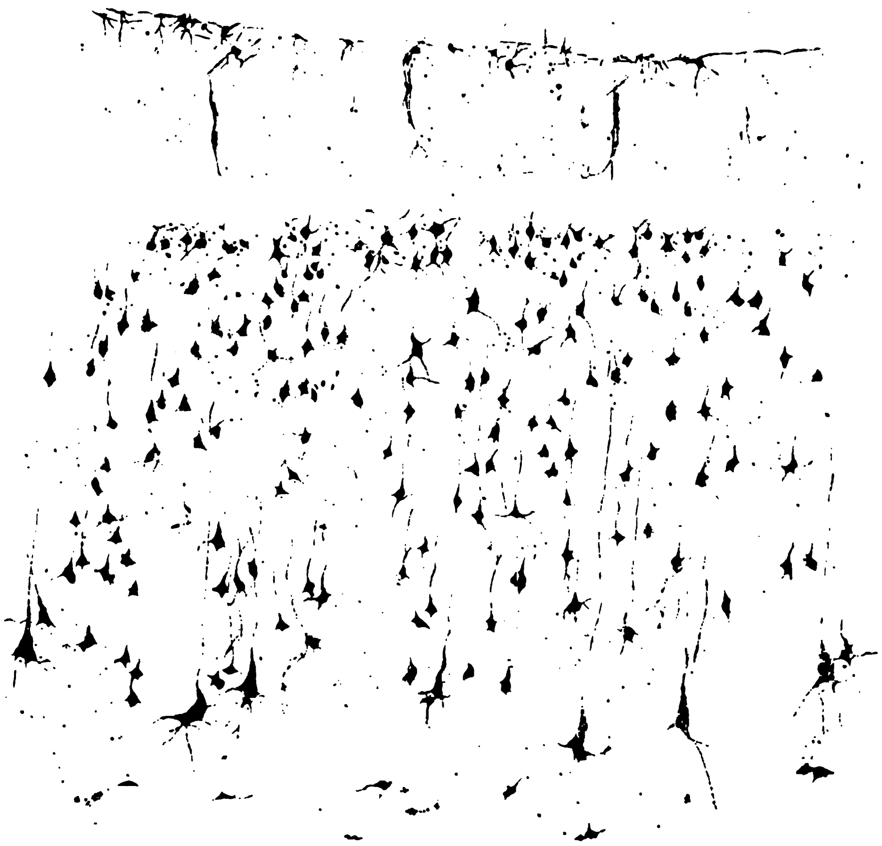
West Newman & Co. lith.



From Six-laminated Cortex of Sheep.
Section carried down to fifth layer.
x 55

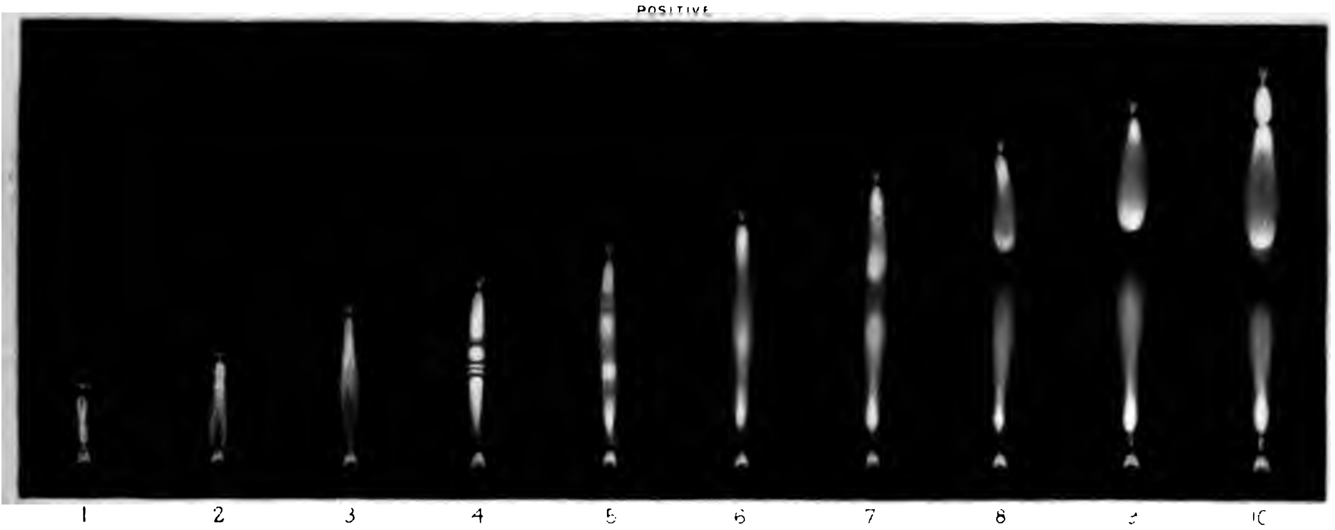


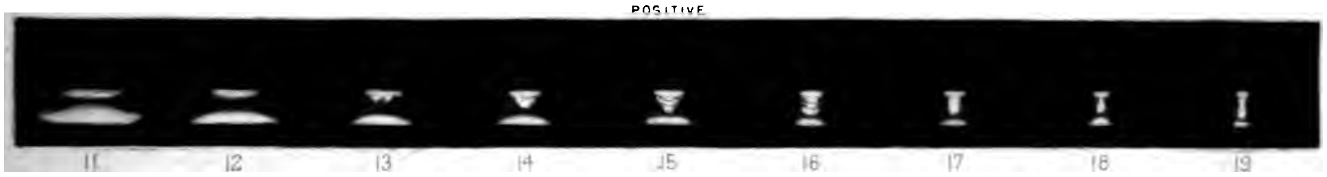
Ganglionic Cells as grouped in the
Limbic Lobe of the Pig.
x 270

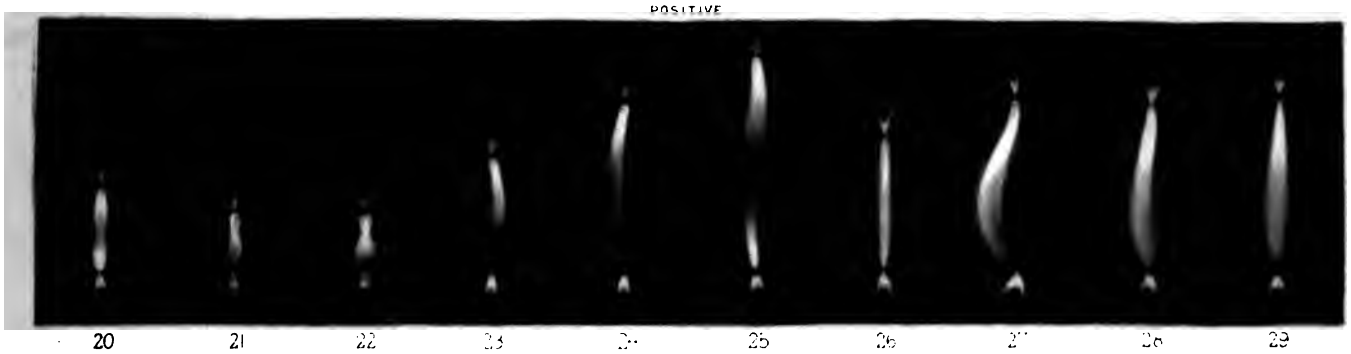


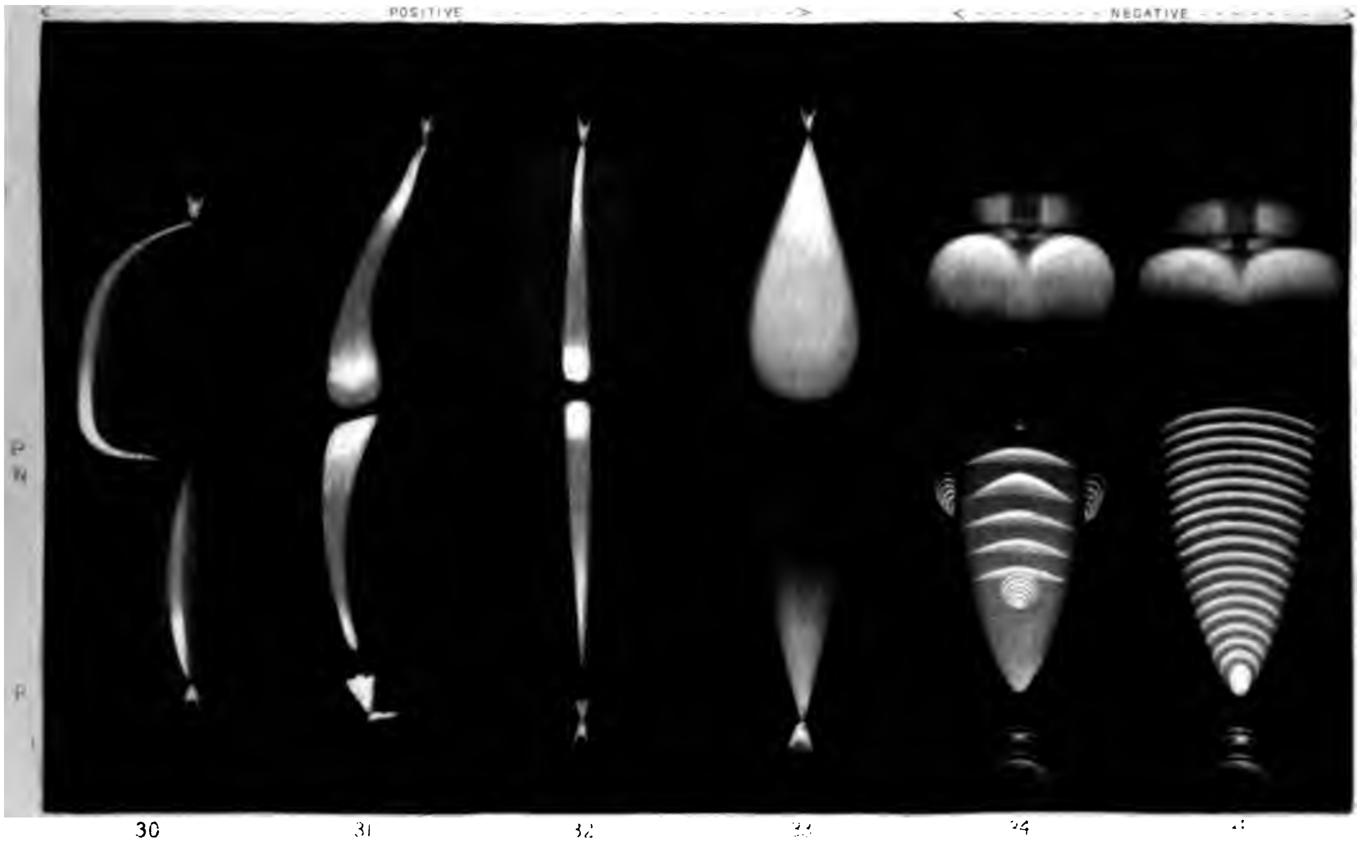
Section through the Cortex Cerebri of Pig.
From a Sulcus in the Limbic Lobe.
x 98

West. Newman.



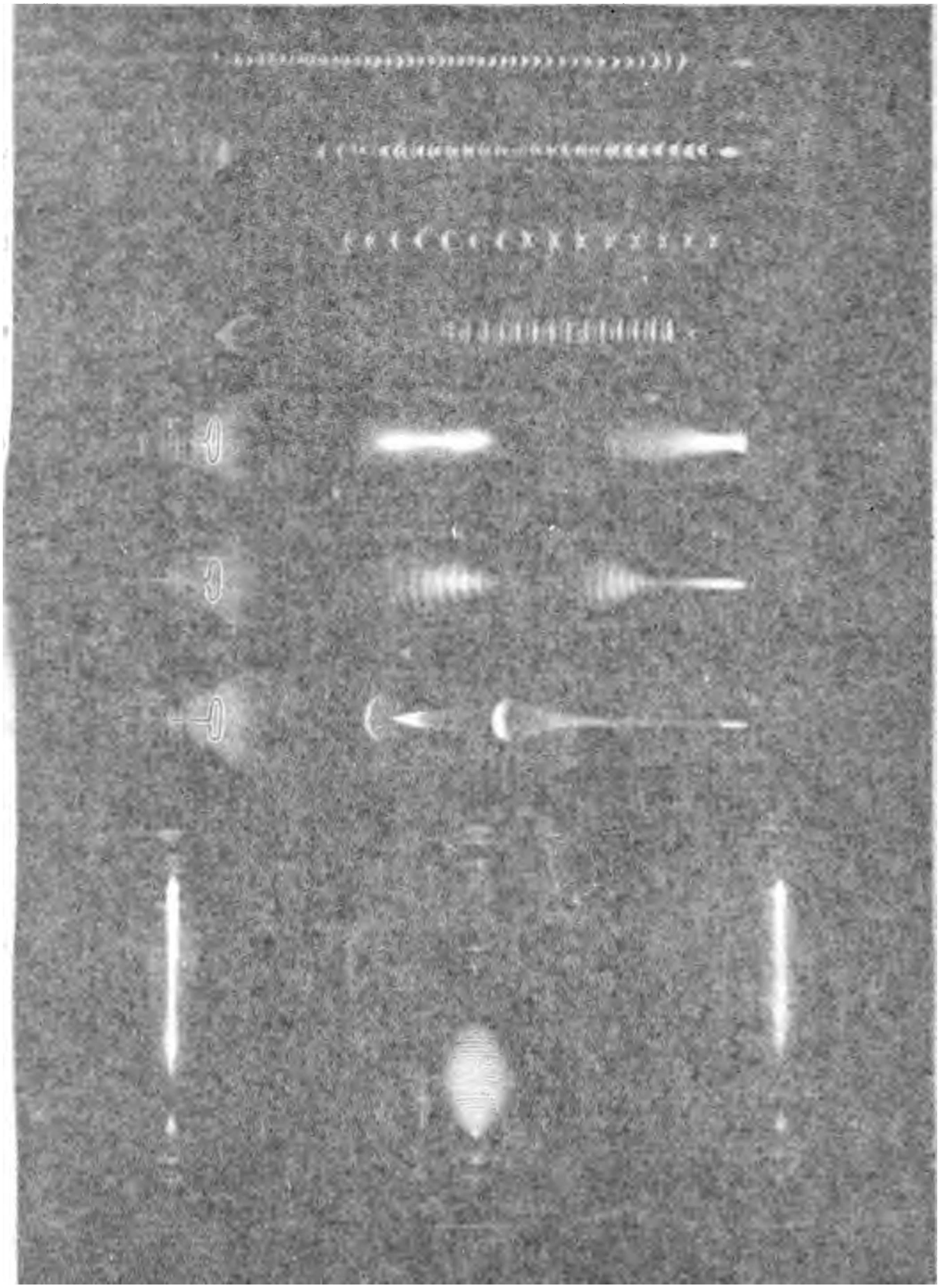


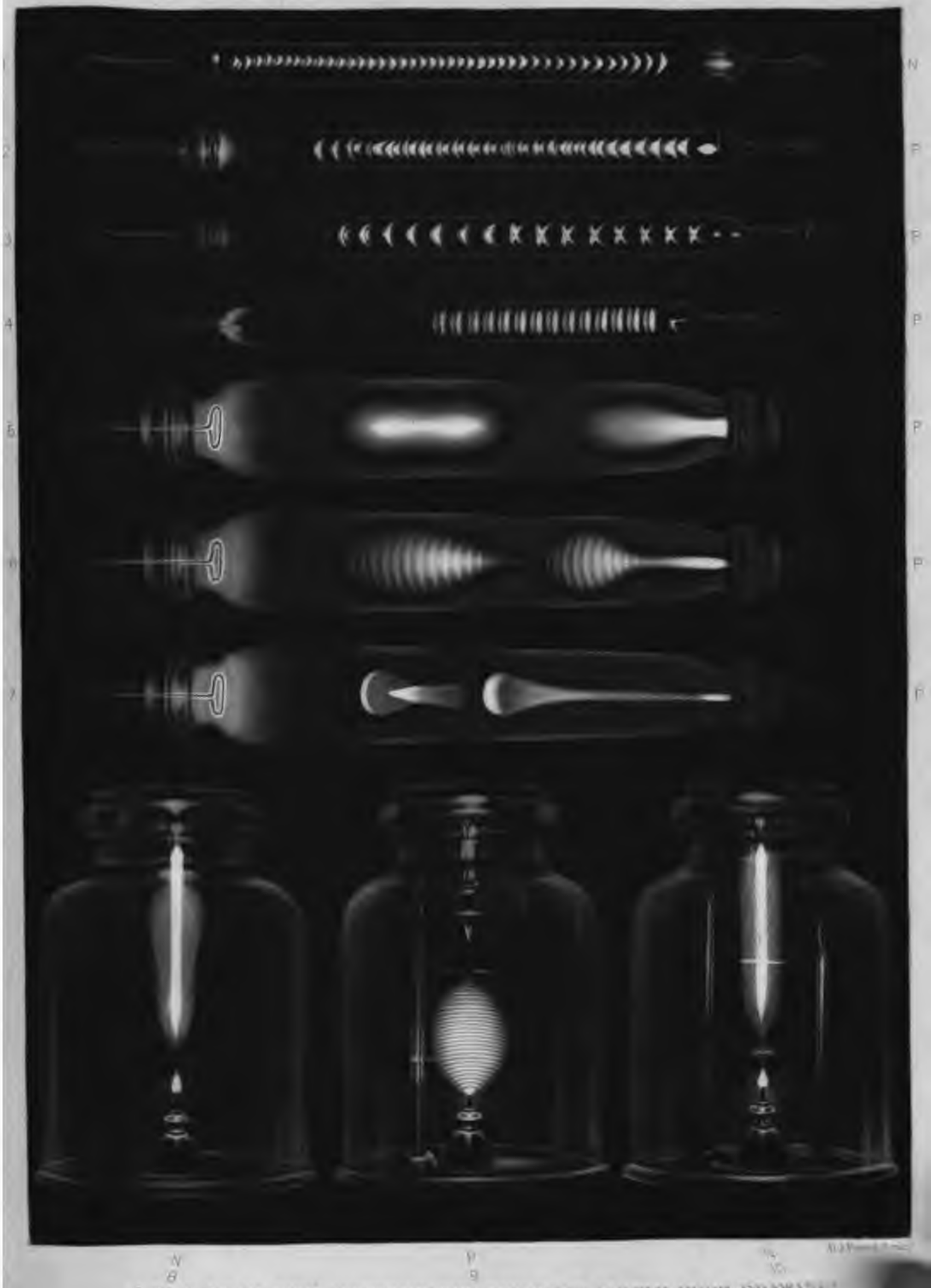




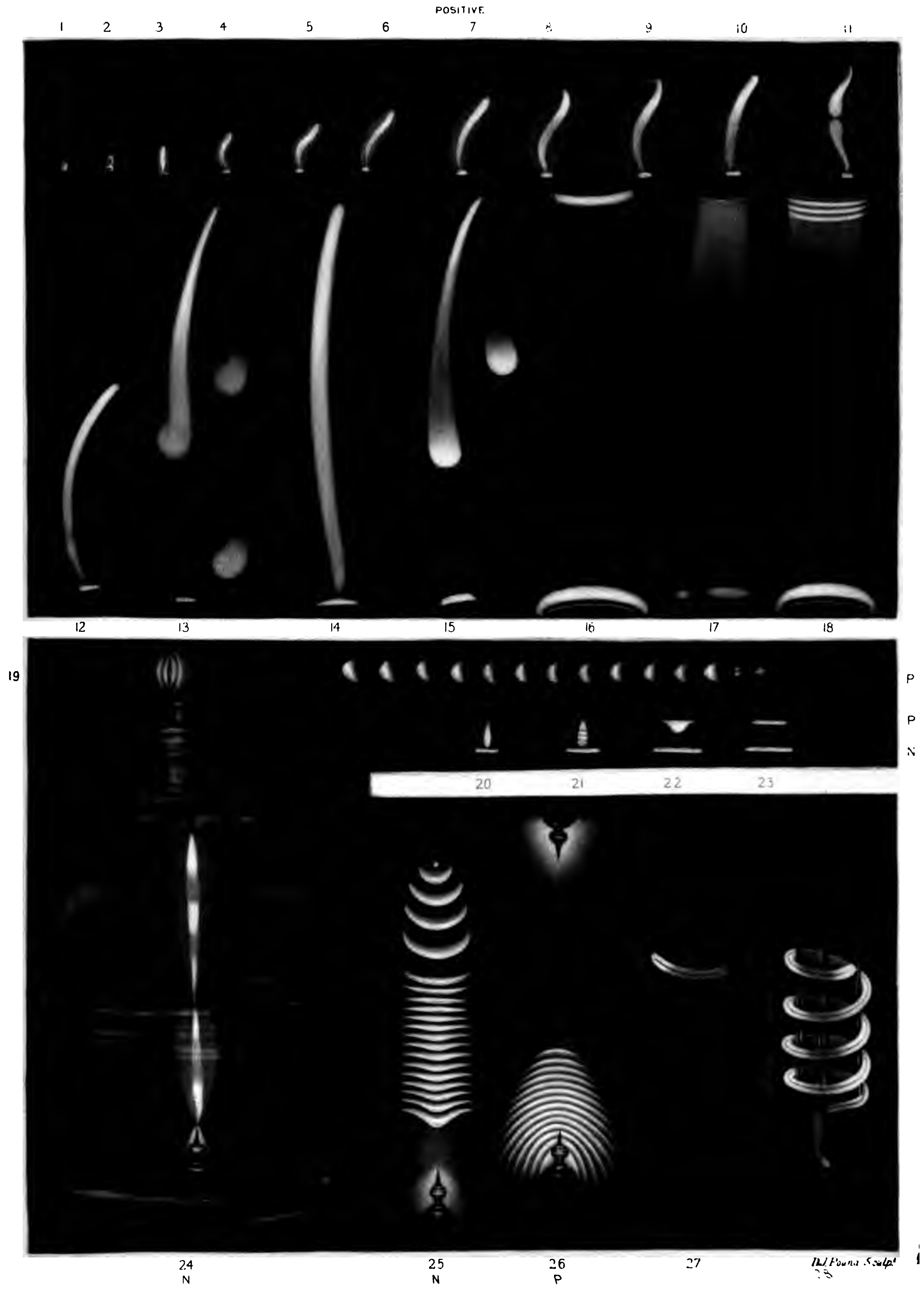
D.J. Pound Sculpt.

1 TO 29 FROM PHOTOGRAPHS. 30 TO 35 FROM DRAWINGS.



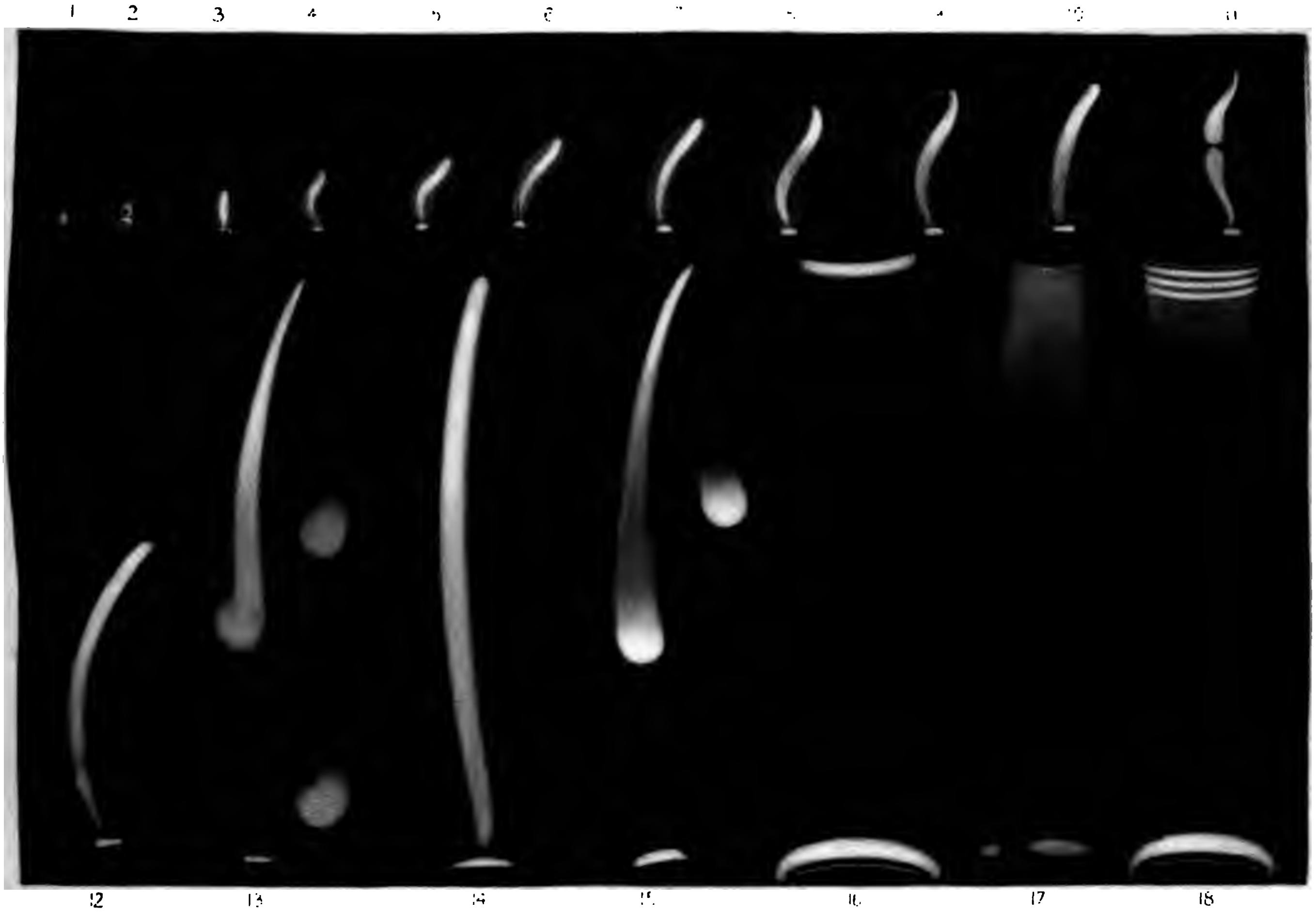


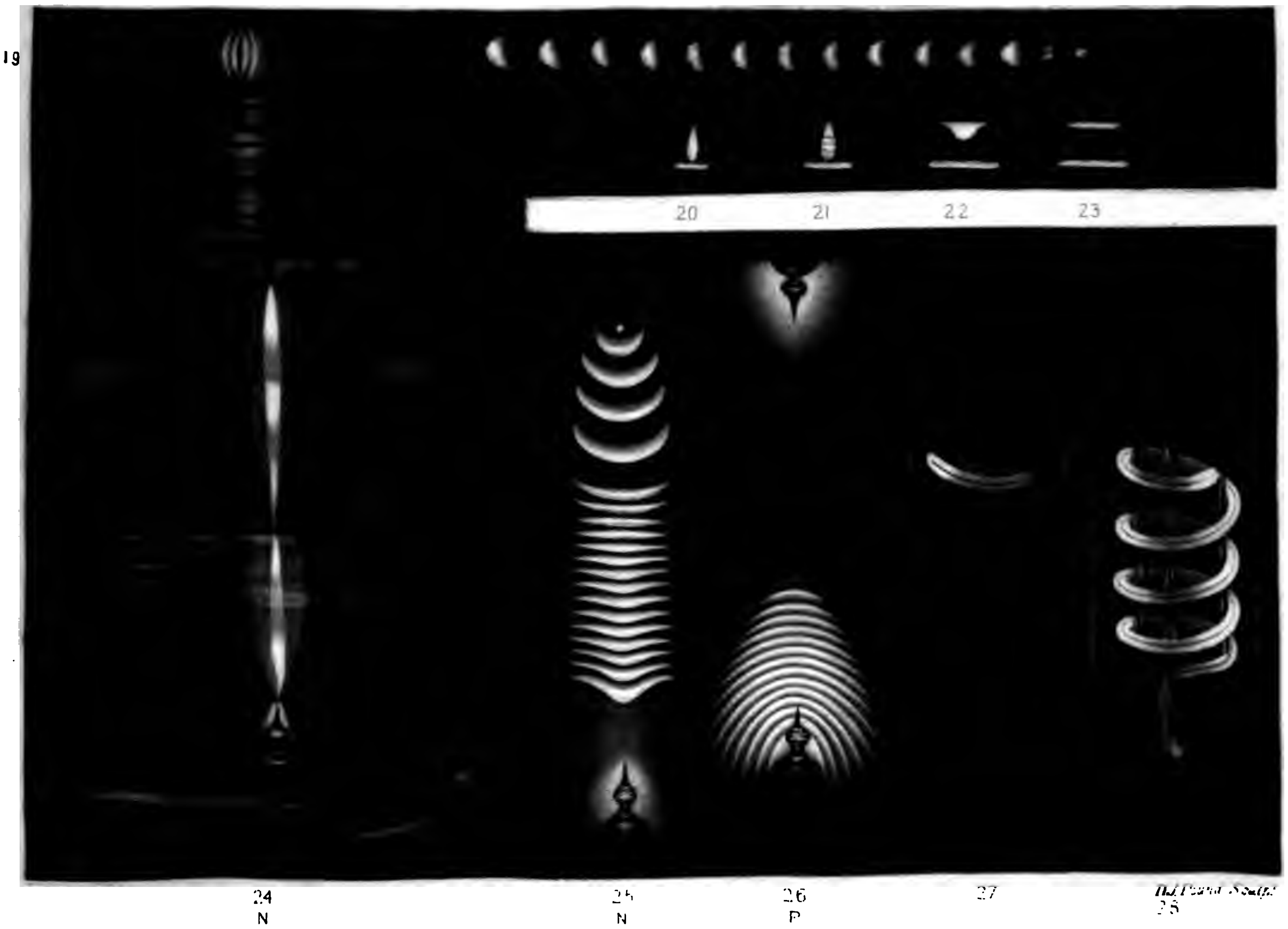
1 TO 5, 8 & 10 FROM PHOTOGRAPHS THE REST PARTLY FROM DRAWINGS



19, 27, 28 FROM PHOTOGRAPHS, 1 TO 18, 24 TO 26 FROM DRAWINGS.

De La Rue & Müller. *Phil. Trans. 1880. Plate 10.*





H.J.Pearl Sculp.

19, 22, 23 FROM PHOTOGRAPHS, 1 TO 18, 24 TO 28 FROM DRAWINGS

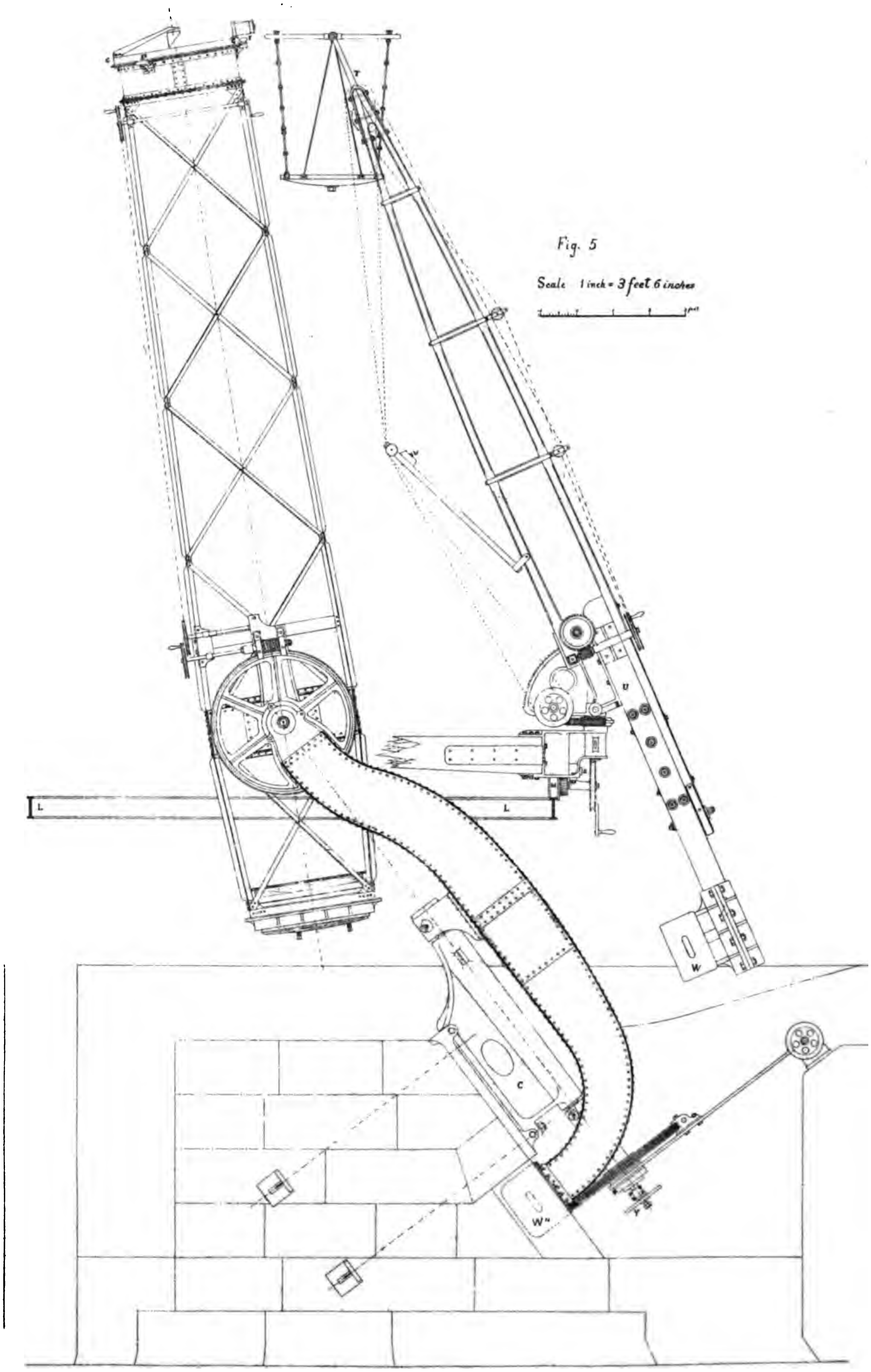


Fig. 5

Scale 1 inch = 3 feet 6 inches

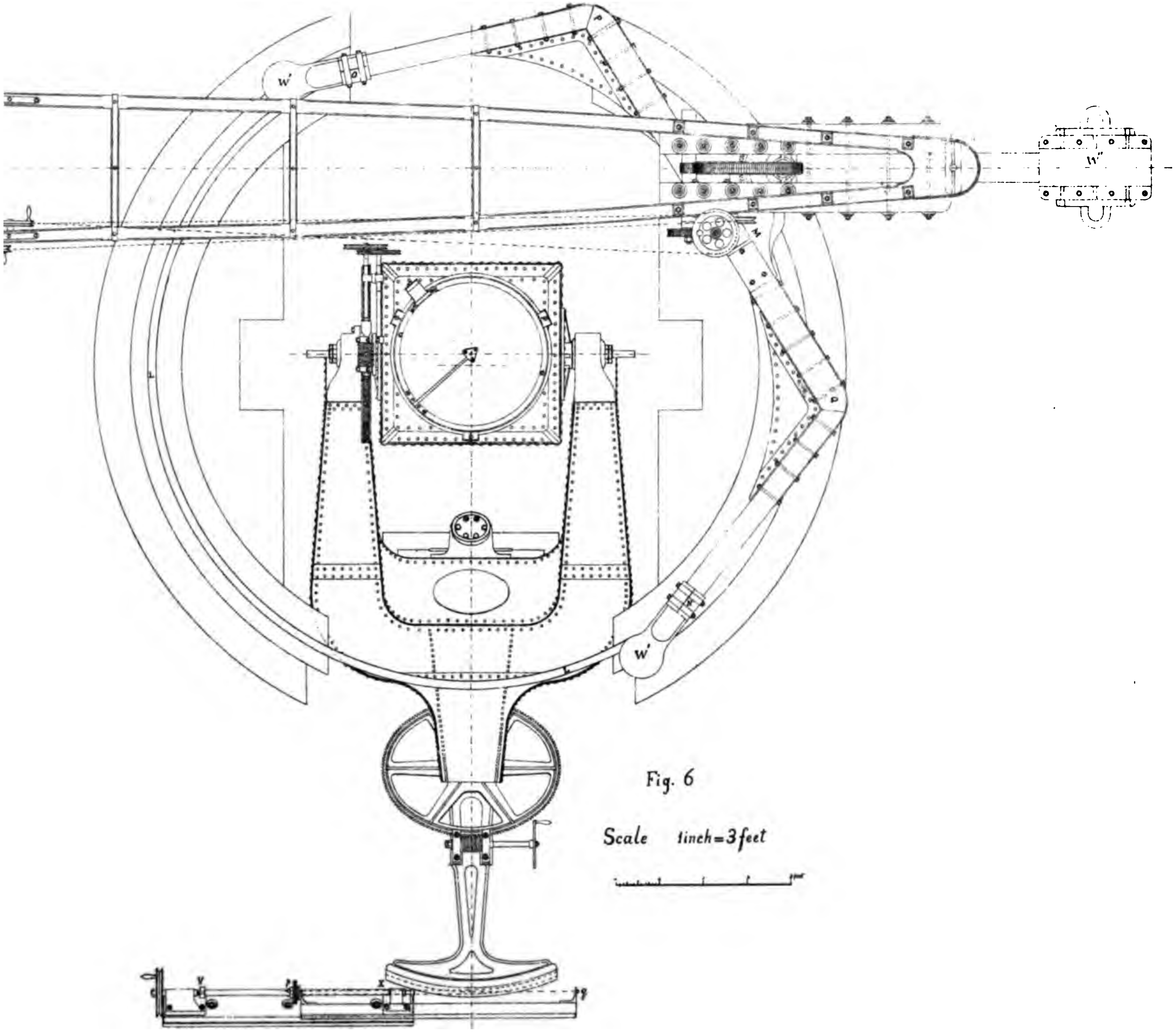


Fig. 6

Scale 1 inch = 3 feet

RA
TA
IVE
RD
SIT
ES
BRA
STA
NIV
ity
alif
be

Stanford University
Stanford, Cali

Return this book on or be